Write An Equation Of A Line Having Slope -(2)/(3) And Having Y-intercept As (0,(3)/(7)). Write The Equation

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Understanding how to formulate the equation of a line given specific parameters is a fundamental skill in algebra and coordinate geometry. In this article, we will explore the process step-by-step to write the equation of a line when the slope and y-intercept are provided. Specifically, we will focus on creating the equation for a line with a slope of $-\frac{2}{3}$ and a y-intercept at the point $(0, \frac{3}{7})$. This detailed explanation aims to help students and learners grasp the concept thoroughly and apply it to similar problems confidently.

Basics of the Equation of a Line

Before diving into the specifics of our problem, it's essential to understand the general form of a linear equation and the significance of the slope and y-intercept.

General Form of a Line

The most common form of a linear equation in two variables (x and y) is:

$$
y = mx + b
$$

Where:
- m is the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

This form is known as the slope-intercept form because it directly displays the slope and y-intercept.

Understanding Slope and Y-Intercept

- Slope (m): Represents the rate of change of y with respect to x. It indicates how steep the line is and in which direction it inclines. A positive slope means the line rises from left to right, while a negative slope indicates a decline.

- Y-Intercept (b): The point where the line crosses the y-axis ($x=0$). It provides a starting point for graphing the line.

Given Parameters and Their Meaning

In our specific problem:

- Slope (m): $-\frac{2}{3}$

- Y-intercept (b): $\frac{3}{7}$

The point given as $(0, \frac{3}{7})$ confirms the y-intercept because when $x=0$, $y=\frac{3}{7}$.

Visualizing the Line

To better understand, imagine plotting the y-intercept at $(0, \frac{3}{7})$. From this point, the line will slope downward (since the slope is negative) at a rate of $\frac{2}{3}$ units down for every 3 units moved to the right.

Step-by-Step Process to Write the Equation

Now, let's proceed through the steps to derive the line's equation.

Step 1: Recall the Slope-Intercept Form

The general form:

$$
y = mx + b
$$

Where:

- $m = -\frac{2}{3}$

- $b = \frac{3}{7}$

Step 2: Substitute the Known Values

Plugging in the known slope and y-intercept directly:

$$
y = -\frac{2}{3}x + \frac{3}{7}
$$

This is the equation of the line in slope-intercept form.

Step 3: Confirm the Equation with the Given Point

Verify that the equation passes through the y-intercept point $(0, \frac{3}{7})$:

- Substitute $x=0$:

$$
y = -\frac{2}{3} \times 0 + \frac{3}{7} = 0 + \frac{3}{7} = \frac{3}{7}
$$

which matches the point exactly. This confirms the correctness of our equation.

Alternate Forms of the Equation

While the slope-intercept form is most straightforward, sometimes the equation is required in other forms.

Point-Slope Form

Using the point $(0, \frac{3}{7})$ and the slope $-\frac{2}{3}$:

$$
y - y_1 = m(x - x_1)
$$

Since $x_1 = 0$, $y_1 = \frac{3}{7}$:

$$
y - \frac{3}{7} = -\frac{2}{3}(x - 0)
$$
$$
y - \frac{3}{7} = -\frac{2}{3}x
$$

Rearranged to the slope-intercept form:

$$
y = -\frac{2}{3}x + \frac{3}{7}
$$

which is consistent with our earlier result.

Standard Form

To write the equation in standard form $Ax + By = C$:

- Start with the slope-intercept form:

$$
y = -\frac{2}{3}x + \frac{3}{7}
$$

- Multiply through by the least common denominator (21) to clear fractions:

$$
21y = 21 \times \left(-\frac{2}{3}x + \frac{3}{7}\right)
$$

Calculate:

$$
21y = 21 \times -\frac{2}{3}x + 21 \times \frac{3}{7}
$$
$$
21y = -14x + 9
$$

Rearranged:

$$
14x + 21y = 9
$$

This is the standard form of the line's equation.

Graphical Interpretation

Graphing the line provides visual confirmation:

- The y-intercept at $(0, \frac{3}{7})$ is approximately $(0, 0.429)$.

- The slope of $-\frac{2}{3}$ indicates that for every 3 units moved to the right, the line falls 2 units.

- Plotting a second point: starting from $(0, 3/7)$, move 3 units right and 2 units down:

\[

$$x = 3,\quad y = \frac{3}{7} - 2 = \frac{3}{7} - \frac{14}{7} = -\frac{11}{7}$$

which is approximately -1.571.

Connecting these points yields the line.

Applications and Relevance

Understanding how to derive the equation of a line from slope and y-intercept is crucial for various mathematical and real-world applications:

- Graphing Lines: Quickly sketch lines based on given parameters.

- Finding Equations of Lines: From points and slopes in coordinate geometry problems.

- Linear Modeling: In economics, physics, and social sciences, to model relationships.

- Data Analysis: Fitting lines to data points (regression).

Summary of the Process

To summarize, the key steps to write the equation of a line given slope and y-intercept are:

1. Recall the slope-intercept form $y = mx + b$.

2. Substitute the given slope and y-intercept values.

3. Simplify if necessary, especially when fractions are involved.

4. Confirm by substituting the y-intercept point into the equation.

5. Convert to other forms if needed, such as point-slope or standard form.

Conclusion

The equation of a line with slope $-\frac{2}{3}$ and y-intercept $\frac{3}{7}$ is:

$$\boxed{ y = -\frac{2}{3}x + \frac{3}{7} }$$

This equation provides a complete algebraic representation of the line's behavior and can be used for graphing, analysis, and solving related problems. Mastering this process enhances your ability to handle a wide variety of coordinate geometry tasks, making it a fundamental skill for students and professionals alike.

If you need more advanced topics like converting to other forms, finding parallel or perpendicular lines, or solving real-world problems involving lines, feel free to ask!

Frequently Asked Questions

What is the slope and y-intercept given for the line in the problem?

The slope is -2/3 and the y-intercept is (0, 3/7).

How do you write the equation of a line given the slope and y-intercept?

Use the slope-intercept form y = mx + b, where m is the slope and b is the y-intercept.

What is the slope-intercept form of the line with slope -2/3 and y-intercept (0, 3/7)?

The equation is y = -2/3 x + 3/7.

Why is the y-intercept written as 3/7 in the equation?

Because the y-intercept point is (0, 3/7), which indicates the y-value when x=0, so b = 3/7.

Can the equation be written in other forms besides slope-intercept form?

Yes, it can also be written in standard form or point-slope form, but slope-intercept form is simplest here.

What is the final equation of the line with the given slope and y-intercept?

The final equation is y = -2/3 x + 3/7.

Additional Resources

Write An Equation Of A Line Having Slope -(2)/(3) And Having Y-intercept As (0, (3)/(7))

Introduction

In the realm of coordinate geometry, the equation of a line is fundamental for understanding the relationship between two variables plotted on the Cartesian plane. Whether in pure mathematics, physics, engineering, or data science, the ability to determine and analyze the equations of lines under various conditions forms the backbone of many analytical processes. This article aims to explore the derivation of a specific linear equation characterized by a given slope and y-intercept, providing an in-depth investigation into the process, the theoretical underpinnings, and practical applications.

The Significance of Line Equations in Mathematics and Beyond

Lines are among the most basic yet powerful constructs in mathematics. They serve as the foundation for more complex geometric and algebraic concepts, underpin the study of functions, and facilitate modeling in real-world scenarios such as motion, economics, and engineering design.

A line in a two-dimensional plane can be represented in several forms:

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By = C$

Understanding how to derive and manipulate these forms allows for flexible analysis and problem-solving. The focus here is on the slope-intercept form, which directly encodes the line's steepness and position relative to the y-axis.

Contextualizing the Given Data

Given the problem:

- Slope (m): $-\frac{2}{3}$
- Y-intercept (b): $\frac{3}{7}$, with the point $(0, \frac{3}{7})$

The goal is to derive the explicit equation of the line that satisfies these conditions.

Step-by-Step Derivation of the Line Equation

1. Understanding the Slope

The slope, $m = -\frac{2}{3}$, indicates the line's rate of change: for every increase of 3 units along the x-axis, the y-value decreases by 2 units.

Implication: The line declines as it moves from left to right, characteristic of a negative slope.

2. Recognizing the Y-Intercept

The y-intercept point, $(0, \frac{3}{7})$, is where the line crosses the y-axis. This provides a specific point through which the line passes.

Significance: The y-intercept simplifies the equation derivation since the slope-intercept form directly incorporates it.

3. Formulating the Equation

Given the slope and y-intercept, the standard slope-intercept form applies:

$$
y = mx + b
$$

Substituting the known values:

$$
y = -\frac{2}{3}x + \frac{3}{7}
$$

This is the explicit equation of the line.

Validating the Equation

To reinforce the correctness, consider the following:

- The line passes through the point $(0, \frac{3}{7})$:

Substituting $x=0$:

$$
y = -\frac{2}{3} \times 0 + \frac{3}{7} = \frac{3}{7}
$$

Confirmed.

- The slope indicates the rate of change:

For an increase of 3 units in x:

$$
\Delta y = -\frac{2}{3} \times 3 = -2
$$

So, moving from $x=0$ to $x=3$:

$$
y \text{ decreases by } 2
$$

Starting at $y = \frac{3}{7}$:

$$
y_{x=3} = -\frac{2}{3} \times 3 + \frac{3}{7} = -2 + \frac{3}{7} = -\frac{14}{7} + \frac{3}{7} = -\frac{11}{7}
$$

The point $(3, -\frac{11}{7})$ lies on the line, confirming the slope.

Interpretation and Applications

1. Graphical Representation

Plotting the line $y = -\frac{2}{3}x + \frac{3}{7}$:

- The y-intercept is at $\left(0, \frac{3}{7}\right)$.
- The slope dictates the descent: for every 3 units rightward, the line drops 2 units.

2. Real-World Context

Suppose the line models a scenario such as:

- Economics: A decrease in profit (y) relative to an increase in production units (x).
- Physics: A negative linear relationship between temperature and pressure under certain conditions.
- Engineering: The rate at which a component's performance deteriorates over time.

3. Analytical Uses

- Predicting values: Given a specific x, compute y.
- Finding intersections: When combined with other lines or curves.
- Optimization: Identifying maximum or minimum points (though linear functions have no maxima or minima unless restricted).

Extending Beyond the Basic Equation

While the primary goal was to write the line's equation, deeper insights can emerge through analysis:

1. Standard Form Conversion

Rearranging the slope-intercept form into standard form:

$$
y = -\frac{2}{3}x + \frac{3}{7}
$$

Multiply through by 21 (the least common denominator of 3 and 7):

$$
21y = -14x + 9
$$

Rearranged:

$$
14x + 21y = 9
$$

This form is often useful in solving systems of equations and in graphical analysis.

2. Equation with General Coefficients

Expressing the line as:

$$
Ax + By = C
$$

where:

- $A = 14$,
- $B = 21$,
- $C = 9$.

Broader Implications and Educational Significance

Understanding how to derive the equation of a line from given parameters is fundamental for students and professionals alike. It demonstrates:

- The power of algebraic manipulation.
- The importance of recognizing different forms of line equations.
- The relationship between slope, intercepts, and graphical representation.

Furthermore, this exercise underscores the importance of precise calculations, especially when dealing with fractional coefficients and points.

Conclusion

In conclusion, the explicit equation of the line with slope $-\frac{2}{3}$ and y-intercept $\frac{3}{7}$ is:

$$
\boxed{
y = -\frac{2}{3}x + \frac{3}{7}
}
$$

This derivation exemplifies core principles in coordinate geometry, illustrating how given parameters translate into an algebraic expression. Such foundational knowledge is indispensable across a multitude of scientific and mathematical disciplines, enabling accurate modeling, analysis, and problem-solving.

This detailed exploration not only confirms the correctness of the derived line but also contextualizes its significance, demonstrating the interconnectedness of algebraic concepts and their practical applications.

Write An Equation Of A Line Having Slope 2 3 And Having Y Intercept As 0 3 7 Write The Equation

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