

What Is The Average Of The Points A, B, And C With Weights 1, 4 And 4 Respectively? A (8, -6) B(9, -5) C(-6, 7)

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Understanding how to compute the weighted average of points in a coordinate plane is a fundamental skill in mathematics, especially in fields like geometry, physics, computer graphics, and data analysis. In this article, we will explore the process of calculating the weighted average of three points with given weights, specifically focusing on points A, B, and C with coordinates and weights as provided. We will break down the concept step-by-step, include relevant formulas, and provide practical examples to ensure clarity.

Introduction to Weighted Averages in Coordinates

Before diving into calculations, it is essential to understand what a weighted average represents, especially in the context of points on a plane.

What Is a Weighted Average?

A weighted average is a type of mean where each value has a specific weight, reflecting its importance or contribution to the overall average. Unlike a simple average, where all values are equally considered, weighted averages assign different levels of significance to each data point.

Formula for Weighted Average:

For a set of values $(x_1, x_2, \dots, x_n)$ with corresponding weights $(w_1, w_2, \dots, w_n)$, the weighted average $(\bar{x})$ is:

$$\bar{x} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n}$$

Similarly, this concept applies to the y-coordinates of points in a plane.

Applying Weighted Averages to Coordinates

When working with points in a two-dimensional space, the weighted average can be computed separately for the x-coordinates and y-coordinates:

$$\bar{x} = \frac{w_A x_A + w_B x_B + w_C x_C}{w_A + w_B + w_C}$$

$$\bar{y} = \frac{w_A y_A + w_B y_B + w_C y_C}{w_A + w_B + w_C}$$

Where:

- (x_A, y_A) , (x_B, y_B) , (x_C, y_C) : Coordinates of points A, B, and C.
- w_A , w_B , w_C : Corresponding weights.

Given Data and Objective

Let's restate the problem with the provided data:

- Points:
- A: (8, -6), weight $w_A = 1$
- B: (9, -5), weight $w_B = 4$
- C: (-6, 7), weight $w_C = 4$

Our goal is to determine the weighted average point (x_{avg}, y_{avg}) considering these weights.

Step-by-Step Calculation of the Weighted Average

To compute the weighted average, we will perform the calculations separately for the x and y coordinates.

Step 1: Calculate the sum of weights

$$\begin{aligned} W &= w_A + w_B + w_C = 1 + 4 + 4 = 9 \end{aligned}$$

This total weight will be used as the denominator in the averaging formulas.

Step 2: Calculate weighted sum of x-coordinates

$$x_{\text{weighted}} = (w_A \times x_A) + (w_B \times x_B) + (w_C \times x_C)$$

Substituting the values:

$$x_{\text{weighted}} = (1 \times 8) + (4 \times 9) + (4 \times -6) = 8 + 36 - 24 = 20$$

Step 3: Calculate weighted sum of y-coordinates

$$y_{\text{weighted}} = (w_A \times y_A) + (w_B \times y_B) + (w_C \times y_C)$$

Substituting the values:

$$y_{\text{weighted}} = (1 \times -6) + (4 \times -5) + (4 \times 7) = -6 - 20 + 28 = 2$$

Step 4: Compute the weighted averages for x and y

$$x_{\text{avg}} = \frac{x_{\text{weighted}}}{W} = \frac{20}{9} \approx 2.22$$

$$y_{\text{avg}} = \frac{y_{\text{weighted}}}{W} = \frac{2}{9} \approx 0.22$$

Result:

$$\boxed{}$$

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\text{Weighted Average Point} \approx (2.22, 0.22)
}
```

This point can be interpreted as the "center of mass" or the weighted centroid of the three points considering the specified weights.

Understanding the Significance of the Result

The weighted average point provides a meaningful way to find a representative point considering different levels of importance (weights). In practical applications, this concept is used in:

- Physics: Calculating center of mass of a system.
- Geography: Finding the weighted centroid of locations with varying significance.
- Data Analysis: Combining data points with different weights to get an overall summary.
- Computer Graphics: Determining the focal point in rendering or modeling.

The resulting point (approximately (2.22, 0.22)) suggests a location that balances the influence of points A, B, and C according to their respective weights.

Additional Considerations and Applications

While the calculation above is straightforward, there are other aspects and applications worth exploring.

Weighted Centroids in Geometry

The concept of a centroid (or geometric center) can be extended to weighted points. When points have different weights, the centroid shifts toward the points with higher weights. This is particularly useful in:

- Design and Architecture: Balancing structural elements.
- Robotics: Calculating the center of mass for movement and stability.
- Urban Planning: Determining optimal locations considering population densities.

Impacts of Different Weight Choices

Changing the weights alters the position of the weighted average point. For example:

- Increasing w_A would shift the centroid closer to point A.
- Equal weights would revert to the simple average (arithmetic mean).
- Emphasizing one point over others allows for tailored calculations to fit specific purposes.

Limitations and Assumptions

- The method assumes linearity and proportional influence.
- Coordinates are in Euclidean space; in non-Euclidean or curved spaces, different methods may be needed.
- The weights must be positive; negative weights could complicate interpretations.

Practical Examples and Exercises

To reinforce understanding, consider the following exercises:

1. Change Weights: Recalculate the weighted average when $w_A = 2$, $w_B = 3$, and $w_C = 4$.
2. Add a Fourth Point: Include a point D at (0, 0) with a weight of 2, and recalculate the weighted average.
3. Visualize the Points: Plot points A, B, C, and the weighted centroid to see their relative positions.

Summary

Calculating the weighted average of points in a 2D plane involves applying the principles of weighted means separately to the x and y coordinates. For the given points:

- A: (8, -6), weight = 1
- B: (9, -5), weight = 4
- C: (-6, 7), weight = 4

The weighted average point is approximately (2.22, 0.22), representing a

balance point that accounts for the importance (weights) assigned to each original point.

This approach is widely applicable across multiple disciplines, providing insights into the distribution, influence, and central tendency of spatial data. Understanding and applying weighted averages equips professionals and students with a powerful tool for analysis in both theoretical and practical contexts.

References and Further Reading

- "Mathematics for Engineers," by Anthony Croft, Robert Davison.
- "Introduction to Geometry," by H.S.M. Coxeter.
- Online resources on weighted averages and centroid calculations.
- Tutorials on coordinate geometry and vector calculus.

Closing Note: Mastery of weighted averages in coordinate systems enhances problem-solving skills and supports advanced applications in science, engineering, and data science. Practice with different datasets and scenarios to deepen understanding and proficiency.

Frequently Asked Questions

How do you calculate the weighted average of points A, B, and C with weights 1, 4, and 4 respectively?

To calculate the weighted average of points A, B, and C, multiply each point's coordinates by their respective weights, sum these products, and then divide by the total weight ($1 + 4 + 4 = 9$).

What are the coordinates of point A, B, and C in the given problem?

Point A is at (8, -6), point B at (9, -5), and point C at (-6, 7).

How do you compute the weighted average for the x-coordinates of points A, B, and C?

Multiply each x-coordinate by its weight, sum these products, then divide by the total weight (9). Specifically: $(81 + 94 + (-6)4) / 9$.

What is the formula for finding the weighted average of the y-coordinates?

Similarly, multiply each y-coordinate by its weight, sum them, and divide by the total weight: $(-61 + (-5)4 + 74) / 9$.

Calculate the weighted average point of A, B, and C with weights 1, 4, and 4.

First, compute the x-coordinate: $(81 + 94 + (-6)4) / 9 = (8 + 36 - 24) / 9 = 20 / 9 \approx 2.22$. Then, compute the y-coordinate: $(-61 + (-5)4 + 74) / 9 = (-6 - 20 + 28) / 9 = 2 / 9 \approx 0.22$. So, the weighted average point is approximately $(2.22, 0.22)$.

Why are the weights 1, 4, and 4 used in calculating the average?

The weights 1, 4, and 4 assign different levels of importance to each point, with points B and C being more influential than point A in the average calculation.

What is the significance of calculating a weighted average of points in coordinate geometry?

Calculating a weighted average allows us to find a central point that reflects the relative importance or influence of each individual point based on their assigned weights.

Can this method be used to find the centroid of a set of points with different weights?

Yes, the weighted average formula is used to find the centroid or center of mass of points when each point has an associated weight, representing its significance or mass.

What is the final result of the weighted average calculation for the given points and weights?

The weighted average point is approximately $(2.22, 0.22)$, calculated as $(20/9, 2/9)$.

Additional Resources

What Is The Average Of The Points A, B, And C With Weights 1, 4 And 4 Respectively? A $(8, -6)$, B $(9, -5)$, C $(-6, 7)$

Introduction

In the realm of mathematics, understanding how to compute weighted averages of points in a coordinate plane is fundamental for various applications, ranging from data analysis to engineering design. The problem at hand involves calculating the weighted average (or weighted centroid) of three points—A, B, and C—with specified weights of 1, 4, and 4, respectively.

This investigation aims to dissect the process thoroughly, elucidate the conceptual underpinnings, and demonstrate the step-by-step methodology to arrive at the precise coordinates of the weighted average point. Such an exploration not only clarifies the specific calculation but also offers broader insights into the principles governing weighted means in multidimensional spaces.

Contextual Framework: Understanding Weighted Averages in Geometry

Before delving into the calculations, it is essential to establish a clear understanding of what a weighted average represents in a geometric context.

Definition of Weighted Average

Given a set of points $(P_1, P_2, \dots, P_n)$ with corresponding weights $(w_1, w_2, \dots, w_n)$, the weighted average (or weighted centroid) (P_{avg}) is calculated as:

$$P_{\text{avg}} = \frac{\sum_{i=1}^n w_i P_i}{\sum_{i=1}^n w_i}$$

Expressed component-wise in the (x) and (y) directions:

$$x_{\text{avg}} = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$$
$$y_{\text{avg}} = \frac{\sum_{i=1}^n w_i y_i}{\sum_{i=1}^n w_i}$$

This approach assigns different levels of importance to each point, proportional to their weights, which influences the position of the average point accordingly.

Data Specification and Problem Details

The points provided are:

- Point A: (8, -6), with weight $(w_A = 1)$
- Point B: (9, -5), with weight $(w_B = 4)$
- Point C: (-6, 7), with weight $(w_C = 4)$

Our goal is to compute:

$$\boxed{\text{Weighted average point } P_{\text{avg}} = (x_{\text{avg}}, y_{\text{avg}})}$$

with the weights as specified.

Step-by-Step Calculation Process

1. Summation of Weights

First, compute the total weight:

$$W_{\text{total}} = w_A + w_B + w_C = 1 + 4 + 4 = 9$$

This total will serve as the denominator for both the (x) and (y) coordinate calculations.

2. Calculation of the (x) -Coordinate

Applying the weighted average formula:

$$x_{\text{avg}} = \frac{w_A x_A + w_B x_B + w_C x_C}{W_{\text{total}}}$$

Substituting the known values:

$$x_{\text{avg}} = \frac{(1)(8) + (4)(9) + (4)(-6)}{9}$$

Calculating numerator step-by-step:

- $(1)(8) = 8$
- $(4)(9) = 36$

$$- (4)(-6) = -24$$

Adding these:

$$8 + 36 - 24 = 20$$

Thus,

$$x_{\text{avg}} = \frac{20}{9} \approx 2.222\ldots$$

3. Calculation of the y -Coordinate

Similarly,

$$y_{\text{avg}} = \frac{w_A y_A + w_B y_B + w_C y_C}{W_{\text{total}}}$$

Substituting values:

$$y_{\text{avg}} = \frac{(1)(-6) + (4)(-5) + (4)(7)}{9}$$

Calculating numerator step-by-step:

$$\begin{aligned} - (1)(-6) &= -6 \\ - (4)(-5) &= -20 \\ - (4)(7) &= 28 \end{aligned}$$

Adding these:

$$-6 - 20 + 28 = 2$$

Therefore,

$$y_{\text{avg}} = \frac{2}{9} \approx 0.222\ldots$$

Final Result: The Weighted Average Point

Combining the results:

```
\[
\boxed{
P_{\text{avg}} \approx (2.222..., 0.222...)
}
```

or, precisely,

```
\[
P_{\text{avg}} = \left( \frac{20}{9}, \frac{2}{9} \right)
\]
```

which simplifies to approximately (2.22, 0.22) in decimal form.

Broader Implications and Applications

This calculation exemplifies a foundational principle in spatial analytics: the weighted centroid. Such a point effectively represents the "center of mass," considering the relative importance (weights) assigned to each constituent point. This concept has numerous applications:

- Geographical and Urban Planning: Determining optimal locations based on weighted population centers.
- Physics and Engineering: Calculating centers of mass for composite objects.
- Data Science: Computing weighted means for clustering or aggregation tasks.

Practical Considerations

While the example involves just three points, the methodology extends effortlessly to larger datasets, with the calculation relying on straightforward summations and divisions. The key is ensuring that the weights accurately reflect the significance of each point, and that the data points are correctly identified with their respective weights.

Potential Variations and Advanced Topics

- Weighted averages in higher dimensions: Extending calculations to three or more dimensions.
- Dynamic weights: Adjusting weights based on changing factors, such as time or external conditions.
- Weighted medians and other measures: For data that is not symmetrically distributed or contains outliers.

Conclusion

The process of calculating the weighted average of points in a two-dimensional space, especially with specified weights, is a vital technique in both theoretical and applied mathematics. In this specific case, the weighted centroid of points A, B, and C with weights 1, 4, and 4 respectively is approximately at (2.22, 0.22), reflecting a balanced point influenced more heavily by points B and C.

This detailed exploration underscores the importance of meticulous calculation, conceptual clarity, and understanding the implications of weight assignments. Whether applied in geographic modeling, physics, or data analysis, the principles illustrated here serve as a foundation for more complex spatial reasoning and decision-making processes.

References

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