DETAILS ZILLDIFFEQMODAP11 25.015. Solve The Given Differential Equation By Using An Appropriate Substitution.

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Introduction to Differential Equations and Their Solutions

Differential equations are fundamental in describing various phenomena in science, engineering, and mathematics. They relate a function to its derivatives, providing insights into how systems evolve over time or space. Solving differential equations involves finding functions that satisfy given relationships, often requiring techniques such as substitution, separation of variables, or integration factors.

In many cases, differential equations can be transformed into simpler forms through appropriate substitutions. These substitutions often involve recognizing the structure of the equation—such as a product of functions, a composite function, or a specific pattern—and replacing variables to reduce the equation to a standard form that is easier to solve.

This article focuses on solving a particular differential equation labeled as ZILLDIFFEQMODAP11 25.015 by employing an appropriate substitution. We will explore the step-by-step process, underlying principles, and the mathematical reasoning behind the solution.

Understanding the Given Differential Equation

Before applying any substitution, it is crucial to analyze the structure of the differential equation. Typically, such equations involve derivatives of a function $y(x)$, and their form can suggest suitable methods for solution.

Suppose the differential equation given is of the form:

$$
\frac{dy}{dx} = f(x, y)
$$

or more specifically, perhaps it involves a product or a composite function that hints at substitution.

For example, consider an equation resembling:

$$
\frac{dy}{dx} = y \cdot g(x, y)
$$

or

$$
\frac{dy}{dx} = \frac{f(x)}{y}
$$

which are classic candidates for substitution methods such as the substitution $v = y^n$, $u = y/x$, or similar.

In the absence of the explicit form, we will assume that the differential equation resembles a form that can be simplified via an appropriate substitution, such as:

$$
\frac{dy}{dx} = \frac{A y + B}{C y + D}
$$

or involves nonlinear terms that can be linearized through substitution.

Identifying the Appropriate Substitution

The key to solving the differential equation efficiently lies in identifying the substitution that simplifies the equation. The common strategies include:

1. Substituting for the function y or its powers

- For equations involving y^n, use $v = y^{n}$.
- For equations involving y and y', substitution can linearize the equation.

2. Substituting for a composite function

- If the equation involves y/x, set $u = y/x$ (the homogeneous substitution).
- For equations with exponential or logarithmic functions, substitutions such as $z = \ln y$ may be helpful.

3. Recognizing Bernoulli or homogeneous equations

- For Bernoulli equations of the form $y' + P(x) y = Q(x) y^n$, substitution $v = y^{1-n}$ reduces the equation to linear form.

Once the type of equation is identified, the substitution can be systematically applied.

Step-by-Step Solution Using an Appropriate Substitution

Suppose the differential equation is:

$$
\frac{dy}{dx} = \frac{(a y + b)}{(c y + d)}
$$

where a, b, c, d are constants. This form suggests a substitution to simplify the ratio.

Step 1: Recognize the structure

The right side is a ratio of linear functions in y. A substitution of the form:

$$
u = c y + d
$$

can be beneficial because:

$$
\frac{du}{dx} = c \frac{dy}{dx}
$$

Expressing y in terms of u:

$$
y = \frac{u - d}{c}
$$

and substituting into the original differential equation simplifies the ratio.

Step 2: Rewrite the differential equation in terms of u

Given $y = (u - d)/c$:

$$
\frac{dy}{dx} = \frac{1}{c} \frac{du}{dx}
$$

Substituting into the original:

$$
\frac{1}{c} \frac{du}{dx} = \frac{a \left(\frac{u - d}{c}\right) + b}{u}
$$

Simplify numerator:

$$
\frac{a (u - d)}{c} + b
$$

which leads to:

$$
\frac{1}{c} \frac{du}{dx} = \frac{\frac{a (u - d)}{c} + b}{u}
$$

Multiply both sides by c:

$$
\frac{du}{dx} = c \cdot \frac{\frac{a (u - d)}{c} + b}{u}
$$

Simplify numerator:

$$
\frac{a (u - d)}{c} + b = \frac{a u - a d + b c}{c}
$$

Thus:

$$
\frac{du}{dx} = c \cdot \frac{\frac{a u - a d + b c}{c}}{u} = c \cdot \frac{a u - a d + b c}{c u} = \frac{a u - a d + b c}{u}
$$

Now, the differential equation in u:

$$
\frac{du}{dx} = \frac{a u - a d + b c}{u}
$$

which simplifies to:

$$
\frac{du}{dx} = a - \frac{a d - b c}{u}
$$

Step 3: Rearrange into a separable form

Rewrite as:

$$
\frac{du}{dx} = a - \frac{m}{u}
$$

where $m = a d - b c$ is a constant.

Expressed as:

$$
\frac{du}{dx} = a - \frac{m}{u}
$$

or equivalently:

$$
\frac{u}{a u - m} du = dx
$$

which is obtained by multiplying both sides by u:

$$
u \frac{du}{dx} = a u - m
$$

Rearranged as:

$$
u du = (a u - m) dx
$$

but more straightforwardly, rewrite the initial as:

$$
\frac{du}{dx} = a - \frac{m}{u}
$$

or:

$$
u \frac{du}{dx} = a u - m
$$

which suggests the substitution:

$$
v = u^2

\]

or directly separate variables:

\[
\frac{du}{a - \frac{m}{u}} = dx
\]

Solving the Separated Equation

From:

\[
\frac{du}{a - \frac{m}{u}} = dx
\]

we can write:

\[
\frac{du}{\frac{a u - m}{u}} = dx
\]

which simplifies to:

\[
\frac{u}{a u - m} du = dx
\]

Now, integrate both sides:

\[
\int \frac{u}{a u - m} du = \int dx
\]

Step 4: Perform the integral

Let’s focus on the integral:

\[
I = \int \frac{u}{a u - m} du
\]

Use substitution:

\[

$$
w = a u - m \Rightarrow dw = a du \Rightarrow du = \frac{dw}{a}
$$

Express u in terms of w:

$$
u = \frac{w + m}{a}
$$

Substitute into the integral:

$$
I = \int \frac{\frac{w + m}{a}}{w} \cdot \frac{dw}{a} = \frac{1}{a^2} \int \frac{w + m}{w} dw
$$

Split the integral:

$$
I = \frac{1}{a^2} \int \left(1 + \frac{m}{w}\right) dw = \frac{1}{a^2} \left[\int 1 \, dw + m \int \frac{1}{w} dw \right]
$$

Compute integrals:

$$
I = \frac{1}{a^2} \left[w + m \ln |w| \right] + C
$$

Recall $w = a u - m$, so:

$$
I = \frac{1}{a^2} \left[a u - m + m \ln |a u - m| \right] + C
$$

Therefore,

\[
\int \frac{u}{a u - m} du = \frac{1}{a^2} \left[a u - m + m \ln |a

Frequently Asked Questions

What is the main goal when solving the differential equation using an appropriate substitution?

The main goal is to simplify the differential equation into a form that is easier to integrate, often by reducing it to a separable or linear form through a suitable substitution.

How do you identify the appropriate substitution for a differential equation like ZILLDIFFEQMODAP11 25.015?

You look for patterns such as functions and their derivatives within the equation, often recognizing forms like substitution of a function u = f(x) to reduce the equation's complexity.

Can you provide an example of an appropriate substitution for a typical differential equation?

Yes, for equations involving terms like y' and y^n, substituting u = y^{1-n} can often transform the equation into a linear differential equation.

What are common types of substitutions used in solving differential equations?

Common substitutions include u = y, u = xy', or u = y^n, depending on the structure of the differential equation, to facilitate easier integration.

Why is choosing the right substitution important in solving differential equations like ZILLDIFFEQMODAP11 25.015?

Choosing the correct substitution simplifies the differential equation, making it possible to integrate directly and find the explicit solution efficiently.

What steps should be followed after choosing an appropriate substitution in solving the differential equation?

After choosing the substitution, differentiate it as needed, rewrite the original equation in terms of the new variable, solve the resulting simpler differential equation, and then back-substitute to find the original variables.

How can I verify that my solution to the differential equation is correct?

You can verify by differentiating your solution and substituting it back into the original differential equation to check if it satisfies the equation.

Are there any specific techniques recommended for solving differential equations like 25.015 using substitutions?

Yes, techniques such as substitution of variables, reduction of order, or recognizing homogeneous or exact equations are recommended, depending on the structure of the specific differential equation.

Additional Resources

DETAILS ZILLDIFFEQMODAP11 25.015. Solve The Given Differential Equation By Using An Appropriate Substitution.

Introduction

In the realm of differential equations, solving complex equations often requires ingenuity and strategic substitution techniques. The problem labeled ZILLDIFFEQMODAP11 25.015 is a representative example of such a challenge, demanding a deep understanding of substitution methods to facilitate an elegant solution. This article aims to dissect the problem thoroughly, explore the methodologies involved, and present a comprehensive guide to solving the differential equation using an appropriate substitution.

Context and Significance of the Problem

Differential equations are fundamental in modeling various phenomena across science, engineering, economics, and more. They describe how quantities change concerning each other and are central to understanding dynamic systems.

The specific problem ZILLDIFFEQMODAP11 25.015 is part of a collection designed to test and enhance problem-solving skills in differential equations, especially focusing on substitution strategies. Mastery of these techniques is crucial because:

- Many differential equations are not directly integrable.
- Substitutions can reduce a complex differential equation to a standard form.
- They often reveal underlying structures or symmetries in the equations.

The Differential Equation: Statement and Initial Analysis

While the exact form of ZILLDIFFEQMODAP11 25.015 is not explicitly provided here, typical problems of this nature involve equations that are not straightforward to integrate directly, such as:

\[\frac{dy}{dx} = f(x, y) \]

or more specifically,

\[\text{An equation involving nonlinear terms, powers, or products of } y \text{ and } x. \]

The goal is to identify a substitution that simplifies the form, perhaps transforming the differential equation into a linear, separable, or homogeneous form.

Step 1: Recognizing the Structure of the Equation

Key observations:

- Is the equation homogeneous? (i.e., can it be expressed as a function of y/x)
- Does it involve powers or products of y and x?
- Is it reducible to a Bernoulli or Bernoulli-like form?
- Are there terms suggesting substitution like $u = y/x$, $u = xy$, or $u = y^n$?

Step 2: Identifying the Appropriate Substitution

Based on common patterns, the following substitutions are often effective:

Pattern	Substitution	Rationale
Homogeneous equations	$u = \frac{y}{x}$	Simplifies equations involving y and x as ratios
Equations with y^n	$u = y^{1-n}$	Converts nonlinear to linear form for Bernoulli equations
Products xy or $x^m y^n$	$u = xy$ or $u = y/x$	Exploits symmetry or homogeneity

For the current problem, suppose the differential equation resembles:

$$\frac{dy}{dx} = \frac{y^2}{x} + y$$

This form suggests that the substitution $u = y/x$ could be effective because of the combination of y^2 and y.

Step 3: Implementing the Substitution

Suppose the equation is:

$$\frac{dy}{dx} = \frac{y^2}{x} + y$$

Substitution:

Let

$$u = \frac{y}{x} \Rightarrow y = ux$$

Differentiating both sides with respect to x:

$$\frac{dy}{dx} = u + x \frac{du}{dx}$$

Substitute back into the original differential equation:

$$u + x \frac{du}{dx} = \frac{(ux)^2}{x} + ux$$

Simplify:

$$ u + x \frac{du}{dx} = u^2 x + u x $$

Dividing through by x:

$$ \frac{u}{x} + \frac{du}{dx} = u^2 + u $$

Rearranged:

$$ \frac{du}{dx} + \frac{u}{x} = u^2 + u $$

which is a Bernoulli differential equation of the form:

$$ \frac{du}{dx} + P(x) u = Q(x) u^n $$

with:

- $P(x) = \frac{1}{x}$
- $Q(x) = 1$
- $n = 2$

Step 4: Solving the Transformed Differential Equation

Bernoulli Equation Approach:

For

$$ \frac{du}{dx} + P(x) u = Q(x) u^n $$

the standard substitution:

$$ v = u^{1-n} $$

transforms it into a linear differential equation. Here $n=2$, so:

$$ v = u^{1-2} = u^{-1} $$

Differentiating:

$$ v = u^{-1} \Rightarrow \frac{dv}{dx} = -u^{-2} \frac{du}{dx} $$

Rearranged:

$$ \frac{du}{dx} = -u^2 \frac{dv}{dx} $$

Substitute into the Bernoulli form:

$$ -u^2 \frac{dv}{dx} + \frac{u}{x} = u^2 + u $$

Divide through by u^2:

$$ -\frac{dv}{dx} + \frac{1}{x u} = 1 + \frac{1}{u} $$

But since $v = u^{-1}$, then:

$$ \frac{1}{u} = v $$

and

$$ \frac{1}{x u} = \frac{v}{x} $$

So, the equation becomes:

$$ -\frac{dv}{dx} + \frac{v}{x} = 1 + v $$

Rearranged:

$$ -\frac{dv}{dx} + \frac{v}{x} - v = 1 $$

or

$$ -\frac{dv}{dx} + \left(\frac{1}{x} - 1 \right) v = 1 $$

Multiply through by -1:

$$ \frac{dv}{dx} - \left(\frac{1}{x} - 1 \right) v = -1 $$

which simplifies to:

$$ \frac{dv}{dx} - \left(\frac{1}{x} - 1 \right) v = -1 $$

This is now a linear differential equation in v.

Step 5: Solving the Linear Equation in v

The standard form:

$$ \frac{dv}{dx} + P(x) v = Q(x) $$

where:

- $P(x) = -\left(\frac{1}{x} - 1 \right) = -\frac{1}{x} + 1$
- $Q(x) = -1$

The integrating factor:

$$ \mu(x) = e^{\int P(x) dx} $$

Calculate:

$$ \int P(x) dx = \int \left(-\frac{1}{x} + 1 \right) dx = -\ln |x| + x + C $$

Ignoring the constant as it cancels in the integrating factor:

$$ \mu(x) = e^{ -\ln |x| + x } = e^{x} \cdot e^{ -\ln |x| } = e^{x} \cdot \frac{1}{|x|} $$

Assuming $x > 0$:

$$ \mu(x) = \frac{e^{x}}{x} $$

Multiply the entire linear equation by $\mu(x)$:

$$ \frac{e^{x}}{x} \frac{dv}{dx} + \frac{e^{x}}{x} P(x) v = \frac{e^{x}}{x} Q(x) $$

But by the integrating factor property:

$$ \frac{d}{dx} \left(\mu(x) v \right) = \mu(x) Q(x) $$

Hence,

$$ \frac{d}{dx} \left(\frac{e^{x}}{x} v \right) = - \frac{e^{x}}{x} $$

Integrate both sides:

$$ \frac{e^{x}}{x} v = - \int \frac{e^{x}}{x} dx + C $$

The integral $\int \frac{e^{x}}{x} dx$ is known as the exponential integral function $\text{Ei}(x)$ and does not have an elementary antiderivative, but for the purpose of the solution, we can express the solution in terms of it.

Step 6: Back-Substitution and Final Solution

Recall:

$$ v = u^{-1} $$

and

$$ u = \frac{y}{x} $$

Thus, the solution for y involves:

$$ u = \frac{1}{v} $$

and the relations derived above lead to an implicit solution involving exponential integr

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