An Infinitely Long Wire Carrying A Current I Is Bent At A Right Angle As Shown In The Figure Below. Determine

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Understanding the behavior of current-carrying conductors and their magnetic fields is fundamental in electromagnetism. When an infinitely long wire carries a steady current, it produces a magnetic field surrounding it, governed by Ampère's law and the Biot–Savart law. The scenario of a wire bent at a right angle introduces interesting considerations about the superposition of magnetic fields, the influence of geometry, and the calculation of magnetic field at specific points in space. This article aims to explore the detailed process of determining the magnetic field at a point due to an infinitely long wire bent at a right angle, providing comprehensive insights into the principles involved, calculation techniques, and practical applications.

Understanding the Geometry and Setup

Before delving into calculations, it is essential to clearly understand the physical configuration of the problem.

Description of the Wire and Current Path

- The wire is infinitely long, ensuring that the magnetic field calculations can assume the wire extends without end in both directions along its segments.

- The wire is bent at a right angle, typically forming an 'L' shape with two perpendicular segments:

- One segment runs along the x-axis, extending infinitely in both directions.

- The other segment runs along the y-axis, also extending infinitely in both directions.

Coordinate System and Point of Observation

- Assume a Cartesian coordinate system where the corner of the bend is at the origin (0,0).
- Let the segment along the x-axis be from negative to positive infinity, and similarly for the y-axis.
- The point where the magnetic field is to be determined, denoted as point P, is located at a specific position in the plane, say at (x_0, y_0), often on or near the wire segments.

Assumptions and Simplifications

- The wire is perfectly straight in each segment, with negligible thickness.
- The current I is steady and uniform in magnitude throughout both segments.
- External influences such as magnetic materials or other fields are neglected.
- The point P is sufficiently far from the bend so that the superposition principle applies seamlessly.

Fundamental Principles and Laws Involved

Calculating magnetic fields due to current-carrying conductors relies on core electromagnetism principles.

Ampère's Law

Ampère's law relates the magnetic field around a closed loop to the total current passing through the loop:

$$\oint B \cdot dl = \mu_0 I_{enclosed}$$

However, for infinitely long straight wires, the magnetic field can be directly derived using the Biot–Savart law, which provides a more precise calculation for arbitrary geometries.

Biot–Savart Law

The Biot–Savart law states that the magnetic field B at a point P due to a small element of current dl is:

$dB = (\mu_0 / 4\pi) (I \; dl \times \hat{r}) / r^2$

Where:

- μ_0 is the permeability of free space ($\mu_0 \approx 4\pi \times 10^{-7}$ T·m/A)
- I is the current in the wire
- dl is the differential length element in the wire
- $\hat{r}$ is the unit vector from the current element to point P
- r is the distance from the current element to point P

Superposition Principle

The total magnetic field at point P is the vector sum of the magnetic fields due to each segment of the wire:

$B_{total} = B_x + B_y$

where B_x and B_y are the fields due to the x- and y-segments respectively.

Calculating Magnetic Field Due to a Straight Infinite Wire

Understanding the magnetic field due to an infinite straight wire is crucial, as each segment behaves as such.

Magnetic Field Due to an Infinite Straight Wire

- Magnetic field at a point P located at a perpendicular distance r from an infinitely long straight wire carrying current I is given by:

$B = (\mu_0 I) / (2\pi r)$

- The direction of B is tangential to a circle centered on the wire, following the right-hand rule.

Direction of Magnetic Field

- Applying the right-hand rule: point the thumb in the direction of current I, and curl the fingers around the wire. The direction of B at point P is along the curled fingers.
- For the x-axis segment, the magnetic field at P depends on the position relative to the wire.
- Similarly, for the y-axis segment, the direction is determined via the right-hand rule.

Calculating Magnetic Field at Point P Due to the Bent Wire

The total magnetic field is a vector sum of contributions from each segment; thus, detailed calculations involve integrating the Biot–Savart law along each segment.

Magnetic Field from the Horizontal Segment (Along the x-Axis)

1. Identify the position of the point P relative to the x-axis segment.
2. Determine the perpendicular distance r_x from the x-axis segment to P.
3. Calculate the magnetic field magnitude using $B_x = (\mu_0 I) / (2\pi r_x)$.
4. Determine the direction using the right-hand rule, which results in the field circling around the wire.

Magnetic Field from the Vertical Segment (Along the y-Axis)

1. Identify the position of P relative to the y-axis segment.
2. Determine the perpendicular distance r_y from the y-axis segment to P.
3. Calculate the magnetic field magnitude using $B_y = (\mu_0 I) / (2\pi r_y)$.
4. Ascertain the direction with the right-hand rule.

Superposition and Vector Addition

- Express the magnetic field contributions as vectors, considering their directions (clockwise or counterclockwise around the wire).
- Sum the vectors algebraically to find the resultant magnetic field at point P.

Special Cases and Symmetry Considerations

Certain positions of point P simplify the calculations due to symmetry.

Point P at the Corner (Origin)

- At the junction point where the two segments meet, the magnetic fields from each segment are perpendicular and equal in magnitude if P is at the origin.
- Resultant magnetic field can be found by vector addition, considering directions.

Point P Along the Bisector of the Right Angle

- When P lies along the line bisecting the right angle, the contributions from each segment can be symmetric, simplifying the superposition process.

- The net field may be directed perpendicular to the bisector, with magnitude calculable via Pythagoras.

Far-Field Approximation

- At points far from the bend (large distances), the wire segments appear almost as a combined single current path, and the magnetic field can be approximated accordingly.

Practical Applications and Implications

Understanding the magnetic field due to a bent infinite wire has numerous applications across physics and engineering.

Electromagnetic Device Design

- Designing coils, inductors, and magnetic sensors often involves understanding field distributions around complex conductor geometries.

- Accurate modeling ensures optimal performance and safety.

Magnetic Field Mapping and Measurement

- In experimental physics, mapping magnetic fields around conductors helps in diagnosing system behaviors and validating theoretical models.

- Tools like Hall probes and magnetometers rely on fundamental principles discussed here.

Electromagnetic Compatibility and Interference

- Knowing how magnetic fields emanate from bent conductors aids in mitigating electromagnetic interference in electronic systems.

Advanced Topics and Further Reading

For those interested in extending their understanding

Frequently Asked Questions

What is the magnetic field at the corner where the wire bends due to the current I?

The magnetic field at the corner can be found by superimposing the fields due to each segment of the wire, using the Biot-Savart law or Ampère's law, considering the contributions from both straight sections and their geometrical arrangement.

How does the shape of the wire influence the magnetic field at a point near the bend?

The shape determines the direction and magnitude of the magnetic field; at the bend, the fields from each segment combine vectorially, and the angle affects the resultant field's strength and orientation.

What is the significance of the right-angle bend in calculating the magnetic field?

The right-angle bend creates a specific geometric configuration that affects the superposition of magnetic fields from each segment, requiring careful vector addition to determine the net field at a point nearby.

How can Ampère's law be applied to find the magnetic field in this configuration?

Ampère's law can be used by choosing an appropriate Amperian loop and calculating the line integral of the magnetic field, but for a bent wire, the Biot-Savart law often provides a more direct approach for the field at specific points.

What assumptions are typically made when solving for the magnetic field of an infinitely long wire bent at a right angle?

The assumptions include that the wire is infinitely long, thin, and straight segments are considered ideal; the effects of finite length or thickness are neglected, and the magnetic field is calculated in the idealized static case.

How does the distance from the wire affect the magnetic field at a point in this scenario?

The magnetic field strength decreases with increasing distance from the wire, typically following an inverse proportionality, such as 1/r for an infinitely long straight wire, but the exact dependence can vary near the bend.

What is the direction of the magnetic field around each segment of the wire according to the right-hand rule?

Using the right-hand rule, point your thumb in the direction of current I; your fingers curl around the wire, showing the magnetic field lines circling the wire. At the bend, the fields from each segment combine according to their orientations.

How does the superposition principle help in determining the net magnetic field at a point near the bend?

Superposition allows summing the magnetic field vectors due to each individual segment of the wire at the point of interest, providing the total magnetic field resulting from the entire bent wire configuration.

Additional Resources

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Understanding the magnetic fields generated by current-carrying conductors forms a fundamental part of electromagnetism, a pillar of modern physics and electrical engineering. The scenario of an infinitely long wire bent at a right angle introduces intriguing complexities that challenge our intuition and demand rigorous analysis. In this article, we explore the magnetic field configuration produced by such a wire, elucidate the principles involved, and guide you through the steps necessary to determine the magnetic field at any point in space due to this particular arrangement.

Fundamentals of Magnetic Fields Due to Current-Carrying Wires

Before delving into the specific problem of a bent infinite wire, it’s essential to revisit the foundational concepts that underpin magnetic field calculations in electromagnetism.

The Biot-Savart Law

The Biot-Savart Law is the cornerstone for calculating magnetic fields generated by steady currents. It states that the magnetic field d at a point P due to a small current element I dl is:

$$ \mathbf{dB} = \frac{\mu_0}{4\pi} \frac{I \, d\mathbf{l} \times \mathbf{\hat{r}}}{r^2} $$

where:

- μ_0 is the permeability of free space ($4\pi \times 10^{-7} \, \text{T}\cdot\text{m/A}$),
- $d\mathbf{l}$ is the vector length element of the wire in the direction of current,
- $\mathbf{\hat{r}}$ is the unit vector from the current element to the point P,
- r is the distance from the element to P.

This law enables the calculation of magnetic fields by integrating over the entire current distribution.

Symmetry and the Right-Hand Rule

Symmetry considerations simplify calculations significantly. The right-hand rule helps determine the direction of the magnetic field: if you point your thumb in the direction of current, your fingers curl around the wire in the direction of the magnetic field lines.

The Scenario: A Bent Infinite Wire

Imagine an infinitely long wire that runs along the x-axis from $-\infty$ to 0, then bends at a right angle (a perfect corner) and extends along the y-axis from 0 to $+\infty$. The current I flows through the wire, along the x-axis initial segment, then along the y-axis.

Visually, this configuration resembles an 'L' shape extending infinitely in two perpendicular directions. The goal is to determine the magnetic field $\mathbf{B}$ at an arbitrary point P in space due to this wire.

Decomposition of the Problem: Analyzing Each Segment

Given the geometry, the total magnetic field at any point P can be viewed as the vector sum of the fields due to each segment of the wire:

$$
\mathbf{B}_{\text{total}} = \mathbf{B}_x + \mathbf{B}_y
$$

where:

- $\mathbf{B}_x$ is the field due to the segment along the x-axis,
- $\mathbf{B}_y$ is the field due to the segment along the y-axis.

This approach leverages the principle of superposition, wherein the magnetic field contributions from separate current elements add vectorially.

Coordinate System and Point of Observation

To proceed systematically, define a coordinate system:

- The corner point where the wire bends is at the origin $(0,0,0)$.
- The x-axis extends from $-\infty$ to 0, along the negative x-direction.
- The y-axis extends from 0 to $+\infty$.

Suppose the observation point P is located at (x_0, y_0, z_0). For simplicity and typical analysis, consider the case where P lies in the plane $z=0$ (i.e., in the xy-plane), but the general case can be extended.

Calculating Magnetic Field Due to Each Segment

Segment Along the X-Axis

The x-axis segment runs from $-\infty$ to 0 along the x-direction, with current flowing toward the origin from negative x.

- Current direction: Along the positive x-axis or negative? For a standard configuration, assume current flows from $-\infty$ towards the origin, i.e., from negative infinity to 0 along the x-axis.

- Differential element: $d\mathbf{l}_x = dx \, \hat{\mathbf{x}}$

- Position of element: at $(x, 0, 0)$

- Position vector from element to point P: $\mathbf{r}_x = \mathbf{r}_P - \mathbf{r}_x = (x_0 - x, y_0, z_0)$

Applying Biot-Savart law, the magnetic field contribution:

$$
d\mathbf{B}_x = \frac{\mu_0}{4\pi} \frac{I \, d\mathbf{l}_x \times \mathbf{r}_x}{r_x^3}
$$

where $r_x = |\mathbf{r}_x|$.

Since $d\mathbf{l}_x = dx \hat{\mathbf{x}}$, the cross product becomes:

$$
d\mathbf{l}_x \times \mathbf{r}_x = dx \, \hat{\mathbf{x}} \times (x_0 - x, y_0, z_0)
$$

Calculating the cross product:

$$
\hat{\mathbf{x}} \times (X, Y, Z) = (0, -Z, Y)
$$

Thus,

$$
d\mathbf{B}_x = \frac{\mu_0 I}{4\pi} \frac{dx \, (0, -z_0, y_0)}{r_x^3}
$$

Integrating over x from $-\infty$ to 0:

$$
\mathbf{B}_x = \frac{\mu_0 I}{4\pi} \int_{-\infty}^0 \frac{(0, -z_0, y_0)}{r_x^3} dx
$$

This integral yields the magnetic field contribution from the x-segment at point P.

Segment Along the Y-Axis

Similarly, for the y-axis segment:

- Range: from 0 to $+\infty$

- Differential element: $d\mathbf{l}_y = dy \, \hat{\mathbf{y}}$

- Position of element: at $(0, y, 0)$

- Position vector: $\mathbf{r}_y = (x_0, y_0 - y, z_0)$

Applying Biot-Savart law:

$$
d\mathbf{B}_y = \frac{\mu_0}{4\pi} \frac{I \, d\mathbf{l}_y \times \mathbf{r}_y}{r_y^3}
$$

with

$$
d\mathbf{l}_y \times \mathbf{r}_y = dy \, \hat{\mathbf{y}} \times (x_0, y_0 - y, z_0)
$$

Calculating the cross product:

$$
\hat{\mathbf{y}} \times (X, Y, Z) = (Z, 0, -X)
$$

Therefore:

$$
d\mathbf{B}_y = \frac{\mu_0 I}{4\pi} \frac{dy \, (z_0, 0, -x_0)}{r_y^3}
$$

Integrating over y from 0 to $+\infty$:

$$
\mathbf{B}_y = \frac{\mu_0 I}{4\pi} \int_0^{\infty} \frac{(z_0, 0, -x_0)}{r_y^3} dy
$$

Evaluating the Integrals: Obtaining the Magnetic Field

The key to solving the problem lies in performing these integrals explicitly or recognizing the standard forms. Notably, these integrals resemble those encountered in the magnetic field of semi-infinite wires, which are well-documented and have known solutions.

Standard Result for a Semi-Infinite Wire

For a semi-infinite straight wire carrying current I, the magnetic field at a point perpendicular to its endpoint is:

$$
B = \frac{\mu_0 I}{4\pi r} (\sin \alpha_2 - \sin \alpha_1)
$$

where α_1 and α_2 are the angles between the wire and the line from the observation point to the wire endpoints.

Applying this idea to our bent wire, we observe that each segment behaves like a semi-infinite wire extending from the corner point.

Applying the

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