

A Given Copper Wire Is 7.5m Long And Has A Circular Cross-section Of Diameter 0.5mm. Calculate The Resistance

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Understanding the resistance of a copper wire is crucial in many electrical and electronic applications. Whether designing circuits, selecting appropriate wiring for electrical installations, or conducting experiments in physics, knowing how to calculate the resistance helps ensure safety, efficiency, and performance. In this article, we will explore the detailed process to calculate the resistance of a copper wire with specified dimensions: 7.5 meters in length and a circular cross-section with a diameter of 0.5 mm. We will also delve into the fundamental concepts of electrical resistance, material properties of copper, and the mathematical steps involved in deriving the resistance value.

Fundamentals of Electrical Resistance

What Is Electrical Resistance?

Electrical resistance is a measure of how much a material opposes the flow of electric current. It is a property that depends on the material's nature, its physical dimensions, and temperature. Resistance is measured in ohms (Ω), named after Georg Simon Ohm, who formulated Ohm's law.

Factors Affecting Resistance

Resistance in a wire is influenced by:

- **Material properties:** Different materials have different resistivities.
- **Length of the wire:** Longer wires have higher resistance.
- **Cross-sectional area:** Thicker wires have lower resistance.
- **Temperature:** Resistance varies with temperature, typically increasing with higher temperatures for metals like copper.

Understanding the Properties of Copper

Why Copper?

Copper is widely used in electrical wiring due to its excellent electrical conductivity, ductility, and relatively low cost. Its resistivity makes it ideal for transmitting electrical current efficiently.

Resistivity of Copper

Resistivity (denoted as ρ) is a fundamental property that indicates how strongly a material opposes the flow of electric current. For copper at room temperature (approximately 20°C), the resistivity is:

- **Resistivity (ρ):** $1.68 \times 10^{-8} \Omega \cdot \text{m}$

Knowing the resistivity allows us to calculate the resistance of any piece of copper wire, given its dimensions.

Calculating the Resistance of the Copper Wire

Step 1: Gather Known Data

The problem provides:

- **Length of the wire (L):** 7.5 meters
- **Diameter of the wire (d):** 0.5 mm
- **Resistivity of copper (ρ):** $1.68 \times 10^{-8} \Omega \cdot \text{m}$

Step 2: Convert All Measurements to SI Units

To maintain consistency, all measurements should be in meters and meters squared:

- Diameter: $0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ meters} = 5 \times 10^{-4} \text{ meters}$
- Radius (r): Since the wire has a circular cross-section, its radius is half of the diameter:

- $r = d / 2 = 5 \times 10^{-4} \text{ m} / 2 = 2.5 \times 10^{-4} \text{ m}$

Step 3: Calculate Cross-Sectional Area (A)

The cross-sectional area of a circular wire is given by:

$$A = \pi r^2$$

Calculating:

$$A = \pi \times (2.5 \times 10^{-4} \text{ m})^2$$

$$A = \pi \times 6.25 \times 10^{-8} \text{ m}^2$$

$$A \approx 3.1416 \times 6.25 \times 10^{-8} \text{ m}^2$$

$$A \approx 1.9635 \times 10^{-7} \text{ m}^2$$

This is the cross-sectional area in square meters.

Step 4: Use Resistance Formula

The resistance (R) of a uniform wire is given by:

$$R = \rho \times (L / A)$$

Where:

- R = resistance in ohms (Ω)
- ρ = resistivity of copper ($\Omega \cdot \text{m}$)
- L = length of the wire (m)
- A = cross-sectional area (m^2)

Substituting the known values:

$$R = 1.68 \times 10^{-8} \Omega \cdot \text{m} \times (7.5 \text{ m} / 1.9635 \times 10^{-7} \text{ m}^2)$$

Calculating:

$$R = (1.68 \times 10^{-8} \Omega \cdot \text{m}) \times (7.5 / 1.9635 \times 10^{-7})$$

First, compute the denominator:

$$7.5 / 1.9635 \times 10^{-7} \approx 3.821 \times 10^7$$

Now, multiply:

$$R \approx 1.68 \times 10^{-8} \times 3.821 \times 10^7$$

$$R \approx (1.68 \times 3.821) \times 10^{-8+7}$$

$$R \approx 6.422 \times 10^{-1} \Omega$$

So,

$$R \approx 0.642 \, \Omega$$

Therefore, the resistance of the copper wire is approximately 0.642 ohms.

Additional Considerations and Practical Aspects

Effect of Temperature

The resistance value calculated assumes room temperature ($\sim 20^\circ\text{C}$). Copper's resistance increases with temperature, approximately by 0.4% per degree Celsius. If the wire operates at higher temperatures, adjust the resistance accordingly:

$$R_{\text{temp}} = R_{20^\circ\text{C}} \times [1 + \alpha \times (T - 20^\circ\text{C})]$$

Where:

- $\alpha \approx 0.0039 / ^\circ\text{C}$ for copper
- T = operating temperature in Celsius

Impact of Manufacturing Tolerances

In real-world scenarios, manufacturing tolerances may cause slight variations in the diameter and resistivity, thus affecting resistance. When precision is essential, measuring the actual dimensions and resistivity for the specific wire batch is recommended.

Importance of Proper Insulation and Safety

While calculating resistance is important for designing and analyzing circuits, safety considerations such as insulation, proper grounding, and adhering to electrical standards must always be observed during installation and usage.

Summary of Key Steps in Resistance Calculation

1. Identify the dimensions of the wire: length and diameter.
2. Convert all measurements to SI units.
3. Calculate the cross-sectional area using the radius.
4. Use the resistivity of copper to find the resistance with the formula $R = \rho \times (L / A)$.
5. Adjust for temperature effects if necessary.

In conclusion, the resistance of a copper wire measuring 7.5 meters in length with a diameter of 0.5 mm is approximately 0.642 ohms at room temperature. This calculation exemplifies how fundamental physics principles and mathematical formulas intersect to provide practical insights in electrical engineering and physics.

Practical Applications of Resistance Calculations

Designing Electrical Circuits

Knowing the resistance helps in selecting appropriate wire gauges to prevent overheating and ensure efficient power transmission.

Material Selection in Engineering

Engineers compare resistivities of different materials to optimize performance based on application requirements.

Physics Education and Experiments

Calculations like these are fundamental in teaching concepts related to Ohm's law, resistivity, and material science.

Additional Resources

- Resistivity of Common Materials: An extensive list for various metals and alloys.
- Electrical Safety Standards: Guidelines for wiring and installation.
- Physics Textbooks: For in-depth understanding of resistance and related concepts.

By mastering the calculation of resistance, students and professionals alike can enhance their understanding of electrical systems, leading to safer and more efficient designs.

Frequently Asked Questions

How do I calculate the resistance of a copper wire with a given length and diameter?

To calculate the resistance, use the formula $R = \rho (L / A)$, where ρ is the resistivity of copper (approximately $1.68 \times 10^{-8} \Omega \cdot m$), L is the length of the wire, and A is the cross-sectional area. First, find A using the diameter, then substitute into the formula.

What is the cross-sectional area of a copper wire with a diameter of 0.5mm?

The cross-sectional area $A = \pi (d/2)^2$. Converting 0.5mm to meters (0.0005m), $A = \pi (0.00025)^2 \approx 1.96 \times 10^{-7} \text{ m}^2$.

What is the resistance of a 7.5m long copper wire with a diameter of 0.5mm?

Using $R = \rho (L / A)$, with $\rho \approx 1.68 \times 10^{-8} \Omega \cdot \text{m}$, $L = 7.5\text{m}$, and $A \approx 1.96 \times 10^{-7} \text{ m}^2$, the resistance $R \approx 1.68 \times 10^{-8} (7.5 / 1.96 \times 10^{-7}) \approx 0.642 \Omega$.

Why is the resistivity of copper important in calculating resistance?

Resistivity (ρ) is a material property that indicates how strongly a material opposes the flow of electric current. It is essential in resistance calculations because it allows us to determine how much a specific material will resist electrical flow based on its dimensions.

How does increasing the diameter of the copper wire affect its resistance?

Increasing the diameter increases the cross-sectional area, which decreases the resistance since R is inversely proportional to A . Therefore, a thicker wire will have lower resistance.

Additional Resources

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Introduction

Copper has long been celebrated as one of the most reliable and efficient conductors of electricity, making it a staple in electrical wiring and numerous electronic applications. Its excellent electrical conductivity, affordability, and ductility have cemented its place in modern infrastructure. When designing electrical systems, understanding the resistance of copper wires is crucial, as resistance impacts efficiency, power loss, and safety.

This detailed investigation aims to analyze a specific scenario: a copper wire that measures 7.5 meters in length with a circular cross-section of diameter 0.5 millimeters. We will explore how to accurately calculate its electrical resistance, delve into the fundamental principles involved, and contextualize the significance of these calculations within real-world electrical engineering practices.

Understanding the Fundamental Concepts

Electrical Resistance and Its Significance

Electrical resistance (R) quantifies how strongly a material opposes the flow of electric current. It is defined by Ohm's Law:

$$R = \frac{V}{I}$$

where V is voltage and I is current. For resistive materials like copper wires, resistance depends on the material's properties and physical dimensions.

High resistance in a wire leads to voltage drops and power dissipation as heat, reducing efficiency and potentially causing safety hazards. Conversely, low resistance indicates efficient conduction.

Factors Influencing Resistance

Several factors influence the resistance of a wire:

- Material resistivity (ρ): intrinsic property; copper's resistivity at room temperature is approximately $1.68 \times 10^{-8} \, \Omega \cdot m$.
- Length (L): longer wires have higher resistance.
- Cross-sectional area (A): thicker wires have lower resistance.
- Temperature: increased temperature generally raises resistivity.

In our investigation, the primary focus will be on the physical dimensions and material properties at standard conditions.

Step 1: Gathering Known Data

Let's compile the given parameters:

Parameter	Value	Units	Notes
Length of wire	7.5	meters	Given length of the copper wire
Diameter of wire	0.5	millimeters	Cross-sectional diameter
Resistivity of copper (ρ)	1.68×10^{-8}	$\Omega \cdot m$	Standard room temperature value

Step 2: Converting Units to SI

To perform calculations accurately, all measurements must be in SI units.

- Diameter: $0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m} = 5 \times 10^{-4} \text{ m}$

- Radius: $r = \frac{d}{2} = 2.5 \times 10^{-4} \text{ m}$

Step 3: Calculating the Cross-Sectional Area

Since the wire has a circular cross-section, the area (A) is given by:

$$A = \pi r^2$$

Substituting the radius:

$$A = \pi \times (2.5 \times 10^{-4})^2$$

Calculating:

$$A = \pi \times 6.25 \times 10^{-8} \approx 3.1416 \times 6.25 \times 10^{-8}$$

$$A \approx 1.9635 \times 10^{-7} \text{ m}^2$$

This represents the cross-sectional area in square meters.

Step 4: Applying the Resistance Formula

The resistance of a uniform wire is calculated using the fundamental formula:

$$R = \frac{\rho L}{A}$$

Where:

- (ρ) = resistivity of copper

- (L) = length of wire

- (A) = cross-sectional area

Plugging in the known values:

$$R = \frac{(1.68 \times 10^{-8}) \times 7.5}{1.9635 \times 10^{-7}}$$

Calculating numerator:

$$(1.68 \times 10^{-8}) \times 7.5 = 1.26 \times 10^{-7}$$

Now, compute resistance:

$$R = \frac{1.26 \times 10^{-7}}{1.9635 \times 10^{-7}} \approx 0.641$$

Result:

$$\boxed{\text{Resistance of the wire} \approx 0.641 \, \Omega}$$

This indicates that under standard conditions, the 7.5-meter copper wire with a 0.5 mm diameter has approximately 0.641 ohms of resistance.

Deep Dive: Factors Affecting Resistance in Practical Scenarios

While the above calculation provides a theoretical resistance value, practical considerations often influence the actual resistance experienced in real-world applications.

Temperature Effects

Copper's resistivity varies with temperature. At higher temperatures, resistivity increases, leading to higher resistance. The temperature coefficient of resistivity for copper is approximately $(0.0039 \, \text{per } ^\circ\text{C})$.

If the wire heats up due to current flow or environmental factors, resistance could increase by:

$$R_T = R_0 [1 + \alpha (T - T_0)]$$

where:

- (R_0) = resistance at reference temperature (T_0) ,

- α = temperature coefficient,

- T = operating temperature.

Understanding this relationship is essential in designing systems for specific current loads and environmental conditions.

Manufacturing Variations

Actual wire resistances can deviate slightly due to manufacturing imperfections such as:

- Variations in diameter or cross-sectional shape
- Impurities and alloying elements
- Surface oxidation or contamination

These factors, though minor, can influence resistance measurements.

Deepening the Analysis: Implications for Electrical Design

Knowing the resistance of the wire is vital for various engineering decisions:

1. Voltage Drop Calculation: Using Ohm's Law, engineers can determine voltage drops across the wire for given current loads to ensure proper voltage levels at load points.
2. Power Loss Estimation: Power dissipated as heat is:

$$P = I^2 R$$

Minimizing resistance reduces heat generation and improves efficiency.

3. Material Selection and Sizing: The resistance calculation guides the choice of wire gauge and length to meet safety standards and performance criteria.

4. Cost and Safety Considerations: Higher resistance can lead to excessive heating, risking fire hazards, especially over longer distances.

Additional Calculations and Considerations

Suppose a scenario where the wire carries a current of 10 A; the power dissipated due to resistance:

$$P = I^2 R = (10)^2 \times 0.641 \approx 100 \times 0.641 = 64.1 \, \text{W}$$

\]

This indicates significant heat generation, which must be managed through appropriate insulation and ventilation.

Conclusion

The calculation of resistance for a specific copper wire—7.5 meters long with a 0.5 mm diameter—demonstrates the practical application of fundamental physics principles in electrical engineering. The derived resistance of approximately 0.641 ohms underscores copper's excellent conductive properties and highlights the importance of precise measurements and considerations in system design.

Understanding these calculations aids engineers and technicians in optimizing wiring configurations, ensuring safety, and improving overall system efficiency. As electrical systems become increasingly complex, such foundational knowledge remains indispensable for advancing reliable and efficient electronic infrastructure.

References

- Resistivity of Copper, National Institute of Standards and Technology (NIST).
- Electrical Resistance and Conductivity, Electrical Engineering Fundamentals.
- Standard Wire Gauge and Material Properties, ASTM Standards.
- Temperature Coefficient of Copper Resistivity, Physics of Materials.

This comprehensive review emphasizes the importance of detailed analysis in electrical resistance calculations, illustrating how fundamental principles translate into practical engineering insights.

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