Two 5.0-cm-diameter Metal Disks Separated By A0.61-mm-thick Piece Of Pyrex Glass Are Charged To A Potential

Introduction

Two 5.0-cm-diameter metal disks separated by a 0.61-mm-thick piece of Pyrex glass are charged to a potential—a scenario that encapsulates fundamental principles of electrostatics, dielectric materials, and capacitor behavior. Understanding this setup involves exploring the physics of conductors, dielectrics, electric fields, and potential energy storage. This article provides an in-depth analysis of the system, including the physical configuration, the electrical properties involved, the calculation of electric fields and potentials, and the implications for energy storage and material behavior.

Physical Configuration and Basic Assumptions

Geometry and Dimensions

- Metal disks: two identical circular disks, each with a diameter of 5.0 cm.
- Separation: the disks are separated by a 0.61-mm-thick Pyrex glass sheet.
- Shape: assumed to be parallel, flat disks aligned along a common axis.
- Material of the disks: conductive, capable of maintaining a uniform potential.
- Dielectric material: Pyrex glass, characterized by its dielectric constant.

Material Properties

- Metal Disks:
  - Conductivity: ideal conductors with negligible resistance.
  - Charge distribution: assumed to be uniform over the surface.
  - Potential: each disk can be charged to a specified potential difference.

- Pyrex Glass:
  - Dielectric constant (relative permittivity): approximately 4.7 to 5.0 at room temperature.
  - Thickness: 0.61 mm (0.00061 m).
  - Behavior: acts as an insulator, affecting the electric field distribution and capacitance.

Fundamental Concepts in Electrostatics

Capacitance of Parallel Plate Capacitors

The basic model for this setup is a parallel plate capacitor, which consists of two conductive plates separated by a dielectric medium. The capacitance C is a measure of a capacitor's ability to store charge and is given by:

$$
C = \frac{\varepsilon_0 \varepsilon_r A}{d}
$$

where:

- ε_0: vacuum permittivity ($8.854 \times 10^{-12} \text{ F/m}$)
- ε_r: relative permittivity (dielectric constant of Pyrex)
- A: area of the plates
- d: separation between plates

Electric Field and Potential Difference

The electric field E between the plates of a capacitor is related to the potential difference V and the separation d:

$$
E = \frac{V}{d}
$$

In the presence of a dielectric, the electric field is modified by the dielectric constant, leading to a reduction in the field compared to a vacuum.

Charge and Potential Relationship

The charge Q stored on the plates relates to the capacitance and potential difference via:

$$
Q = C V
$$

Calculating the Capacitance of the System

Area of the Metal Disks

The area A of each disk with diameter $D = 5.0\, \text{cm}$ is:

$$
A = \pi \left(\frac{D}{2}\right)^2 = \pi \times (2.5\, \text{cm})^2 = \pi \times (0.025\, \text{m})^2 \approx 1.9635 \times 10^{-3} \, \text{m}^2
$$

Dielectric Constant of Pyrex

Using an approximate relative permittivity ε_r of 4.9 for Pyrex glass:

$$
\varepsilon_r \approx 4.9
$$

Capacitance Calculation

Applying the parallel plate capacitor formula:

$$
C = \frac{\varepsilon_0 \varepsilon_r A}{d}
$$

$$
C = \frac{(8.854 \times 10^{-12} \text{ F/m}) \times 4.9 \times 1.9635 \times 10^{-3} \text{ m}^2}{0.61 \times 10^{-3} \text{ m}}
$$

Calculating numerator:

$$
8.854 \times 10^{-12} \times 4.9 \times 1.9635 \times 10^{-3} \approx 8.854 \times 4.9 \times 1.9635 \times 10^{-12-3}
$$

$$
= (8.854 \times 4.9 \times 1.9635) \times 10^{-15}
$$

$$
\approx (8.854 \times 4.9) \times 1.9635 \times 10^{-15}
$$

\[

$$
= (43.34) \times 1.9635 \times 10^{-15} \approx 85.14 \times 10^{-15} = 8.514 \times 10^{-14}
$$

Dividing by $0.61 \times 10^{-3} = 6.1 \times 10^{-4}$:

$$
C \approx \frac{8.514 \times 10^{-14}}{6.1 \times 10^{-4}} \approx 1.397 \times 10^{-10} \text{ F}
$$

Thus, the capacitance:

$$
C \approx 0.14\, \text{nF}
$$

Charging the System to a Potential

Potential Difference Applied

Suppose the disks are charged to a specific potential V. The magnitude of the charge stored is then:

$$
Q = C V
$$

For example, if the potential difference is 100 V:

$$
Q = 1.397 \times 10^{-10} \times 100 = 1.397 \times 10^{-8} \text{ C}
$$

Electric Field Distribution

Within the dielectric, the electric field is uniform and given by:

$$
E = \frac{V}{d}
$$

For a 100 V potential:

$$
E = \frac{100\, \text{V}}{0.61 \times 10^{-3}\, \text{m}} \approx 1.639 \times 10^{5} \text{ V/m}
$$

This high electric field indicates a strong potential for dielectric stress.

Implications of Dielectric Properties

Dielectric Strength of Pyrex

- Pyrex has a dielectric strength of approximately 10–20 MV/m.

- The calculated electric field (~164,000 V/m) is below the dielectric strength, indicating the system can withstand such voltages without dielectric breakdown under ideal conditions.

Effect on Capacitance and Stored Energy

The presence of the dielectric increases the capacitance compared to vacuum. The stored energy U in the capacitor is given by:

$$
U = \frac{1}{2} C V^2
$$

For the example with $V=100\, \text{V}$:

$$
U = \frac{1}{2} \times 1.397 \times 10^{-10} \times (100)^2 \approx 6.985 \times 10^{-9} \text{ J}
$$

Charge Distribution and Potential on the Conductors

Surface Charge Density

- The surface charge density σ (charge per unit area) is:

$$
\sigma = \frac{Q}{A} \approx \frac{1.397 \times 10^{-8}}{1.9635 \times 10^{-3}} \approx 7.11 \times 10^{-6} \text{ C/m}^2
$$

- This uniform charge distribution assumes no edge effects, valid for large, flat disks.

Potential Uniformity

- Due to the high conductivity of the metal disks, the potential remains uniform across each disk's surface.
- The potential difference is maintained by an external power supply or charge separation process.

Effects of Material and System Limitations

Dielectric Limitations and Breakdown