A Proton Is Held Fixed At The Origin. A Second Proton Is Placed On The X Axis At X = 4.5 M And Then Released.

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Introduction to Coulombic Interactions Between Protons

Understanding the behavior of charged particles, such as protons, in electrostatic fields is fundamental in physics. When analyzing the dynamics of two protons along a straight line, especially when one is held fixed and the other is released, Coulomb's law provides the essential foundation. This scenario exemplifies how electrostatic forces influence particle motion, energy transfer, and potential interactions at a distance.

This article explores the physical principles, mathematical modeling, and implications of a proton held stationary at the origin with a second proton initially placed at 4.5 meters along the x-axis and subsequently released. We will delve into the forces involved, equations of motion, energy considerations, and potential outcomes, providing a comprehensive understanding suited for students and enthusiasts of classical electromagnetism.

Coulomb's Law and Force Between Two Protons

Fundamental Principles

Coulomb's law describes the electrostatic force between two point charges. For two charges q_1 and q_2, separated by a distance r, the magnitude of the force is given by:

$$
F = \frac{k |q_1 q_2|}{r^2}
$$

where:

- F is the magnitude of the electrostatic force,
- k is Coulomb's constant ($8.9875 \times 10^9 \, \text{N·m}^2/\text{C}^2$),
- q_1 and q_2 are the charges,
- r is the distance between the charges.

Charges of Protons

Protons have a fundamental charge:

\[

$$
q_p = +1.602 \times 10^{-19} \, \text{C}
$$

Since both charges are positive, the force between the two protons is repulsive.

Force Calculation at $r = 4.5\, \text{m}$

Applying Coulomb's law:

$$
F = \frac{k q_p^2}{r^2}
$$

At $r = 4.5\, \text{m}$:

$$
F = \frac{(8.9875 \times 10^9) \times (1.602 \times 10^{-19})^2}{(4.5)^2}
$$

Calculating numerator:

$$
(8.9875 \times 10^9) \times (2.566 \times 10^{-38}) \approx 2.304 \times 10^{-28}
$$

Calculating denominator:

$$
(4.5)^2 = 20.25
$$

Thus,

$$
F \approx \frac{2.304 \times 10^{-28}}{20.25} \approx 1.137 \times 10^{-29} \, \text{N}
$$

This extremely small force illustrates the weak but persistent electrostatic repulsion at macroscopic distances.

Mathematical Modeling of Proton Motion

Initial Conditions and Assumptions

- Proton 1 is fixed at the origin ($x=0$), immovable due to either a high mass or an external constraint.
- Proton 2 is placed at $x = 4.5\, \text{meters}$.
- Proton 2 is released from rest at this position.
- The system is isolated with no external forces other than electrostatic interactions.

- Proton 2 moves along the x-axis under the influence of Coulomb's repulsive force.

Equation of Motion for Proton 2

The motion of the moving proton can be described by Newton's second law:

$$
m_p \frac{d^2 x}{dt^2} = F_x
$$

where:

- m_p is the proton mass ($1.6726 \times 10^{-27} \, \text{kg}$),
- F_x is the x-component of the electrostatic force.

Since both protons are aligned along the x-axis, the force acts directly along this line, and the magnitude simplifies to:

$$
F_x = \frac{k q_p^2}{x^2}
$$

with the direction away from the fixed proton at the origin, meaning the force is repulsive and acts in the positive x-direction when $x > 0$.

Differential Equation of Motion

The equation becomes:

$$
m_p \frac{d^2 x}{dt^2} = \frac{k q_p^2}{x^2}
$$

or

$$
\frac{d^2 x}{dt^2} = \frac{k q_p^2}{m_p x^2}
$$

This nonlinear second-order differential equation governs the motion of proton 2.

Energy Conservation in Proton Dynamics

Total Mechanical Energy

Since no external work is done on the system and electrostatic forces are conservative, the total energy remains constant:

\[

$$
E = \text{Kinetic Energy} + \text{Potential Energy}
$$

Expressed as:

$$
E = \frac{1}{2} m_p v^2 + U(x)
$$

where:

- $v = \frac{dx}{dt}$ is the velocity of proton 2,
- $U(x)$ is the electrostatic potential energy between the two protons.

Potential Energy Expression

The electrostatic potential energy for two point charges:

$$
U(x) = \frac{k q_p^2}{x}
$$

Since the fixed proton is at $x=0$, the potential energy diverges as $x \to 0$, but for $x > 0$, it remains finite.

Initial Conditions

- At $t=0$, $x = 4.5\, \text{m}$,
- $v = 0$ (initially at rest),
- Total energy:

$$
E = U(4.5) = \frac{k q_p^2}{4.5}
$$

Dynamics and Predicted Behavior

Repulsive Force and Proton Acceleration

As proton 2 is released, it experiences a repulsive force pushing it away from the fixed proton at the origin. The initial acceleration:

$$
a(0) = \frac{F}{m_p} = \frac{k q_p^2}{m_p r^2}
$$

which, using the earlier calculations, yields:

\[

$$
a(0) \approx \frac{2.304 \times 10^{-28}}{1.6726 \times 10^{-27} \times (4.5)^2} \approx 0.0064\, \text{m/s}^2
$$

a small but continuous acceleration away from the origin.

Motion Over Time

- Proton 2 accelerates outward, increasing its velocity.
- As it moves further away, the force diminishes ($\propto 1/x^2$), and acceleration decreases.
- The proton continues to move away indefinitely, asymptotically approaching a constant velocity as the force approaches zero at large distances.

Energy Considerations

- The initial potential energy converts into kinetic energy as the proton accelerates.
- Since the potential energy decreases with increasing x, the kinetic energy increases correspondingly.
- Proton 2's velocity at a given position x:

$$
v(x) = \sqrt{\frac{2}{m_p} \left(E - U(x) \right)} = \sqrt{\frac{2}{m_p} \left(\frac{k q_p^2}{4.5} - \frac{k q_p^2}{x} \right)}
$$

which simplifies to:

$$
v(x) = \sqrt{\frac{2k q_p^2}{m_p} \left(\frac{1}{4.5} - \frac{1}{x} \right)}
$$

This expression indicates that as $x \to \infty$, $v(x) \to \sqrt{\frac{2k q_p^2}{m_p} \times \frac{1}{4.5}}$.

Numerical Solution and Simulation

Numerical Methods

Given the nonlinear differential equation, numerical methods such as Runge-Kutta or Euler's method are employed to simulate the proton's trajectory over time.

Sample Computation Steps

1. Initialize $x = 4.5\, \text{m}$, $v=0$.
2. Calculate acceleration a from Coulomb's law.
3. Update velocity: $v_{new} = v + a \times \Delta t$.
4. Update position: $x_{new} = x + v_{new} \times \Delta t$.
5. Repeat for successive time steps until a specified condition is met (e.g., $x \to \infty$ or

simulation time expires).

Expected Results

- Proton 2 accelerates outward from \(4.5\, \text{m} \).
- Its velocity increases gradually, approaching a terminal velocity asymptotically.
- The time taken to reach certain distances can be estimated, illustrating the dynamics governed by electrostatics.

Real-World Implications and Applications

Particle Physics and Coulomb Repulsion

- Understanding proton interactions informs nuclear physics, particle

Frequently Asked Questions

What is the initial setup of the proton problem involving two protons?

A proton is fixed at the origin, and a second proton is placed at x = 4.5 meters on the x-axis and then released.

What forces act on the second proton when it is released near the fixed proton?

The second proton experiences a Coulomb electrostatic repulsion from the fixed proton at the origin.

Will the second proton accelerate towards or away from the fixed proton?

It will accelerate away from the fixed proton due to electrostatic repulsion.

What is the significance of the initial position x = 4.5 m for the second proton?

This initial position determines the initial potential and kinetic energy, influencing the subsequent motion of the second proton.

How can the motion of the second proton be analyzed

theoretically?

By applying Coulomb's law and solving the equations of motion, considering the electrostatic force between the two protons.

What would happen if the second proton is given an initial velocity towards the fixed proton?

It might collide with the fixed proton if the initial velocity is sufficient to overcome the electrostatic repulsion, or it may oscillate or escape depending on the initial conditions.

How does the concept of potential energy relate to the proton's motion in this scenario?

The potential energy is highest when the protons are close and decreases as they move apart, influencing the kinetic energy and speed of the second proton.

What real-world applications or phenomena relate to this two-proton interaction model?

This model helps in understanding atomic interactions, nuclear forces, and behaviors in plasma physics and particle accelerators.

Additional Resources

Proton Dynamics in a Simplified Coulomb System: An In-Depth Analysis

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Introduction

When exploring the fundamental interactions that govern atomic and subatomic systems, the behavior of protons under electrostatic forces offers a window into the core principles of physics. Imagine a scenario where a proton is held fixed at the origin, acting as a stationary charge, and a second proton is placed at a specific point along the x-axis—say, at $x = 4.5\, \text{m}$—and then released. This seemingly simple setup encapsulates rich physics, illustrating concepts like Coulomb's law, potential energy landscapes, force interactions, and the resulting motion of particles governed by classical mechanics.

In this article, we will dissect this scenario thoroughly, adopting an expert perspective akin to a detailed product review. We will explore the initial conditions, analyze the forces involved, derive the equations of motion, examine possible outcomes, and interpret the results in the context of fundamental physics principles. Our goal is to provide an exhaustive, accessible, and insightful discussion suitable for students, educators, and science enthusiasts alike.

Setting the Stage: The Physical System

The Fixed Proton at the Origin

The fixed proton at the origin ($x=0$) acts as the primary source of electrostatic influence. Its role is critical—it creates an electrostatic potential landscape that influences the second proton's motion. Since the fixed proton is immovable, perhaps anchored by an external force or idealized as infinitely massive, it effectively becomes a static point charge in the system.

The Mobile Proton at $x=4.5\, \text{m}$

The second proton is introduced at $x=4.5\, \text{m}$ along the positive x-axis. Initially at rest (assuming no initial velocity), this proton is released and allowed to respond solely to the electrostatic force exerted by the stationary proton. This setup is a classic example of a two-body Coulomb problem, simplified by fixing one particle.

Fundamental Principles and Physical Laws

Coulomb's Law

The core force governing the interaction is Coulomb's law, which states:

$$
F = \frac{k \, |q_1 q_2|}{r^2}
$$

where:

- F is the magnitude of the electrostatic force,

- $k \approx 8.9875 \times 10^9\, \text{N·m}^2/\text{C}^2$ (Coulomb's constant),

- q_1 and q_2 are the charges (for protons, $q = +e \approx 1.602 \times 10^{-19}\, \text{C}$),

- r is the distance between the charges.

Since both particles are protons with positive charges, the force is repulsive, pushing the second proton away from the fixed one.

Mathematical Modeling of the System

Force Expression

The electrostatic force exerted on the mobile proton at position x is:

$$
F(x) = \frac{k e^2}{x^2}
$$

The direction of the force is along the x-axis, repulsive from the fixed proton at the origin, so:

$$
F(x) = - \frac{k e^2}{x^2} \hat{x}
$$

where the negative sign indicates the force points towards decreasing x (away from the origin), depending on the coordinate convention.

Equation of Motion

Applying Newton's second law:

$$
m \frac{d^2 x}{dt^2} = F(x)
$$

where:

- m is the mass of the proton ($\approx 1.673 \times 10^{-27}\, \text{kg}$),

- $x(t)$ is the position of the mobile proton as a function of time.

Thus:

$$
m \frac{d^2 x}{dt^2} = - \frac{k e^2}{x^2}
$$

This is a nonlinear second-order differential equation describing the proton's motion.

Initial Conditions and Their Implications

- Initial Position:

$$
x(0) = 4.5\, \text{m}
$$

- Initial Velocity:

$$
v(0) = 0\, \text{m/s}
$$

The proton starts at rest, a common baseline for analyzing conservative systems.

Analyzing the Dynamics: Qualitative Expectations

Repulsion and Motion

Since both protons carry positive charge, the electrostatic force is repulsive. This implies:

- The second proton will accelerate away from the fixed proton.

- Its velocity will increase as it moves further from the origin, reducing the force magnitude but increasing kinetic energy.

- There is no restoring force or potential well; the system is unbounded.

Potential Energy Perspective

The electrostatic potential energy between the two protons:

$$
U(x) = \frac{k e^2}{x}
$$

- At $x=4.5\, \text{m}$, the potential energy is $U(4.5)$.

- As the proton moves away, $U(x)$ decreases, converting potential energy into kinetic energy.

- The total mechanical energy:

$$
E = K + U
$$

- Initially, the proton's kinetic energy is zero, so:

$$
E = U(4.5)
$$

- As it moves infinitely far away ($x \to \infty$), the potential energy approaches zero, and the proton's kinetic energy reaches a maximum:

$$
K_\text{max} = E = \frac{k e^2}{4.5}
$$

Quantitative Analysis

Calculating Initial Potential Energy

Using known constants:

$$
k \approx 8.9875 \times 10^9\, \text{N·m}^2/\text{C}^2
$$

$$
e \approx 1.602 \times 10^{-19}\, \text{C}
$$

Calculate $U(4.5\, \text{m})$:

$$
U(4.5) = \frac{k e^2}{4.5}
$$

$$
U(4.5) = \frac{(8.9875 \times 10^9) \times (1.602 \times 10^{-19})^2}{4.5}
$$

$$
U(4.5) \approx \frac{(8.9875 \times 10^9) \times (2.566 \times 10^{-38})}{4.5}
$$

$$
U(4.5) \approx \frac{2.304 \times 10^{-28}}{4.5} \approx 5.12 \times 10^{-29}\, \text{J}
$$

This energy is minuscule, but in the realm of particle physics, it is significant enough to influence the motion.

Final Kinetic Energy and Velocity at Infinity

Assuming no other forces:

$$
K_\infty = U(4.5) \approx 5.12 \times 10^{-29}\, \text{J}
$$

The velocity at infinity:

$$
K_\infty = \frac{1}{2} m v^2
$$

$$
v = \sqrt{\frac{2 K_\infty}{m}}
$$

$$
v = \sqrt{\frac{2 \times 5.12 \times 10^{-29}}{1.673 \times 10^{-27}}} \approx \sqrt{\frac{1.024 \times 10^{-28}}{1.673 \times 10^{-27}}}
$$

$$
v \approx \sqrt{0.0612} \approx 0.247\, \text{m/s}
$$

Thus, the proton attains a final velocity of roughly 0.25 m/s as it escapes infinitely far away—a slow but definitive escape velocity in classical terms.

Numerical Simulation and Trajectory Analysis

Given the nonlinear nature of the force, numerical methods, such as Runge-Kutta algorithms, are ideal for simulating the proton's trajectory over time.

Key steps for simulation:

1. Discretize time into small steps (Δt).

2. Update velocity:

$$
v_{n+1} = v_n + \frac{F(x_n)}{m} \Delta t
$$

3. Update position:

$$
x_{n+1} = x_n + v_{n+1} \Delta t
$$

4. Iterate until the proton reaches a desired distance or time.

Expected Results and Physical Interpretation

- Initial phase:

The proton remains near $x=4.5\, \text{m}$, gradually gaining speed as it repels from the fixed proton.

- Acceleration:

As the distance increases, the force diminishes ($F \propto 1/x^2$), but the proton continues to accelerate, approaching the calculated escape velocity.

- Asymptotic behavior:

The proton continues moving outward, slowing down only as it gains kinetic energy, eventually reaching near-constant velocity at large distances.

- Energy conservation:

The total energy remains constant throughout, with potential energy converting into kinetic energy.

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