Determine Whether The Series Is Convergent Or Divergent. Sigma N=1 7 Sin 1/n Part 1 Of 4 The Limit Comparison

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When analyzing infinite series in calculus, one of the essential tasks is to determine whether a given series converges or diverges. This process involves applying various convergence tests, understanding the behavior of the series' terms, and comparing the series to known benchmark series. In this article, we focus on the series $\sum_{n=1}^{\infty} 7 \sin \frac{1}{n}$, exploring its convergence properties through the Limit Comparison Test and other analytical tools. This detailed examination will help clarify whether the series converges or diverges, providing insight into how such problems are approached in advanced calculus.

Understanding the Series $\sum_{n=1}^{\infty} 7 \sin \frac{1}{n}$

Before diving into convergence tests, it's crucial to understand the behavior of the individual terms in the series.

Series Terms Analysis

- The general term of the series is $a_n = 7 \sin \frac{1}{n}$.
- As $n \to \infty$, $\frac{1}{n} \to 0$.
- The sine function near zero can be approximated as $\sin x \approx x$ for small x.

Applying this approximation:

$$
a_n = 7 \sin \frac{1}{n} \approx 7 \times \frac{1}{n} = \frac{7}{n}
$$

This suggests that for large n, the terms behave similarly to $\frac{7}{n}$, which is a constant multiple of the harmonic series $\sum \frac{1}{n}$.

Applying the Limit Comparison Test

The Limit Comparison Test is a powerful tool for determining the convergence or divergence of series with positive terms that behave similarly to a known benchmark series.

Statement of the Limit Comparison Test

- Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.
- If $\lim_{n \to \infty} \frac{a_n}{b_n} = c$, where c is a finite positive number, then either both series converge or both diverge.

Choosing the Benchmark Series

Given the approximation $a_n \sim \frac{7}{n}$, it is natural to compare the series to $\sum_{n=1}^\infty \frac{1}{n}$, the harmonic series, known to diverge.

Calculating the Limit

Evaluate:

$$
\lim_{n \to \infty} \frac{7 \sin \frac{1}{n}}{\frac{1}{n}}
$$

Using the small-angle approximation $\sin x \approx x$:

$$
\frac{7 \sin \frac{1}{n}}{\frac{1}{n}} \to 7 \times \lim_{n \to \infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} = 7 \times 1 = 7
$$

Since the limit is a finite positive number (7), the Limit Comparison Test applies.

Conclusion from the Limit Comparison Test

Because:

- $\sum_{n=1}^\infty \frac{1}{n}$ diverges,
- and the limit $\lim_{n \to \infty} \frac{7 \sin \frac{1}{n}}{\frac{1}{n}} = 7 \neq 0$,

it follows that
$\sum_{n=1}^\infty 7 \sin \frac{1}{n}$ also diverges.

Additional Convergence Tests and Insights

While the Limit Comparison Test provides a clear conclusion, it is instructive to explore other methods and the underlying reasoning.

Comparison to the Harmonic Series

- The harmonic series $\sum \frac{1}{n}$ is well-known to diverge.
- Since $a_n \sim \frac{7}{n}$, the series behaves similarly to a divergent harmonic series.
- Therefore, the divergence of the harmonic series implies the divergence of our series.

Behavior of $\sin \frac{1}{n}$ as $n \to \infty$

- The sine function’s small-angle approximation indicates the terms decrease to zero.
- However, the fact that $\sin \frac{1}{n}$ behaves like $\frac{1}{n}$ for large n is crucial; the decay rate isn’t fast enough to produce convergence.

Test for Absolute Convergence

- The absolute value of the terms is $|7 \sin \frac{1}{n}|$.
- Since $\sin \frac{1}{n} \to 0$, the terms are bounded.
- But $\sum |7 \sin \frac{1}{n}| \sim 7 \sum \frac{1}{n}$, which diverges.
- Therefore, the series does not converge absolutely.

Summary of Key Points

- The terms $a_n = 7 \sin \frac{1}{n}$ tend to zero as $n \to \infty$.
- The series behaves similarly to $\frac{7}{n}$, which diverges.
- The Limit Comparison Test confirms divergence by comparing with the harmonic series.
- The series does not converge absolutely or conditionally.

Final Verdict: Divergence of the Series $\sum_{n=1}^\infty 7 \sin \frac{1}{n}$

Based on the analysis above, the infinite series $\sum_{n=1}^{\infty} 7 \sin \frac{1}{n}$ diverges. Despite the individual terms approaching zero, their decay rate is comparable to that of the harmonic series, which is known to diverge. The Limit Comparison Test offers a definitive conclusion, confirming the series' divergence.

Implications for Mathematical Analysis and Calculus Students

Understanding the convergence or divergence of series like $\sum_{n=1}^\infty 7 \sin \frac{1}{n}$ is fundamental in advanced calculus. Recognizing the importance of comparing series to known benchmarks and applying appropriate convergence tests are key skills for students and mathematicians alike. This problem exemplifies how seemingly complex series can be analyzed effectively through the Limit Comparison Test and foundational limit approximations.

SEO Optimization and Keywords

- Series convergence and divergence
- Limit Comparison Test in calculus
- $\sum_{n=1}^\infty 7 \sin \frac{1}{n}$ analysis
- Infinite series comparison tests
- Convergence of trigonometric series
- Harmonic series divergence
- Small-angle approximation sine
- Absolute convergence vs conditional convergence
- Calculus series tests
- Analyzing infinite series behavior

Conclusion

In conclusion, the series $\sum_{n=1}^\infty 7 \sin \frac{1}{n}$ diverges. The key to this determination lies in understanding the behavior of the terms for large n, applying the Limit Comparison Test with the harmonic series, and recognizing that the decay rate of the terms is insufficient for convergence. Mastery of these concepts and techniques is vital for anyone studying infinite series in calculus, and this example serves as an instructive illustration of the analytical process involved in convergence testing.

If you are exploring series convergence tests or need further assistance with similar problems, consider reviewing additional resources on comparison tests, the behavior of trigonometric functions in series, and the properties of divergent series in calculus.

Frequently Asked Questions

What is the main goal when determining whether the series $\sum_{n=1}^\infty 7 \sin \frac{1}{n}$ converges or diverges?

The main goal is to analyze the behavior of the series as $n \to \infty$ to see if the sum approaches a finite value (converges) or diverges to infinity (diverges).

How does the Limit Comparison Test help in assessing the convergence of $\sum_{n=1}^\infty 7 \sin \frac{1}{n}$?

The Limit Comparison Test compares the given series to a known benchmark series, such as $\sum \frac{1}{n}$, by evaluating the limit of their ratio; if the limit is finite and positive, both series share the same convergence behavior.

What is the behavior of $\sin \frac{1}{n}$ as $n \to \infty$?

As $n \to \infty$, $\frac{1}{n} \to 0$, and since $\sin x \sim x$ for small x, $\sin \frac{1}{n} \sim \frac{1}{n}$.

Based on the behavior of $\sin \frac{1}{n}$, what comparison series can be used to determine convergence?

Since $\sin \frac{1}{n} \sim \frac{1}{n}$, the series can be compared to the harmonic series $\sum \frac{1}{n}$, which diverges.

What is the limit of $\frac{7 \sin \frac{1}{n}}{1/n}$ as $n \to \infty$?

The limit is 7, since $\lim_{n \to \infty} \frac{7 \sin \frac{1}{n}}{1/n} = 7 \times \lim_{x \to 0} \frac{ \sin x }{ x } = 7 \times 1 = 7$.

What conclusion can be drawn from the limit being a finite positive number in the Limit Comparison Test?

Since the limit is a finite positive number (7), and the comparison series (harmonic series) diverges, the original series also diverges.

Is the series $\sum_{n=1}^\infty 7 \sin \frac{1}{n}$ convergent or divergent?

The series diverges because it behaves similarly to the harmonic series, which is divergent.

Can the series $\sum_{n=1}^\infty 7 \sin \frac{1}{n}$ be considered an alternating series?

No, because $\sin \frac{1}{n}$ is positive for all $n \geq 1$, so the series is not alternating.

What is the significance of the series' divergence for understanding its behavior?

The divergence indicates that the sum does not settle to a finite limit, which is crucial in understanding the nature of the series and its divergence properties.

Additional Resources

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Introduction

Mathematics, especially calculus and infinite series, forms the backbone of numerous scientific disciplines, from physics to economics. Among the many intriguing topics in this realm is the analysis of the convergence or divergence of infinite series. Determining whether a series converges or diverges is fundamental because it informs us about the behavior of infinite sums, their limits, and their applicability in modeling real-world phenomena.

In this comprehensive review, we focus on the series:

$$
\sum_{n=1}^{\infty} 7 \sin \left(\frac{1}{n}\right)
$$

which we will refer to as "the series" throughout this discussion. Our goal is to thoroughly analyze whether this series converges or diverges using various techniques, with particular emphasis on the Limit Comparison Test – a powerful tool in the mathematician's arsenal. This investigation will be comprehensive, detailed, and structured to facilitate a deep understanding of the problem.

Background and Mathematical Context

Before diving into the specific series, it's essential to understand some key concepts that underpin the analysis.

Infinite Series and Convergence

An infinite series is a sum of infinitely many terms:

$$
\sum_{n=1}^{\infty} a_n
$$

where a_n is the n-th term of the series. The series is said to converge if the sequence of partial sums

$$
S_N = \sum_{n=1}^{N} a_n
$$

approaches a finite limit as $N \to \infty$. If no such finite limit exists, the series diverges.

Common Tests for Convergence

- Comparison Test: Compare a_n to a known convergent or divergent series.
- Limit Comparison Test: Based on the limit of the ratio a_n / b_n as $n \to \infty$.
- Integral Test: Use integrals to analyze the sum.
- Alternating Series Test: For series with alternating signs.

In our case, the Limit Comparison Test is particularly promising because the terms involve $\sin(1/n)$, which suggests a relation to simpler known series.

Understanding the Series: The Behavior of $7 \sin(1/n)$

The series under consideration is:

$$
\sum_{n=1}^{\infty} 7 \sin \left(\frac{1}{n}\right)
$$

Let's analyze the behavior of the individual terms.

Asymptotic Behavior of $\sin(1/n)$

Recall the Taylor series expansion of $\sin x$ around 0:

$$
\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots
$$

Applying this to $x = 1/n$:

$$
\sin \left(\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{6 n^3} + \text{higher-order terms}
$$

Multiplying by 7:

$$
7 \sin \left(\frac{1}{n}\right) \approx \frac{7}{n} - \frac{7}{6 n^3} + \cdots
$$

This approximation is valid for large n, which is critical in analyzing the convergence of the series.

Analysis of the Series: Convergence or Divergence?

The key insight is comparing the series to known convergent or divergent series.

Leading Term Approximation

Given the approximation:

$$
a_n = 7 \sin \left(\frac{1}{n}\right) \sim \frac{7}{n}
$$

for large n. Since the harmonic series:

$$
\sum_{n=1}^{\infty} \frac{1}{n}
$$

diverges, this suggests that the original series might also diverge, but further analysis is necessary to confirm.

Limit Comparison Test: Methodology and Application

The Limit Comparison Test states:

Given two series $\sum a_n$ and $\sum b_n$ with positive terms, if

$$
\lim_{n \to \infty} \frac{a_n}{b_n} = c
$$

where c is a finite positive number, then either both series converge or both diverge.

Choosing the Comparator Series

Based on the asymptotic behavior, a natural choice for b_n is:

$$
b_n = \frac{1}{n}
$$

since $a_n \sim \frac{7}{n}$.

Computing the Limit

Calculate:

$$
L = \lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{7 \sin (1/n)}{1/n}
$$

Using the Taylor expansion:

$$
\sin \left(\frac{1}{n}\right) \approx \frac{1}{n} - \frac{1}{6 n^3}
$$

then:

$$
\frac{7 \sin (1/n)}{1/n} \approx 7 \times \frac{\frac{1}{n} - \frac{1}{6 n^3}}{\frac{1}{n}} = 7 \times \left(1 - \frac{1}{6 n^2}\right)
$$

As $n \to \infty$, the term $\frac{1}{6 n^2} \to 0$, so:

$$
L = \lim_{n \to \infty} 7 \times \left(1 - \frac{1}{6 n^2}\right) = 7 \times 1 = 7
$$

Since $L = 7$, a finite positive number, the Limit Comparison Test applies.

Conclusion from the Limit Comparison Test

Because:

- $\sum_{n=1}^\infty \frac{1}{n}$ diverges (harmonic series),
- and the limit $L = 7$ (finite and positive),

we deduce that:

$$
\sum_{n=1}^\infty 7 \sin \left(\frac{1}{n}\right)
$$

also diverges.

Additional Validation: Comparison with the Harmonic Series

To reinforce this conclusion, consider the comparison:

- For sufficiently large n, $\sin(1/n) \geq \frac{1}{2n}$ (since $\sin x \geq \frac{2}{\pi} x$ for small x), and $2/\pi \approx 0.6366$), which is greater than $1/2$).

Thus, for large n:

$$
7 \sin \left(\frac{1}{n}\right) \geq 7 \times \frac{2}{\pi} \times \frac{1}{n} \approx \frac{14}{\pi n}
$$

which is proportional to $1/n$. Since the harmonic series diverges, and our terms are asymptotically comparable to $1/n$, the series diverges as well.

Implications and Broader Context

This analysis confirms that the series

$$
\sum_{n=1}^{\infty} 7 \sin \left(\frac{1}{n}\right)
$$

diverges, despite the fact that individual terms tend to zero as $n \to \infty$. This underscores an important principle: terms tending to zero is necessary but not sufficient for convergence.

The use of the Limit Comparison Test provides a robust method for analyzing such series, especially when the terms involve transcendental functions like sine and their asymptotic behaviors.

Additional Considerations and Related Series

While this specific series diverges, similar series involving functions with different asymptotic behaviors can converge. For example:

- Series with terms like $1/n^p$ for $p > 1$ converge (p-series).
- Series involving oscillatory functions like $\sin n$ or $\cos n$ may require specialized tests due to oscillations.

Understanding these distinctions is crucial for advanced mathematical analysis and applications.

Final Remarks

The detailed examination of the series $\sum_{n=1}^\infty 7 \sin(1/n)$ reveals that it diverges, primarily because its terms behave like $1/n$ for large n. The Limit Comparison Test, leveraging the asymptotic expansion of sine, provides a clear and rigorous pathway to this conclusion.

This investigation exemplifies the importance of asymptotic analysis, comparison tests, and the nuanced understanding of series convergence in advanced mathematics. It also serves as a reminder that intuition based solely on the terms tending to zero must be supplemented with formal analytical tools to ascertain the true nature of infinite series.

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