Use The Origin As The Center Of Dilation And The Given Scale Factor To Find The Coordinates Of The Vertices

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Understanding how to manipulate geometric figures through dilation is a fundamental skill in coordinate geometry. When the center of dilation is the origin (0,0), and the scale factor is provided, it becomes straightforward to determine the new positions of a figure's vertices after dilation. This process involves applying a simple mathematical rule to each vertex: multiplying its coordinate values by the scale factor. This article explores the methodology in detail, providing step-by-step instructions, examples, and insights to help you master this essential concept.

Fundamentals of Dilation in Coordinate Geometry

What Is Dilation?

Dilation is a transformation that enlarges or reduces a figure by a scale factor relative to a fixed point called the center of dilation. The figure's size changes, but its shape remains similar. When the center of dilation is the origin, the process simplifies because the transformation applies directly to the coordinates of each vertex.

Center of Dilation: The Origin

Choosing the origin as the center of dilation means that every point is scaled relative to (0,0). This choice simplifies calculations because the transformation involves just multiplying the coordinates by the scale factor, without needing to account for shifts or translations.

Scale Factor

The scale factor, denoted usually as 'k', determines the degree of dilation:

- **k > 1**: The figure enlarges (dilation by a factor greater than 1).
- **0 < k < 1**: The figure reduces in size (dilation by a fraction).
- **k < 0**: The figure is reflected across the origin and scaled, which may flip it across axes depending on the value.

Mathematical Procedure for Finding New Coordinates

Step 1: Identify Original Coordinates

Begin with the set of vertices of the original figure, each expressed as an ordered pair (x, y). For example:

- Vertex A: (x_A, y_A)
- Vertex B: (x_B, y_B)
- Vertex C: (x_C, y_C)

Step 2: Know Your Scale Factor

Obtain the given scale factor 'k' from the problem. Ensure you understand whether it enlarges or reduces the figure and whether it introduces reflection.

Step 3: Apply the Dilation Formula

Since the origin is the center, each vertex's new coordinates are calculated as:

$$(x_{new}, y_{new}) = (k \quad x, k \quad y)$$

Apply this formula to each vertex individually:

Step 4: Compute The New Coordinates

- For vertex A: (x_A, y_A) ⍰ $(k\,x_A, k\,y_A)$
- For vertex B: (x_B, y_B) ⍰ $(k\,x_B, k\,y_B)$
- For vertex C: (x_C, y_C) ⍰ $(k\,x_C, k\,y_C)$

Step 5: Interpret and Plot the Result

Once the new coordinates are calculated, plot the vertices on the coordinate plane. Connect the points to visualize the dilated figure, which will be similar in shape to the original but scaled according to the factor 'k'.

Examples Illustrating the Method

Example 1: Simple Dilation with a Positive Scale Factor

Suppose a triangle has vertices:

- A (2, 3)

- B (4, 1)
- C (3, 5)

The scale factor is 3.

Solution:

1. Calculate new vertex A:
    - $x_A = 2$, $y_A = 3$
    - $x_{A,new} = 3\ 2 = 6$
    - $y_{A,new} = 3\ 3 = 9$
    - New A: (6, 9)

2. Calculate new vertex B:
    - $x_B = 4$, $y_B = 1$
    - $x_{B,new} = 3\ 4 = 12$
    - $y_{B,new} = 3\ 1 = 3$
    - New B: (12, 3)

3. Calculate new vertex C:
    - $x_C = 3$, $y_C = 5$
    - $x_{C,new} = 3\ 3 = 9$
    - $y_{C,new} = 3\ 5 = 15$
    - New C: (9, 15)

The dilated triangle has vertices at (6, 9), (12, 3), and (9, 15).

Example 2: Dilation With a Negative Scale Factor

Given a quadrilateral with vertices:

- P (1, 2)
- Q (3, 4)

- R (5, 2)
- S (3, 0)

The scale factor is -2.

Solution:

1. Vertex P:
    - (1, 2) ⍰ (-2 1, -2 2) = (-2, -4)

2. Vertex Q:
    - (3, 4) ⍰ (-2 3, -2 4) = (-6, -8)

3. Vertex R:
    - (5, 2) ⍰ (-2 5, -2 2) = (-10, -4)

4. Vertex S:
    - (3, 0) ⍰ (-2 3, -2 0) = (-6, 0)

The resulting figure is reflected across the origin and scaled by a factor of 2, resulting in vertices at (-2, -4), (-6, -8), (-10, -4), and (-6, 0).

Additional Considerations and Tips

Handling Coordinates with Negative Values

The dilation formula applies uniformly regardless of whether coordinate values are positive or negative. Multiplying negative values by a positive scale factor results in scaled negatives, reflecting the figure across the axes as needed.

Visualizing the Transformation

Plotting both the original and the dilated figures on a graph can help in visualizing the effect of the transformation, understanding how the figure expands or contracts relative to the origin.

Using Software Tools

Graphing calculators, coordinate geometry software, or online graphing tools can automate the process, allowing for quick visualization and verification of the dilation process.

Summary

To summarize, when the center of dilation is the origin, finding the coordinates of the vertices after dilation is a straightforward process involving multiplying each coordinate by the given scale factor. This method ensures accuracy and simplifies the process, especially when dealing with complex figures or multiple vertices. Remember to consider the sign of the scale factor, as negative values introduce reflection across the origin, affecting the orientation of the figure.

Conclusion

1, the figure enlarges.
- If $0 < |k| < 1$, the figure reduces.
- If $k = 1$, the figure remains unchanged.
- If $k = -1$, the figure is reflected and scaled.

In coordinate geometry, dilation can be represented algebraically, which simplifies the process of finding the new vertices once the center and scale factor are known.

The Special Case: Center at the Origin

When the center of dilation is at the origin (0,0), the transformation rules become especially straightforward. This is because the origin acts as a natural reference point, simplifying calculations. The transformation formula for a point (x, y) under dilation centered at the origin is:

$$
(x', y') = (k \times x, \; k \times y)
$$

where:

- (x, y) are the original coordinates,
- (x', y') are the new coordinates after dilation,
- k is the scale factor.

This simplicity makes the origin a popular choice in many problems and real-world applications.

Applying Dilation with the Origin as Center and a Given Scale Factor

The process of finding the new coordinates after dilation involves clear, step-by-step procedures.

Step 1: Identify the Original Coordinates of Vertices

Begin by listing the original coordinates of each vertex of the figure. For example, a triangle might have vertices at:

$$
A(x_1, y_1), \quad B(x_2, y_2), \quad C(x_3, y_3)
$$

Ensure these are correctly identified, as any mistake here will propagate through the entire transformation.

Step 2: Determine the Scale Factor (k)

The scale factor should be provided in the problem. If not, it needs to be deduced from the context, such as ratios of lengths or other given information. Remember:

- $k > 0$ results in an enlargement or reduction without reflection.
- $k < 0$ results in an enlargement or reduction combined with a reflection through the origin.

Step 3: Apply the Dilation Formula to Each Vertex

For each vertex (x, y), multiply both coordinates by the scale factor:

$$
x' = k \times x
$$
$$
y' = k \times y
$$

This step is straightforward but crucial; it ensures proportional scaling relative to the origin.

Step 4: List the New Coordinates

The transformed vertices are:

$$
A' (k \times x_1, \; k \times y_1)
$$
$$
B' (k \times x_2, \; k \times y_2)
$$
$$
C' (k \times x_3, \; k \times y_3)
$$

and so on for polygons with more vertices.

Examples and Practice Problems

To solidify your understanding, let's explore a practical example and then outline some practice problems to test your skills.

Example: Dilation of a Triangle at the Origin

Suppose you have a triangle with vertices:

$$
A(2, 3), \quad B(4, 1), \quad C(1, -2)
$$

and you are asked to dilate this triangle about the origin with a scale factor of 3.

Solution:

1. Original coordinates:

$$
A(2, 3), \quad B(4, 1), \quad C(1, -2)
$$

2. Scale factor:

$$
k = 3
$$

3. Apply the dilation formula:

- $A' = (3 \times 2, 3 \times 3) = (6, 9)$
- $B' = (3 \times 4, 3 \times 1) = (12, 3)$
- $C' = (3 \times 1, 3 \times -2) = (3, -6)$

Result:

Original triangle vertices are scaled to:

$$
A'(6, 9), \quad B'(12, 3), \quad C'(3, -6)
$$

This new triangle is proportionally larger, maintaining the same shape, but scaled uniformly about the origin.

Special Considerations and Common Mistakes

While the process appears straightforward, several nuances can impact the accuracy of your results.

Handling Negative Scale Factors

A negative scale factor reflects the figure across the origin, producing a mirror image. For example, with $k = -2$:

- The point (x, y) becomes $(-2x, -2y)$.
- This results in a scaled and reflected figure.

Tip: Always pay attention to the sign of the scale factor, as it affects the orientation.

Ensuring Correct Coordinates

- Double-check the original coordinates.
- Confirm the scale factor before applying.
- Apply the multiplication carefully to avoid arithmetic errors.

Applying to Non-Polygon Figures

While the focus here is on vertices of polygons, the same principles apply to other figures, such as circles or line segments, where scaling involves multiplying the relevant lengths or radius.

Practical Tips for Using the Origin as the Center of Dilation

- Visualize the Transformation: Plot the original figure on a coordinate plane to understand how it will change.
- Use Software Tools: Geometry software like GeoGebra can help visualize dilation and verify calculations.
- Understand the Relationship Between Scale Factor and Coordinates: Recognize that multiplying each coordinate by the scale factor ensures proportionality.
- Practice with Diverse Figures: Apply the method to various polygons and shapes to develop fluency.

Conclusion

Using the origin as the center of dilation combined with a given scale factor provides a powerful, elegant method for transforming figures in coordinate geometry. By understanding the fundamental formula $(x', y') = (k \times x, k \times y)$, practicing with real examples, and paying close attention to the sign and magnitude of the scale factor, you can accurately determine the new coordinates of any figure's vertices.

This technique is not only mathematically robust but also highly applicable in fields like computer graphics, engineering design, and mathematical modeling. Mastering it enhances your ability to manipulate geometric figures with precision and confidence, making it an essential skill in your mathematical toolkit.

Remember: The key to success lies in careful calculation, visualization, and consistent practice. With these principles, you will confidently handle dilation problems centered at the origin with any given scale factor.

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