A Sample Of Ice At 0C Is Added To 100. G Of Water At 33C. The Mixture Is Stirred Gently Until The Temperature

A Sample Of Ice At 0°C Is Added To 100 G Of Water At 33°C. The Mixture Is Stirred Gently Until The Temperature

The process of mixing ice with warm water is a classic example used to illustrate principles of heat transfer, phase changes, and thermodynamics. When a sample of ice at 0°C is introduced into 100 grams of water at 33°C, energy exchange occurs until thermal equilibrium is achieved. Understanding how the temperature of the mixture evolves requires analyzing the heat transfer from the warm water to the ice, the melting of the ice, and the final temperature of the resulting mixture. This article explores the detailed thermal processes involved, the calculations necessary to determine the final temperature, and the underlying physics principles governing the scenario.

Fundamental Concepts Involved

Heat Transfer and Conservation of Energy

In an isolated system with no heat exchange with the environment, the principle of conservation of energy applies. The heat lost by the warm water equals the heat gained by the ice (including the heat needed for melting and warming). This ensures that the total energy remains constant during the process, allowing us to set up equations to solve for the final temperature.

Phase Change: Melting of Ice

Ice at 0°C exists in a solid phase. When it absorbs heat equal to its latent heat of fusion, it melts into water at 0°C. The latent heat of fusion (L_f) for ice is approximately 334 J/g. This phase transition is crucial because it involves energy absorption without a temperature change during melting.

Heat Capacity and Temperature Changes

Once melted, the water formed from the ice will warm from 0°C to the final temperature. The specific heat capacity of water (c) is approximately 4.18 J/g·°C. Similarly, the warm water at 33°C will cool down as it transfers heat to the ice and the newly melted water.

Step-by-Step Analysis of the Process

Step 1: Determine the Heat Transfer from Warm Water

The initial warm water at 33°C contains a certain amount of thermal energy. When cooled to the final temperature T_f, it releases heat given by:

- $Q_{hot} = m_{hot} \times c \times (T_{initial\_hot} - T_{final})$

Where:

- $m_{hot} = 100$ g
- $c = 4.18$ J/g·°C
- $T_{initial\_hot} = 33$°C
- $T_{final} = T_f$ (unknown)

Step 2: Determine the Heat Required to Melt the Ice

The ice at 0°C absorbs heat to melt into water at 0°C:

- $Q_{melt} = m_{ice} \times L_f$

Where:

- m_{ice} = unknown (depends on the final temperature and whether all the ice melts)
- $L_f = 334$ J/g

Step 3: Heat to Warm the Melted Ice to Final Temperature

If all the ice melts, the resulting water at 0°C must be warmed to T_f:

- $Q_{warm\_ice} = m_{ice} \times c \times (T_f - 0°C)$

Step 4: Establish the Energy Balance Equation

The heat lost by the warm water equals the heat gained by the ice (melting plus warming):

- $Q_{hot} = Q_{melt} + Q_{warm\_ice}$

Expressed mathematically:

$100 \times 4.18 \times (33 - T_f) = m_{ice} \times 334 + m_{ice} \times 4.18 \times (T_f - 0)$

Calculating the Final Temperature and Ice Mass

Assuming All the Ice Melts

To proceed, we consider that the entire 0°C ice melts into water at 0°C. This is a common assumption unless the temperature calculations suggest otherwise.

Step 1: Set Up the Equation

Let m_{ice} be the mass of ice in grams.

Then, the energy balance becomes:

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100 × 4.18 × (33 - T_f) = m_ice × 334 + m_ice × 4.18 × (T_f - 0)
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Step 2: Simplify the Equation

Calculating the known quantities:

$$418 \times (33 - T_f) = m_{ice} \times 334 + 4.18 \times m_{ice} \times T_f$$

Expressed as:

$$418 \times (33 - T_f) = m_{ice} \times (334 + 4.18 \times T_f)$$

Step 3: Solve for m_ice in terms of T_f

Rearranged:

$$m_{ice} = [418 \times (33 - T_f)] / [334 + 4.18 \times T_f]$$

Step 4: Find T_f for a Given Ice Mass

Suppose we want to find the temperature after all the ice has melted, and the system reaches equilibrium. The maximum amount of ice that can melt depends on the initial heat available from the warm water.

In practice, for 100 g of water at 33°C, the maximum heat available is:

$$Q_{hot} = 418 \times (33) \approx 13{,}794 \text{ J}$$

This energy can melt a certain amount of ice:

$$m_{ice\_max} = Q_{hot} / 334 \approx 13{,}794 / 334 \approx 41.3 \text{ g}$$

Therefore, if the initial ice mass is less than or equal to approximately 41.3 g, all the ice can melt, and the final temperature will be above 0°C. If more ice is added, some ice will remain unmelted, and the final temperature will be exactly 0°C.

Final Temperature When All Ice Melts

Calculating the Equilibrium Temperature

If all 41.3 g of ice melts, the total mass of water after melting and mixing becomes:

- m_{total} = 100 g + 41.3 g = 141.3 g

The heat released by cooling the warm water from 33°C to T_f is:

$$Q\_hot = 418 \times 33 \approx 13,794 \text{ J}$$

The heat needed to warm the melted ice (now at 0°C) to T_f is:

$$Q\_cold = 41.3 \times 4.18 \times (T\_f - 0) = 41.3 \times 4.18 \times T\_f$$

The final temperature T_f can be found by setting the heat lost by warm water equal to the heat gained by the melted ice:

$$13,794 = 41.3 \times 4.18 \times T\_f$$

$$T\_f = 13,794 / (41.3 \times 4.18) \approx 13,794 / 172.5 \approx 79.9°C$$

Since this temperature exceeds 33°C, which is the initial temperature of the warm water, it indicates an inconsistency because heat cannot flow from colder to hotter. Therefore, the system's temperature will settle somewhere between 0°C and 33°C. To accurately determine T_f, iterative or more precise calculations considering the energy balance are necessary, but it generally falls below 33°C, typically around 0°C to 10°C depending on initial ice mass.

Practical Implications and Real-World Applications

Understanding Thermal Equilibrium in Mixing Processes

- In everyday life, mixing hot and cold water can be used to achieve desired temperature conditions.
- In industrial applications, precise calculations of heat transfer are essential for process control.

Refrigeration and Cooling Systems

- Ice melting is fundamental in cooling systems, where phase change absorbs heat efficiently.
- Designing effective cooling relies on understanding heat transfer during phase changes.

Environmental and Climate Studies

- Studying ice melting helps model climate change effects, especially polar ice cap melting.
- Thermodynamic principles aid in predicting environmental impacts of temperature fluctuations.

Conclusion

The scenario

Frequently Asked Questions

What is the initial temperature of the water in the problem?

The initial temperature of the water is 33°C.

What is the temperature of the ice sample before it is added?

The ice sample is at 0°C.

What is the total mass of water involved in the problem?

The mass of water is 100 grams.

What process occurs when ice at 0°C is added to the water at 33°C and stirred?

Heat exchange occurs, leading to melting of ice and cooling of water until thermal equilibrium is reached.

What is the primary goal of the problem?

To find the final temperature of the mixture after the ice has melted and the system has reached thermal equilibrium.

Which principle is used to solve this problem?

The principle of conservation of energy, specifically heat transfer between the ice and water.

What additional information is needed to calculate the final temperature?

The latent heat of fusion of ice and the specific heat capacity of water are needed to perform the calculation.

Additional Resources

Understanding the Thermal Equilibrium of Ice and Water: A Detailed Analysis

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Introduction

The process of mixing ice at 0°C with water at a higher temperature, such as 33°C, is a classic problem in thermodynamics that illustrates the principles of heat transfer, phase change, and energy conservation. This scenario provides insight into how temperature equilibrates in a system involving both liquid water and solid ice, and it involves a variety of physical concepts, including specific heat capacity, latent heat of fusion, and the thermal properties of water and ice. In this comprehensive review, we will explore the problem step-by-step, starting from fundamental principles, examining the energy exchanges involved, and calculating the final temperature of the mixture.

Fundamental Concepts

Specific Heat Capacity

- Definition: The amount of heat required to raise the temperature of 1 gram of a substance by 1°C.
- For Water: Approximately 4.18 J/g°C.
- For Ice: Approximately 2.09 J/g°C.

Latent Heat of Fusion

- Definition: The heat energy required to convert a solid at its melting point into a liquid without changing temperature.
- For Water: Approximately 334 J/g.

Phase Change and Thermodynamics

- When ice at 0°C is added to warmer water, heat flows from the water to the ice.
- The ice absorbs heat to both warm up from 0°C to its melting point (if not already at melting point) and to melt into liquid water.
- The system reaches thermal equilibrium when the temperatures of water and melted ice stabilize.

The Scenario and Given Data

- Mass of ice (m_ice): Let's denote it as M grams.
- Initial temperature of ice (T_ice_initial): 0°C.
- Mass of water (m_water): 100 grams.
- Initial temperature of water (T_water_initial): 33°C.
- Final temperature of the mixture (T_final): To be determined after mixing and stirring.

Assumptions and Simplifications

- No heat loss to the surroundings, i.e., an isolated system.
- The ice is initially at 0°C and is pure.
- The water is at a uniform temperature initially.
- The ice melts completely before the temperature stabilizes (or partially, depending on the energy exchange).
- The process occurs quasi-statically, allowing the use of equilibrium thermodynamics.

Step-by-Step Analysis

1. Heat transfer considerations

Heat will flow from the warm water to the ice until thermal equilibrium is reached. The key is to analyze the energy exchanges:

- The warm water cools down from 33°C to T_final.
- The ice absorbs heat to:

- Warm from 0°C to melting point (if necessary).
- Melt into liquid water at 0°C.
- Warm further (if T_final > 0°C).

- The final mixture reaches a uniform temperature T_final, which is at or below 33°C but above 0°C.

2. Possible scenarios

- Scenario A: The ice absorbs enough heat to melt completely, and the final temperature is above 0°C.
- Scenario B: The ice absorbs enough heat to melt and raise the temperature above 0°C, but the water cools

down significantly.
- Scenario C: The ice only partially melts, or the temperature reaches exactly 0°C, possibly with some ice remaining.

In most practical cases, the system tends toward Scenario A, with complete melting of ice and a final temperature somewhere between 0°C and 33°C.

Quantitative Calculations

1. Heat lost by water

The water cools from 33°C to T_{final}:

$$Q_{water} = m_{water} \times c_{water} \times (T_{water\ initial} - T_{final})$$

$$Q_{water} = 100 \, \text{g} \times 4.18 \, \text{J/g°C} \times (33°C - T_{final})$$

2. Heat gained by ice

The ice absorbs heat in two stages:

- Heating from 0°C to melting point (which is already at 0°C, so no heating needed unless the ice is at a different temperature).
- Melting into water at 0°C:

$$Q_{melt} = m_{ice} \times L_f$$

- Warming the melted ice (if T_{final} > 0°C):

$$Q_{warm} = m_{ice} \times c_{water} \times T_{final}$$

Total heat absorbed by ice:

$$Q_{ice} = Q_{melt} + Q_{warm}$$

which becomes:

$$Q_{ice} = m_{ice} \times 334 \, \text{J/g} + m_{ice} \times 4.18 \, \text{J/g°C} \times T_{final}$$

Establishing the Energy Balance

Assuming the ice melts completely and the final temperature T_{final} is above 0°C, the heat lost by water equals the heat gained by ice:

$$ Q_{water} = Q_{ice} $$

or

$$ 100 \times 4.18 \times (33 - T_{final}) = m_{ice} \times 334 + m_{ice} \times 4.18 \times T_{final} $$

Rearranged:

$$ 418 \times (33 - T_{final}) = m_{ice} \times 334 + m_{ice} \times 4.18 \times T_{final} $$

Exploring the Effect of Ice Mass

The problem, as posed, does not specify the mass of ice. To develop a meaningful analysis, let's consider different scenarios:

Scenario 1: 10 g of ice

Plugging in:

$$ 418 \times (33 - T_{final}) = 10 \times 334 + 10 \times 4.18 \times T_{final} $$

Simplify:

$$ 13854 - 418 T_{final} = 3340 + 41.8 T_{final} $$

Bring all T_{final} terms to one side:

$$ 13854 - 3340 = 418 T_{final} + 41.8 T_{final} $$

$$ 10514 = 459.8 T_{final} $$

Calculate:

$$ T_{final} = \frac{10514}{459.8} \approx 22.88°C $$

Interpretation:

- The final temperature is approximately 22.9°C.
- Since this is above 0°C, the ice melts completely, and the system reaches equilibrium with all ice melted into water.

Scenario 2: 50 g of ice

Similarly:

$$418 \times (33 - T_{final}) = 50 \times 334 + 50 \times 4.18 \times T_{final}$$

$$13854 - 418 T_{final} = 16700 + 209 T_{final}$$

Bring T_{final} terms to one side:

$$13854 - 16700 = 418 T_{final} + 209 T_{final}$$

$$-2856 = 627 T_{final}$$

$$T_{final} = \frac{-2856}{627} \approx -4.55°C$$

Interpretation:

- The negative temperature indicates that the system cannot reach a temperature above 0°C with this amount of ice.
- The final temperature is at or below 0°C, implying that some ice remains unmelted, and the mixture stabilizes at 0°C with partial melting.

General Conclusions from the Calculations

- For small quantities of ice (e.g., 10 g), the final temperature remains well above 0°C, and all ice melts.
- For larger quantities (e.g., 50 g), the energy available from the warm water is insufficient to melt all ice, and the system stabilizes at 0°C with some ice remaining.

Practical Insights and Real-World Implications

1. Thermal Equilibrium and Mixture Stability

In real-life scenarios, the mixture reaches a stable temperature where no net heat transfer occurs. This temperature depends on the initial quantities and temperatures of the substances involved.

2. Effect of Ice Mass

The amount of ice added relative to the water determines whether the entire ice melts or only a portion:

- Small ice quantities: Complete melting, temperature above 0°C.
- Large ice quantities: Partial melting, final temperature at 0°C, with some ice remaining.

3. Energy Conservation Principles

The calculations reinforce the principle of conservation of energy:

- The heat lost by the warm water equals the heat gained by the ice and the melted water.
- The system's final temperature is dictated by the initial energy content and the phase change energy.

Additional Considerations

1. Heat Loss to Surroundings

In practical situations, some heat exchange with the environment occurs, which would slightly alter the final temperature and melting extent.

2. Non-ideal Conditions

Impurities, container heat capacity, and imperfect mixing can influence the actual outcome.

3. Time Dependence

The calculations assume equilibrium is reached instantaneously, but in reality, the process takes time, and temperature gradients may exist during the transient phase.

Summary

- The temperature of a mixture involving ice at 0°C and water at 33°C

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