A Box Contains 10 Cards Of Which 3 Are Of Red Color And 7 Are Of Blue Color. Three Cards Are Chosen Randomly,

A Box Contains 10 Cards Of Which 3 Are Of Red Color And 7 Are Of Blue Color. Three Cards Are Chosen Randomly, this scenario presents an interesting problem in probability theory and combinatorics. Understanding the various outcomes, calculating probabilities, and analyzing the chances of specific events all come into play when dealing with such randomly selected items from a finite set. This article explores the problem in detail, providing definitions, calculations, and insights into the probability of different events involving these cards.

Introduction to the Problem

The problem involves a simple, yet fundamental, scenario in probability. It involves a box containing 10 cards, with a specific color distribution: 3 red cards and 7 blue cards. The task is to analyze the probability of selecting certain combinations when three cards are drawn randomly from the box.

Key Elements of the Scenario

- Total number of cards: 10
- Red cards: 3
- Blue cards: 7
- Number of cards drawn: 3
- Nature of selection: Random, without replacement

Understanding these elements sets the foundation for calculating various probabilities, such as the likelihood of drawing a specific number of red or blue cards, or the probability of certain combinations.

Basic Concepts in Probability and Combinatorics

Before diving into specific calculations, it's essential to review some fundamental concepts:

1. Sample Space

The total number of possible outcomes when selecting three cards from ten is given by the combination formula:

\[
\text{Total outcomes} = \binom{10}{3}

\]

which counts all possible groups of 3 cards regardless of order.

2. Combinations

Since the order in which the cards are drawn does not matter, combinations are used to count the number of ways to choose items:

$$
\binom{n}{k} = \frac{n!}{k!(n - k)!}
$$

where:
- n is the total number of items,
- k is the number of items to choose,
- $!$ denotes factorial.

3. Probabilities

The probability of an event is the ratio of favorable outcomes to the total possible outcomes:

$$
P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}
$$

Total Number of Ways to Select 3 Cards from 10

Calculating the total number of possible combinations:

$$
\binom{10}{3} = \frac{10!}{3! \times 7!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120
$$

There are 120 possible ways to select three cards from the box.

Probabilities of Drawing Red and Blue Cards

Given the composition of the box, various events can occur when selecting three cards:

- All three cards are red
- All three cards are blue
- Exactly two red and one blue

- Exactly one red and two blue

Let's analyze each case.

1. Probability of Drawing All Red Cards (3 Red)

Number of ways to choose 3 red cards from 3 available:

$$
\binom{3}{3} = 1
$$

Number of ways to choose the remaining 0 cards from the 7 blue cards:

$$
\binom{7}{0} = 1
$$

Total favorable outcomes:

$$
1 \times 1 = 1
$$

Probability:

$$
P(\text{all red}) = \frac{1}{120} \approx 0.0083
$$

2. Probability of Drawing All Blue Cards (3 Blue)

Number of ways to choose 3 blue cards from 7:

$$
\binom{7}{3} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35
$$

Number of ways to choose 0 red cards:

$$
\binom{3}{0} = 1
$$

Total favorable outcomes:

$$
35 \times 1 = 35
$$

Probability:

$$
P(\text{all blue}) = \frac{35}{120} \approx 0.2917
$$

3. Probability of Drawing Exactly Two Red and One Blue

Number of ways to choose 2 red cards from 3:

$$
\binom{3}{2} = 3
$$

Number of ways to choose 1 blue card from 7:

$$
\binom{7}{1} = 7
$$

Total favorable outcomes:

$$
3 \times 7 = 21
$$

Probability:

$$
P(\text{2 red, 1 blue}) = \frac{21}{120} = 0.175
$$

4. Probability of Drawing Exactly One Red and Two Blue

Number of ways to choose 1 red card from 3:

$$
\binom{3}{1} = 3
$$

Number of ways to choose 2 blue cards from 7:

$$
\binom{7}{2} = \frac{7 \times 6}{2} = 21
$$

Total favorable outcomes:

$$
3 \times 21 = 63
$$

Probability:

$$
P(\text{1 red, 2 blue}) = \frac{63}{120} = 0.525
$$

Summary of Probabilities

Event	Number of Favorable Outcomes	Probability
All red	1	$\frac{1}{120} \approx 0.0083$
All blue	35	$\frac{35}{120} \approx 0.2917$
Two red, one blue	21	$\frac{21}{120} \approx 0.175$
One red, two blue	63	$\frac{63}{120} \approx 0.525$

These outcomes sum to 120, confirming the consistency of the calculations as probabilities sum to 1.

Expected Values and Mean Number of Red and Blue Cards

In probability, it's often useful to calculate the expected number of red or blue cards in a randomly chosen set.

1. Expected Number of Red Cards in a Draw of 3

Using the linearity of expectation:

$$
E(\text{Red}) = 3 \times \frac{\text{Number of red cards}}{\text{Total cards}} = 3 \times \frac{3}{10} = 0.9
$$

Similarly, for blue:

$$
E(\text{Blue}) = 3 \times \frac{7}{10} = 2.1
$$

This indicates that, on average, out of three randomly selected cards, one expects approximately 0.9 red cards and 2.1 blue cards.

Advanced Analysis: Conditional Probabilities and Events

Beyond simple probabilities, one might be interested in conditional probabilities or specific events.

1. Probability of At Least One Red Card

The complement of drawing no red cards (i.e., all blue):

Number of ways to select 3 blue cards:

$$
\binom{7}{3} = 35
$$

Total outcomes:

$$
120
$$

Probability of no red:

$$
P(\text{no red}) = \frac{35}{120} = 0.2917
$$

Thus, probability of at least one red:

$$
P(\text{at least one red}) = 1 - P(\text{no red}) = 1 - 0.2917 = 0.7083
$$

2. Probability of Exactly Two Blue and One Red

Number of ways:

$$
\binom{7}{2} \times \binom{3}{1} = 21 \times 3 = 63
$$

Probability:

$$
\frac{63}{120} = 0.525
$$

This matches the earlier calculation for 1 red and 2 blue, illustrating the symmetry in combinations.

Practical Applications and Real-World Relevance

Understanding such probability problems is not just academic; it has practical implications in various fields.

1. Card Games and Gambling

Probabilities of drawing certain combinations are fundamental in analyzing strategies and odds in card games, such as poker or blackjack.

2. Quality Control and Sampling

Random sampling from a batch (like the 10 cards) is common in quality control processes, where the probability of selecting defective items (red cards) from a batch is crucial.

3. Decision Making and Risk Assessment

Assessing the likelihood of certain outcomes helps in making informed decisions in investment, insurance, and strategic planning.

Extending the Problem: Variations and Complex Scenarios

While the basic problem involves a straightforward calculation, various extensions can deepen understanding.

1. Drawing with Replacement

If cards are drawn with replacement (after each draw, the card is returned to the box), probabilities change, and calculations involve independence between draws.

2. Larger Sets or Different Distributions

Changing the total number of cards or the ratio of red to blue can lead to more complex probability distributions, such as hypergeometric or binomial distributions.

3. Multiple Draws and Sequential Probabilities

Analyzing probabilities over multiple draws, tracking the composition after each, involves conditional probabilities and more advanced

Frequently Asked Questions

What is the probability of selecting exactly 2 red cards when three cards are drawn randomly from the box?

The probability is calculated as the number of ways to choose 2 red and 1 blue card divided by the total number of ways to choose any 3 cards. That is: (C(3,2) C(7,1)) / C(10,3) = (3 7) / 120 = 21/120 = 7/40.

What is the probability of drawing at least one red card in three random picks?

The probability of at least one red card is 1 minus the probability of drawing no red cards (all blue). The probability of all blue is C(7,3)/C(10,3) = 35/120 = 7/24. Therefore, the probability of at least one red is 1 - 7/24 = 17/24.

What is the probability of selecting all three red cards in the draw?

Since there are only 3 red cards, the probability of selecting all three is the number of ways to choose all red cards divided by total combinations: C(3,3)/C(10,3) = 1/120 120 = 1/120, which simplifies to 1/120.

If one card is drawn and not replaced, what is the probability that the second card drawn is red, given the first was blue?

After drawing a blue card, remaining cards are 3 red and 6 blue. The probability that the second card is red is 3/9 = 1/3.

How many different combinations of 3 cards can be drawn from the box?

The total number of combinations is C(10,3) = 120.

What is the probability of drawing exactly one red card and two blue cards in three draws?

Number of favorable combinations is C(3,1) C(7,2) = 3 21 = 63. Total combinations are C(10,3) = 120. So, probability = 63/120 = 21/40.

Additional Resources

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Introduction

In the realm of probability, combinatorics, and statistical analysis, seemingly simple scenarios often unveil complex insights and rich mathematical structures. One such scenario involves a box containing ten cards—three red and seven blue—and the process of randomly selecting three cards from this collection. While at first glance, this might appear to be a straightforward case of sampling, the underlying principles and calculations reveal a tapestry of probabilistic outcomes, combinatorial considerations, and real-world applications.

This article embarks on an in-depth exploration of this scenario, dissecting the problem from multiple angles, including probability theory, combinatorial enumeration, expected values, conditional probabilities, and potential extensions. Such an investigation not only enhances understanding of basic probability principles but also exemplifies how simple setups can serve as foundational models for more complex systems in fields like statistics, computer science, and decision theory.

Fundamental Setup and Assumptions

Description of the Scenario

- Total number of cards: 10
- Red cards: 3
- Blue cards: 7
- Cards drawn: 3, randomly and without replacement

Assumptions

- The selection process is random and unbiased, meaning each combination of three cards is equally likely.
- The draws are without replacement, so once a card is selected, it is not returned to the box.
- The focus is on analyzing the probabilities of various outcomes related to the colors of the selected cards.

Combinatorial Foundations

Understanding the probabilities involved requires a solid grasp of combinatorial mathematics, particularly the concept of combinations.

Total Number of Ways to Choose 3 Cards

The total number of possible combinations when selecting 3 cards from 10 is given by:

$$
\binom{10}{3} = \frac{10!}{3! \times 7!} = 120
$$

This total forms the denominator in probability calculations for specific events.

Possible Color Combinations

Since each card is either red or blue, the possible distributions of red and blue cards in a 3-card selection are:

Number of Red Cards	Number of Blue Cards	Number of Combinations
0	3	$\binom{3}{0} \times \binom{7}{3} = 1 \times 35 = 35$
1	2	$\binom{3}{1} \times \binom{7}{2} = 3 \times 21 = 63$
2	1	$\binom{3}{2} \times \binom{7}{1} = 3 \times 7 = 21$
3	0	$\binom{3}{3} \times \binom{7}{0} = 1 \times 1 = 1$

Total combinations check: $35 + 63 + 21 + 1 = 120$, confirming the total.

Probabilities of Specific Outcomes

Probability of Drawing a Certain Number of Red Cards

Using the combinatorial counts, the probability of each scenario is:

- 0 Red, 3 Blue:

$$
P(0 \text{ red}) = \frac{35}{120} = \frac{7}{24} \approx 29.17\%
$$

- 1 Red, 2 Blue:

$$
P(1 \text{ red}) = \frac{63}{120} = \frac{21}{40} = 52.5\%
$$

- 2 Red, 1 Blue:

$$
P(2 \text{ red}) = \frac{21}{120} = \frac{7}{40} = 17.5\%
$$

- 3 Red, 0 Blue:

$$
P(3 \text{ red}) = \frac{1}{120} \approx 0.83\%
$$

These probabilities provide a foundational understanding of the likelihood of various color compositions in a random draw.

Expected Number of Red (or Blue) Cards in the Draw

The expected value, or the mean number of red cards in a randomly chosen set of three, can be calculated as:

$$
E(\text{Red}) = 0 \times P(0) + 1 \times P(1) + 2 \times P(2) + 3 \times P(3)
$$

Plugging in the probabilities:

$$
E(\text{Red}) = 0 \times \frac{35}{120} + 1 \times \frac{63}{120} + 2 \times \frac{21}{120} + 3 \times \frac{1}{120} = \frac{63 + 42 + 3}{120} = \frac{108}{120} = 0.9
$$

So, on average, a randomly selected set of three cards contains approximately 0.9 red cards.

Similarly, the expected number of blue cards:

$$
E(\text{Blue}) = 3 - E(\text{Red}) = 3 - 0.9 = 2.1
$$

which aligns with intuitive expectations given the composition of the box.

Conditional Probabilities and Real-World Implications

Probability of a Red Card Given Previous Draws

Suppose a scenario where one or two cards have already been drawn, and the goal is to determine the probability that the next card is red.

- After drawing 1 card:

- If the first card is red (probability $\frac{3}{10}$), remaining red cards: 2, remaining total: 9

- If the first card is blue (probability $\frac{7}{10}$), remaining red cards: 3, remaining total: 9

- Probability the second card is red:

$$
P(\text{second red}) = P(\text{first red}) \times \frac{2}{9} + P(\text{first blue}) \times \frac{3}{9} = \frac{3}{10} \times \frac{2}{9} + \frac{7}{10} \times \frac{3}{9} = \frac{6}{90} + \frac{21}{90} = \frac{27}{90} = \frac{3}{10}
$$

which remains consistent with the initial probability, demonstrating the conditional independence property in this context.

Practical Applications of the Scenario

This seemingly simple problem models various real-world situations:

- Quality control: Selecting samples from a batch with known defect rates.

- Card games and gambling: Calculating probabilities of specific hands.

- Data sampling: Estimating the composition of a population based on a small sample.

- Medical testing: Randomly choosing samples with known prevalence rates.

Understanding the probabilities involved helps in decision-making under uncertainty, risk assessment, and strategic planning.

Advanced Topics and Extensions

Hypergeometric Distribution

The probability calculations align with the hypergeometric distribution, which models the probability of k successes (red cards) in n draws without replacement from a finite population:

$$
P(X = k) = \frac{\binom{K}{k} \binom{N - K}{n - k}}{\binom{N}{n}}
$$

where:

- $N = 10$ (total cards)

- $K = 3$ (red cards)

- $n = 3$ (cards drawn)

- k varies from 0 to 3

This formalizes the earlier calculations and supports deriving other properties such as variance and confidence intervals.

Variance of the Number of Red Cards

The variance of X, the number of red cards in the sample, is:

\[

\text{Var}(X) = n \times \frac{K}{N} \times \left(1 - \frac{K}{N}\right) \times \frac{N - n}{N - 1}
\]

Plugging in the numbers:

\[
\text{Var}(X) = 3 \times \frac{3}{10} \times \frac{7}{10} \times \frac{7}{9} = 3 \times 0.3 \times 0.7 \times \frac{7}{9}
\]

\[
= 3 \times 0.21 \times \frac{7}{9} \approx 3 \times 0.21 \times 0.7778 \approx 3 \times 0.1633 \approx 0.4899
\]

Thus, the standard deviation is approximately \(\sqrt{0.49} \approx 0.7\).

Simulation and Empirical Verification

Monte Carlo simulations can be employed to empirically verify theoretical probabilities. By randomly sampling 10,000 or more instances of 3-card draws, one can observe the frequency distribution of red cards and compare it with theoretical predictions, ensuring robustness of the models.

Broader Implications and Educational Significance

This scenario functions as an excellent pedagogical tool for illustrating core principles of probability, such as:

- The importance of combinatorial enumeration.

- How

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