A Polynomial F(x) Has The Given Zeros Of 7-1, And-3Part A: Using The Factor Theorem, Determine The Polynomial

A Polynomial F(x) Has The Given Zeros Of 7-1, And-3Part A: Using The Factor Theorem, Determine The Polynomial

Understanding how to determine a polynomial from its zeros is a fundamental concept in algebra and polynomial theory. When given specific zeros of a polynomial, such as 7, -1, and -3, we can reconstruct the polynomial using the Factor Theorem, which provides a systematic way to link zeros to factors of the polynomial. This article explores the process of determining the polynomial $F(x)$ with the given zeros, highlighting the use of the Factor Theorem, factoring techniques, and polynomial expansion methods.

Understanding the Zeroes and Their Significance

What Are Zeros of a Polynomial?

Zeros (or roots) of a polynomial are the values of x for which the polynomial evaluates to zero. In other words, if $F(x)$ is a polynomial, then $x = r$ is a zero of $F(x)$ if $F(r) = 0$. These zeros are fundamental in understanding the behavior of the polynomial, including its graph, factors, and solutions to related equations.

Given Zeros: 7, -1, and -3

In this specific problem, the zeros are given as 7, -1, and -3. This means:

- When $x = 7$, $F(7) = 0$
- When $x = -1$, $F(-1) = 0$
- When $x = -3$, $F(-3) = 0$

The goal is to construct a polynomial $F(x)$ that has these zeros.

Using the Factor Theorem to Construct the Polynomial

Introduction to the Factor Theorem

The Factor Theorem states that if r is a zero of a polynomial $F(x)$, then $(x - r)$ is a factor of $F(x)$. Conversely, if $(x - r)$ is a factor of $F(x)$, then r is a zero of the polynomial.

Mathematically:

- If $F(r) = 0$, then $(x - r) \mid F(x)$

This theorem provides a straightforward way to construct a polynomial from known zeros by forming factors corresponding to each zero.

Forming Factors from Zeros

Given the zeros 7, -1, and -3, the corresponding factors are:

- $(x - 7)$
- $(x + 1)$
- $(x + 3)$

The polynomial $F(x)$ will be proportional to the product of these factors:

$$
F(x) = k (x - 7)(x + 1)(x + 3)
$$

where k is a constant coefficient. If the problem does not specify the leading coefficient, the simplest form is to set $k = 1$.

Constructing the Polynomial Step-by-Step

Step 1: Write the Basic Factors

Starting with the factors derived from the zeros:

$$
F(x) = (x - 7)(x + 1)(x + 3)
$$

This expression guarantees that the polynomial has zeros at 7, -1, and -3.

Step 2: Expand the Factors

To express $F(x)$ as a standard polynomial, expand the product:

$$
F(x) = (x - 7)(x + 1)(x + 3)
$$

First, multiply two factors:

$$
(x - 7)(x + 1) = x \cdot x + x \cdot 1 - 7 \cdot x - 7 \cdot 1 = x^2 + x - 7x - 7 = x^2 - 6x - 7
$$

Next, multiply this result by the remaining factor $(x + 3)$:

$$
(x^2 - 6x - 7)(x + 3)
$$

Apply distributive property:

$$
x^2 \times x + x^2 \times 3 - 6x \times x - 6x \times 3 - 7 \times x - 7 \times 3
$$

Calculations:

- $x^2 \times x = x^3$
- $x^2 \times 3 = 3x^2$
- $-6x \times x = -6x^2$
- $-6x \times 3 = -18x$
- $-7 \times x = -7x$
- $-7 \times 3 = -21$

Combine like terms:

$$
x^3 + 3x^2 - 6x^2 - 18x - 7x - 21
$$

Simplify:

$$
x^3 + (3x^2 - 6x^2) + (-18x - 7x) - 21 = x^3 - 3x^2 - 25x - 21
$$

Thus, the polynomial in expanded form is:

\[

F(x) = x^3 - 3x^2 - 25x - 21
\]

Final Polynomial and Its Characteristics

Explicit Form of the Polynomial

The polynomial with zeros at 7, -1, and -3 is:

\[
\boxed{
F(x) = x^3 - 3x^2 - 25x - 21
}
\]

This cubic polynomial satisfies the zeros, as substituting any of the zeros into the polynomial results in zero.

Verifying the Zeros

To verify, substitute each zero:

- For \(x = 7 \):

\[
F(7) = 7^3 - 3(7)^2 - 25(7) - 21 = 343 - 3(49) - 175 - 21 = 343 - 147 - 175 - 21
\]
\[
= (343 - 147) - 175 - 21 = 196 - 175 - 21 = 21 - 21 = 0
\]

- For \(x = -1 \):

\[
F(-1) = (-1)^3 - 3(-1)^2 - 25(-1) - 21 = -1 - 3(1) + 25 - 21 = -1 - 3 + 25 - 21
\]
\[
= (-4) + 25 - 21 = 21 - 21 = 0
\]

- For \(x = -3 \):

\[

$$
F(-3) = (-3)^3 - 3(-3)^2 - 25(-3) - 21 = -27 - 3(9) + 75 - 21 = -27 - 27 + 75 - 21
$$

$$
= (-54) + 75 - 21 = 21 - 21 = 0
$$

All zeros are confirmed.

Additional Considerations

Leading Coefficient and Scaling

While the polynomial derived has a leading coefficient of 1, in some contexts, you might be asked to find a polynomial with a specific leading coefficient. To scale the polynomial accordingly, multiply all coefficients by that constant.

Polynomials of Higher Degree

If the problem involves multiplicities of zeros or additional zeros, the degree of the polynomial increases accordingly. For example, if a zero has multiplicity 2, the corresponding factor appears squared.

Implications of the Polynomial's Roots

Understanding the zeros' implications helps in graphing the polynomial, analyzing its end behavior, and solving related equations.

Conclusion

In conclusion, constructing a polynomial from given zeros involves applying the Factor Theorem and systematic algebraic expansion. For zeros at 7, -1, and -3, the basic factors are $(x - 7)$, $(x + 1)$, and $(x + 3)$. Multiplying these factors yields the cubic polynomial:

$$
\boxed{
F(x) = x^3 - 3x^2 - 25x - 21
}

\]

This polynomial definitively has the specified zeros, confirming the power of the Factor Theorem in polynomial construction. Mastery of these techniques is essential for solving polynomial equations, analyzing their roots, and understanding their graph behavior in algebra and calculus.

Frequently Asked Questions

What is the first step to find the polynomial F(x) with zeros 7 and -1 using the Factor Theorem?

The first step is to express the polynomial as a product of factors corresponding to its zeros, namely (x - 7) and (x + 1).

How does the Factor Theorem help in constructing the polynomial from its zeros?

The Factor Theorem states that if a polynomial has a zero at a point 'a', then (x - a) is a factor of the polynomial. This allows us to write the polynomial as a product of factors based on its zeros.

If the zeros are 7 and -1, what is the basic form of the polynomial F(x)?

The basic form of the polynomial is F(x) = k(x - 7)(x + 1), where k is a non-zero constant.

How do you determine the constant 'k' in the polynomial expression?

You can determine 'k' by using additional information such as a specific point on the polynomial or its value at a certain x-coordinate.

Can the polynomial be quadratic if it has zeros 7 and -1? Why?

Yes, if the zeros are 7 and -1, the polynomial can be quadratic, specifically F(x) = k(x - 7)(x + 1), which is a degree 2 polynomial.

What is the significance of the Polynomial Zero Theorem in this context?

The Polynomial Zero Theorem states that every zero of a polynomial corresponds to a factor of the polynomial, which is fundamental in constructing the polynomial from given zeros.

How does the degree of the polynomial relate to the number

of zeros given?

The degree of the polynomial is at least equal to the number of zeros (counting multiplicities); in this case, with two zeros, the polynomial is at least quadratic.

Additional Resources

A Polynomial F(x) Has The Given Zeros Of 7 - 1, And -3.
Understanding how to determine a polynomial from its zeros is a fundamental skill in algebra and polynomial theory. Whether you're a student preparing for exams or a professional working on mathematical modeling, knowing how to construct a polynomial using the given zeros is essential. In this article, we will explore the process step-by-step, leveraging the Factor Theorem to determine the polynomial \(F(x)\) that has the specified zeros: \(7 - 1\) and \(-3\).

Introduction to Polynomial Zeros and the Factor Theorem

Before diving into the construction of the polynomial, it’s crucial to understand what zeros of a polynomial are and how the Factor Theorem helps in identifying the polynomial from these zeros.

What Are Zeros of a Polynomial?

Zeros (or roots) of a polynomial are values of \(x\) for which the polynomial evaluates to zero. For example, if \(x = a\) makes \(F(a) = 0\), then \(a\) is a zero of \(F(x)\). These zeros can be real or complex, and their multiplicity indicates how many times they appear as roots.

The Factor Theorem

The Factor Theorem provides a direct link between zeros and factors of a polynomial:

- If \(a\) is a zero of \(F(x)\), then \((x - a)\) is a factor of \(F(x)\).
- Conversely, if \((x - a)\) is a factor of \(F(x)\), then \(a\) is a zero of \(F(x)\).

This theorem allows us to construct the polynomial by multiplying out the factors corresponding to each zero.

Clarifying the Given Zeros

The problem states that A Polynomial \(F(x)\) has the given zeros of \(7 - 1\) and \(-3\).

Interpreting the Zero \(7 - 1\)

At first glance, the notation \(7 - 1\) might seem to refer to a single zero or an expression. It's essential to clarify this.

- The expression \(7 - 1\) simplifies to \(6\).
- Therefore, the zeros are 6 and -3.

Key point: The zeros of the polynomial are 6 and -3.

Step-by-Step Guide to Determine $F(x)$

Step 1: Identify the Factors Corresponding to the Zeros

Using the Factor Theorem:

- For zero at 6, the factor is $(x - 6)$.
- For zero at -3, the factor is $(x + 3)$.

Step 2: Form the Polynomial as a Product of Factors

The polynomial $F(x)$ must be divisible by both $(x - 6)$ and $(x + 3)$. Therefore:

$$
F(x) = k \cdot (x - 6)(x + 3)
$$

where k is a non-zero constant. Typically, unless specified, $k = 1$ for the simplest polynomial.

Step 3: Expand the Polynomial

Let's expand the factors:

$$
F(x) = (x - 6)(x + 3)
$$

Using the distributive property (FOIL):

$$
F(x) = x \times x + x \times 3 - 6 \times x - 6 \times 3
$$
$$
F(x) = x^2 + 3x - 6x - 18
$$

Combine like terms:

$$
F(x) = x^2 - 3x - 18
$$

Thus, the polynomial with zeros at 6 and -3 is:

\[
\boxed{
F(x) = x^2 - 3x - 18

}
\]

Additional Considerations

Multiplicity of Zeros

In the above, we assumed each zero has multiplicity 1. If the problem specified multiple roots, we would adjust accordingly:

- For a zero of multiplicity m, include $(x - a)^m$.

Leading Coefficient and General Forms

- If no specific leading coefficient is given, it’s standard to take $k = 1$.
- If you need the polynomial to be monic, set $k = 1$.

Verifying the Zeros

To verify:

$$
F(6) = 6^2 - 3 \times 6 - 18 = 36 - 18 - 18 = 0
$$
$$
F(-3) = (-3)^2 - 3 \times (-3) - 18 = 9 + 9 - 18 = 0
$$

Both zeros check out.

Extending the Analysis: Plotting and Graphical Interpretation

Understanding the behavior of the polynomial can be enhanced by graphing:

- The parabola $F(x) = x^2 - 3x - 18$ opens upward (since the coefficient of x^2 is positive).
- Zero points at $x=6$ and $x=-3$ are the points where the graph crosses the x-axis.
- The y-intercept occurs at $F(0) = -18$.

This graphical insight confirms the algebraic construction.

Summary of the Process

1. Identify the zeros: Given as $7 - 1 = 6$ and -3.
2. Write factors: $(x - 6)$ and $(x + 3)$.
3. Form the polynomial: Multiply factors: $F(x) = (x - 6)(x + 3)$.

4. Expand: Simplify to quadratic form: $F(x) = x^2 - 3x - 18$.
5. Verify: Substitute zeros to confirm roots.

Final Remarks

Constructing a polynomial from its zeros using the Factor Theorem is a straightforward process that combines conceptual understanding with algebraic manipulation. Always ensure clarity in interpreting given data—such as the expression $7 - 1$—to avoid confusion. This approach is fundamental in algebra, calculus, and many applied mathematics fields, laying the groundwork for more advanced polynomial analysis and problem-solving.

If additional zeros or multiplicities are specified, the same principles apply, just with higher powers in the factors. This flexibility makes the process powerful and widely applicable.

References and Further Reading

- Algebra and Trigonometry by Robert F. Blitzer
- Precalculus by James Stewart, Lothar Redlin, and Saleem Watson
- Khan Academy's Polynomial Zeroes and Factor Theorem resources
- Online tools for polynomial graphing and verification

Mastering how to determine a polynomial from its zeros is an essential skill that enhances your understanding of algebraic structures and functions. Practice with different sets of zeros and consider cases with multiplicities to deepen your grasp.

A Polynomial F X Has The Given Zeros Of 7 1 And 3part A Using The Factor Theorem Determine The Polynomial

A Polynomial F X Has The Given Zeros Of 7 1 And 3part A Using The Factor Theorem Determine The Polynomial

Related Articles

- Finish The Sentence: Building On The Ideas Of Advertising CriticSally, Kilbourn States, "Much Of Advertisings
- The Total Force Needed To Drag A Box At Constant Speed Across A Surface With Coefficient Of Kinetic Friction
- Your Teacher Has Asked You To Write A Story For The School Magazine. The Story Must Begin With The Following

Back to Home