A Cylinder Has A Surface Area Of 256 \pi Square Millimeters And A Height Of 8 Millimeters. Find The Diameter.

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Understanding the dimensions and properties of cylinders is fundamental in geometry, particularly when dealing with real-world applications like engineering, manufacturing, and design. In this article, we will delve into a detailed explanation of how to determine the diameter of a cylinder given its surface area and height. We will explore the relevant formulas, step-by-step calculations, and provide practical insights to enhance your understanding of this geometric problem.

Introduction to Cylinder Geometry

A cylinder is a three-dimensional geometric shape characterized by two parallel circular bases connected by a curved surface. The key dimensions of a cylinder include its radius (r), diameter (d), height (h), lateral surface area, and total surface area.

Key components:

- Radius (r): Distance from the center to the edge of the circular base.
- Diameter (d): The full width of the base, equal to twice the radius ($d = 2r$).
- Height (h): The distance between the two bases.
- Lateral Surface Area (LSA): The area of the side surface, calculated as $2\pi rh$.
- Total Surface Area (TSA): The sum of the lateral surface area and the areas of the two bases.

Understanding the Given Data

In our problem, the following data are provided:

- Surface Area (TSA): 256π square millimeters
- Height (h): 8 millimeters

Our goal: to find the diameter (d) of the cylinder.

Formulating the Surface Area Equation

The total surface area (TSA) of a cylinder combines the lateral surface area and the area of the two circular bases:

$$
TSA = 2\pi r h + 2\pi r^2
$$

Where:

- $2\pi r h$ is the lateral surface area.
- $2\pi r^2$ is the combined area of the two bases.

Given that TSA = 256π, and h = 8 mm, substitute these into the formula:

$$
256\pi = 2\pi r \times 8 + 2\pi r^2
$$

Solving for the Radius (r)

Step-by-step, we simplify the equation:

1. Factor out 2π:

$$
256\pi = 2\pi (8r + r^2)
$$

2. Divide both sides by 2π:

$$
\frac{256\pi}{2\pi} = 8r + r^2
$$

$$
\frac{256}{2} = 8r + r^2
$$

$$
128 = 8r + r^2
$$

3. Rearrange into standard quadratic form:

$$
r^2 + 8r - 128 = 0
$$

Calculating the Radius Using Quadratic Formula

The quadratic equation:

$$
r^2 + 8r - 128 = 0
$$

can be solved using the quadratic formula:

$$
r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where:

- $a = 1$
- $b = 8$
- $c = -128$

Calculating the discriminant:

$$
\Delta = b^2 - 4ac = 8^2 - 4 \times 1 \times (-128) = 64 + 512 = 576
$$

Taking the square root:

$$
\sqrt{\Delta} = \sqrt{576} = 24
$$

Substituting into the quadratic formula:

$$
r = \frac{-8 \pm 24}{2}
$$

This yields two solutions:

- $r = \frac{-8 + 24}{2} = \frac{16}{2} = 8$
- $r = \frac{-8 - 24}{2} = \frac{-32}{2} = -16$

Since radius cannot be negative, we discard $r = -16$. Therefore:

$$
r = 8 \text{ mm}
$$

Finding the Diameter

Recall that the diameter d is twice the radius:

$$
d = 2r
$$

Thus:

$$
d = 2 \times 8 = 16 \text{ mm}
$$

Answer: The diameter of the cylinder is 16 millimeters.

Summary of the Calculation Process

To summarize the steps involved in solving for the diameter:

1. Write the surface area formula for a cylinder:

$$
TSA = 2\pi rh + 2\pi r^2
$$

2. Substitute known values:

$$
256\pi = 2\pi \times 8 r + 2\pi r^2
$$

3. Simplify and divide both sides by 2π:

$$
128 = 8r + r^2

4. Rearrange into quadratic form:

$$
r^2 + 8r - 128 = 0
$$

5. Use quadratic formula to solve for r:

$$
r = \frac{-8 \pm 24}{2}
$$

6. Determine the valid radius:

$$
r = 8 \text{ mm}
$$

7. Calculate the diameter:

$$
d = 2r = 16 \text{ mm}
$$

Practical Applications of Cylinder Dimension Calculations

Understanding how to determine the diameter of a cylinder from its surface area and height has numerous practical applications, including:

- Manufacturing: Designing cylindrical components such as pipes, tanks, and rods.
- Engineering: Calculating material requirements and structural stability.
- Product Design: Ensuring precise dimensions for fit and function.
- Education: Enhancing understanding of geometric relationships.

Additional Tips for Solving Cylinder Problems

When approaching similar problems, keep these tips in mind:

- Always clearly identify the known quantities and what needs to be found.

- Recall and write down relevant formulas, including surface area, lateral area, and volume.
- Break down complex problems into smaller steps and verify each calculation.
- Use quadratic formula solutions carefully, considering the physical meaning (e.g., negative radius is invalid).
- Double-check units throughout the calculation to prevent errors.

Conclusion

By applying the surface area formula of a cylinder and solving the resulting quadratic equation, we have determined that the diameter of the cylinder with a surface area of \(256 \pi \) square millimeters and height of 8 millimeters is 16 millimeters. This process underscores the importance of algebraic skills and geometric understanding in solving real-world problems involving three-dimensional shapes. Whether you are a student, engineer, or enthusiast, mastering these calculations enhances your ability to analyze and design cylindrical objects with precision.

FAQs about Cylinder Surface Area and Diameter

Q1: Can the radius be zero?

- No. A radius of zero would mean the cylinder collapses into a line, which isn't a cylinder.

Q2: What if the surface area was given without π?

- If the surface area was provided as a numerical value, you'd need to include π in your calculations and possibly approximate the value for numerical results.

Q3: How does changing the height affect the diameter?

- Increasing the height while keeping the surface area constant would affect the radius and diameter; each case requires recalculating based on the formulas.

Q4: Are there alternative methods to solve for the diameter?

- For this problem, algebraic methods and quadratic formula are most straightforward. Geometric constructions or graphing can also be used for visual understanding.

In summary, understanding the relationship between a cylinder’s surface area, height, and diameter is essential in various fields. With a systematic approach involving formulas, algebra, and careful calculations, solving for unknown dimensions becomes manageable and precise.

Frequently Asked Questions

Given a cylinder with a surface area of 256π mm² and a height of 8 mm, how do you find its diameter?

First, use the surface area formula for a cylinder: $SA = 2\pi r(h + r)$. Substitute $SA = 256\pi$ and $h = 8$, then solve for r, and finally double it to find the diameter.

What is the step-by-step process to determine the diameter of a cylinder when surface area and height are known?

Step 1: Write the surface area formula $SA = 2\pi r(h + r)$. Step 2: Plug in the known values ($SA = 256\pi$, $h = 8$). Step 3: Simplify and solve the resulting quadratic for r. Step 4: Calculate diameter as 2r.

If a cylinder's surface area is 256π mm² and height is 8 mm, what is its radius?

By setting up the equation $2\pi r(8 + r) = 256\pi$, dividing both sides by 2π gives $r(8 + r) = 128$. Solving $r^2 + 8r - 128 = 0$ yields $r = 4\sqrt{2}$ mm, so the diameter is $2 \times 4\sqrt{2} = 8\sqrt{2}$ mm.

How do you verify the diameter of a cylinder after calculating its radius from the surface area?

Once you find the radius r, multiply by 2 to get the diameter: Diameter = 2r. For example, if $r = 8\sqrt{2} / 2 = 4\sqrt{2}$ mm, then diameter = $8\sqrt{2}$ mm.

What formulas are essential to solve for the diameter of a cylinder with known surface area and height?

The key formulas are: Surface area of a cylinder, $SA = 2\pi r(h + r)$, and the relationship Diameter = 2r. These allow you to set up an equation and solve for the radius, then find the diameter.

Additional Resources

A Cylinder Has A Surface Area Of 256 π Square Millimeters And A Height Of 8 Millimeters. Find The Diameter.
This problem involves calculating the diameter of a cylinder when given its total surface area and height. Such questions are common in geometry and often appear in exams or

practical scenarios involving cylindrical objects. Understanding how to approach this problem requires a clear grasp of the formulas for surface area and the relationships between the dimensions of a cylinder.

Understanding the Problem

The problem states:

- Surface Area of the cylinder = 256 π square millimeters
- Height of the cylinder (h) = 8 millimeters
- Goal: Find the diameter (d)

From the given data, our task is to determine the diameter. Since the diameter relates directly to the radius (r = d/2), once we find the radius, we can easily find the diameter.

Key Concepts and Formulas

To solve this problem, it's essential to understand the fundamental formulas related to cylinders:

Surface Area of a Cylinder

The total surface area (SA) of a cylinder is the sum of the lateral surface area and the area of the two circular bases:

$$
\text{SA} = 2\pi r h + 2\pi r^2
$$

Where:
- r = radius of the base
- h = height of the cylinder

The first term, $2\pi r h$, represents the lateral surface area (the side area when you "unwrap" the side).
The second term, $2\pi r^2$, accounts for the areas of the two circular bases.

Step-by-Step Approach to the Solution

Step 1: Write Down the Known Values

From the problem:
- Total surface area: 256π square millimeters
- Height: $h = 8$ millimeters

Expressed mathematically:

$$
2\pi r h + 2\pi r^2 = 256 \pi
$$

Step 2: Simplify the Equation

Divide both sides of the equation by π to make calculations easier:

$$
2 r h + 2 r^2 = 256
$$

Substituting $h = 8$:

$$
2 r \times 8 + 2 r^2 = 256
$$

$$
16 r + 2 r^2 = 256
$$

Step 3: Rearrange the Equation

Rewrite it as a standard quadratic form:

$$
2 r^2 + 16 r - 256 = 0
$$

Divide through by 2 to simplify:

$$
r^2 + 8 r - 128 = 0
$$

Step 4: Solve the Quadratic Equation

The quadratic equation:

$$
r^2 + 8 r - 128 = 0
$$

can be solved using the quadratic formula:

$$
r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where:

- $a = 1$
- $b = 8$
- $c = -128$

Calculate the discriminant:

$$
\Delta = b^2 - 4ac = 8^2 - 4 \times 1 \times (-128) = 64 + 512 = 576
$$

Find the square root:

$$
\sqrt{576} = 24
$$

Compute the roots:

$$
r = \frac{-8 \pm 24}{2}
$$

Step 5: Find the Possible Values of r

Calculate both solutions:

- Positive root:

$$
r = \frac{-8 + 24}{2} = \frac{16}{2} = 8
$$

- Negative root:

$$
r = \frac{-8 - 24}{2} = \frac{-32}{2} = -16
$$

Since the radius cannot be negative, we discard $r = -16$.

Therefore, the radius $r = 8$ millimeters.

Final Step: Calculate the Diameter

Recall:

$$
d = 2r
$$

Substitute $r = 8$:

$$
d = 2 \times 8 = 16 \text{ millimeters}
$$

Summary and Key Takeaways

- The total surface area of a cylinder combines lateral and base areas: $SA = 2\pi r h + 2\pi r^2$.
- By substituting the known values and simplifying, we derive a quadratic equation in r.
- Solving the quadratic yields two solutions, but only the positive radius makes physical sense.
- The diameter is twice the radius: $d = 2r$.

Hence, the diameter of the cylinder is 16 millimeters.

Additional Insights and Practical Tips

- Always double-check the units and ensure consistency throughout calculations.
- When faced with similar problems, it's helpful to write down all formulas clearly and substitute known values step-by-step.
- Remember that the quadratic formula can yield extraneous solutions; always verify if the solution makes sense in the context.

Real-World Applications

Understanding how to find the diameter from surface area and height is crucial in various fields:

- Manufacturing: Determining the size of pipes or containers.
- Design: Calculating dimensions for cylindrical objects like cans or tanks.
- Education: Enhancing problem-solving skills in geometry.

Conclusion

Through a systematic approach involving the fundamental formulas of surface area and quadratic equations, we successfully determined that the diameter of the cylinder is 16 millimeters. This problem exemplifies how algebra and geometry intertwine to solve practical questions involving three-dimensional shapes.

Remember, breaking down complex problems into manageable steps and verifying your solutions is key to mastering geometry challenges like this.

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