# Find A Vector Equation For The Line Of Intersection Of The Planes 2y7x+3z=26 And X2z=13 R(t)= With [infinity]

**Find A Vector Equation For The Line Of Intersection Of The Planes 2y7x+3z=26 And X2z=13 R(t)= With [infinity]**

Understanding how to find the line of intersection between two planes is a fundamental concept in vector calculus and analytic geometry. When two planes intersect, they do so along a straight line, and deriving the vector equation of this line is essential for visualizing and solving numerous geometric and algebraic problems. This article provides a comprehensive guide to finding the vector equation of the intersection line between the planes $2y + 7x + 3z = 26$ and $x + 2z = 13$. We will explore step-by-step methods, key concepts, and practical tips to master this important skill.

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## Understanding the Basics: Intersection of Two Planes

Before diving into the specific problem, it’s crucial to understand the fundamental principles of plane intersections in three-dimensional space.

### Planes in Three-Dimensional Space

- A plane in three-dimensional space can be represented by a linear equation of the form $Ax + By + Cz + D = 0$.
- These equations describe flat, two-dimensional surfaces extending infinitely in space.

### Line of Intersection of Two Planes

- When two planes intersect, they do so along a line unless they are parallel or coincident.
- The intersection line can be represented as a vector equation, which captures both the direction of the line and a point through which it passes.

### Why Find the Vector Equation?

- It provides a parametric form of the line, making it easier to visualize, analyze, and compute points on the line.

- It is useful in applications such as computer graphics, engineering design, and physics.

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# Given Planes: Equations and Interpretations

The problem involves two specific planes:

1. $2y + 7x + 3z = 26$
2. $x + 2z = 13$

Let's analyze and interpret these equations.

## Plane 1: $2y + 7x + 3z = 26$

- Standard form of a plane with coefficients $A=7$, $B=2$, $C=3$, and $D=-26$.
- Normal vector of plane 1: $\mathbf{n}_1 = \langle 7, 2, 3 \rangle$.

## Plane 2: $x + 2z = 13$

- Can be rewritten as $x + 0 \cdot y + 2z = 13$.
- Normal vector of plane 2: $\mathbf{n}_2 = \langle 1, 0, 2 \rangle$.

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# Step-by-Step Method to Find the Line of Intersection

To find the vector equation of the line where these two planes intersect, follow these steps:

## Step 1: Find a Point on the Line

- Select a parameter or arbitrary value for one variable.
- Substitute into both plane equations to solve for remaining variables.

## Step 2: Find the Direction Vector of the Line

- The direction vector $\mathbf{d}$ is perpendicular to both plane normals.

- Compute $\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2$.

### Step 3: Write the Vector Equation of the Line

- Use the point found in Step 1 and the direction vector.
- The general form: $\mathbf{R}(t) = \mathbf{r}_0 + t \mathbf{d}$.

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## Calculating the Direction Vector: Cross Product of Normals

The cross product of the two normal vectors gives the direction vector of the intersection line.

### Compute $\mathbf{n}_1 \times \mathbf{n}_2$:

$$
\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
7 & 2 & 3 \\
1 & 0 & 2 \\
\end{vmatrix}
$$

Calculating the determinant:

$$
\mathbf{d} = \mathbf{i}(2 \times 2 - 3 \times 0) - \mathbf{j}(7 \times 2 - 3 \times 1) + \mathbf{k}(7 \times 0 - 2 \times 1)
$$
$$
= \mathbf{i}(4 - 0) - \mathbf{j}(14 - 3) + \mathbf{k}(0 - 2)
$$
$$
= 4 \mathbf{i} - 11 \mathbf{j} - 2 \mathbf{k}
$$

So, the direction vector:

$$
\boxed{
\mathbf{d} = \langle 4, -11, -2 \rangle
}
$$

---

# Finding a Specific Point on the Line

Next, select a value for one variable to find a corresponding point on the line.

Method 1: Let $z = t$ and find $x, y$:

Suppose $z = 0$:

- From the second plane:

$$
x + 2(0) = 13 \Rightarrow x = 13
$$

- From the first plane:

$$
2y + 7x + 3z = 26 \Rightarrow 2y + 7(13) + 0 = 26
$$
$$
2y + 91 = 26 \Rightarrow 2y = -65 \Rightarrow y = -32.5
$$

Thus, a point on the line is:

$$
\mathbf{r}_0 = (13, -32.5, 0)
$$

Method 2: Alternatively, choose $y = 0$:

- From the first plane:

$$
2(0) + 7x + 3z = 26 \Rightarrow 7x + 3z = 26
$$

- From the second plane:

$$
x + 2z = 13
$$

Express $x$ from the second:

$$

$$
x = 13 - 2z
$$

Substitute into the first:

$$
7(13 - 2z) + 3z = 26
$$

$$
91 - 14z + 3z = 26
$$

$$
91 - 11z = 26
$$

$$
-11z = -65 \Rightarrow z = 5.909...
$$

Find $x$:

$$
x = 13 - 2(5.909...) = 13 - 11.818... = 1.181...
$$

Corresponding point:

$$
\mathbf{r}_0 = (1.181..., 0, 5.909...)
$$

For simplicity, we’ll use the first point:

$$
\boxed{
\mathbf{r}_0 = (13, -32.5, 0)
}
$$

---

# Constructing the Vector Equation of the Line

Now, with the point $\mathbf{r}_0 = (13, -32.5, 0)$ and the direction vector $\mathbf{d} = \langle 4, -11, -2 \rangle$, the line's vector equation is:

$$
\boxed{
\mathbf{R}(t) = \mathbf{r}_0 + t \mathbf{d} = \langle 13, -32.5, 0 \rangle +
$$

t \langle 4, -11, -2 \rangle
}
\]

Expressed component-wise:

\[
x(t) = 13 + 4t
\]
\[
y(t) = -32.5 - 11t
\]
\[
z(t) = 0 - 2t
\]

Parametric equations:

\[
\boxed{
\begin{cases}
x(t) = 13 + 4t \\
y(t) = -32.5 - 11t \\
z(t) = -2t
\end{cases}
}
\]

where \( t \in (-\infty, \infty) \).

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# Summary and Key Takeaways

To summarize, the process of finding the vector equation of the line of intersection between two planes involves:

1. Identifying the normal vectors of each plane.
2. Calculating the cross product of the normal vectors to find the line's direction vector.
3. Finding a specific point on the line by substituting arbitrary values into the plane equations.
4. Constructing the parametric/vector form of the line using the point and the direction vector.

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# Additional Tips for Solving Plane Intersections

- Always double-check your calculations, especially when solving simultaneous equations.
- Use substitution or elimination methods to find points on the line.
- Remember the significance of the cross product in

# Frequently Asked Questions

## How do you find the line of intersection between the planes 2y + 7x + 3z = 26 and x + 2z = 13?

To find the line of intersection, solve the system of equations simultaneously, express two variables in terms of a parameter, and then write the parametric equation of the line.

## What is the first step in deriving the vector equation for the intersection line of two planes?

Identify the normal vectors of the planes and find their cross product to determine the direction vector of the line.

## How do you find a point on the line of intersection of the two planes?

Choose a parameter value (like t=0), then substitute into the equations to solve for the remaining variables, obtaining a specific point on the line.

## What is the purpose of finding the cross product of the plane normals in this problem?

The cross product gives the direction vector of the line of intersection because it is perpendicular to both normals and lies along their intersection.

## How is the parametric equation R(t) formulated once you have a point and a direction vector?

R(t) = P + t v, where P is a point on the line and v is the direction vector, with t being a real parameter.

## Can you provide the vector equation for the line of

## intersection of these planes?

Yes, after calculations, the line can be expressed as R(t) = P + t v, where P is a specific point satisfying both equations, and v is the cross product of the normals.

## What are the normal vectors of the given planes 2y + 7x + 3z = 26 and x + 2z = 13?

The normal vectors are n1 = (7, 2, 3) for the first plane, and n2 = (1, 0, 2) for the second plane.

## How do you compute the direction vector for the line of intersection?

Calculate the cross product of the normal vectors: v = n1 × n2 = (7, 2, 3) × (1, 0, 2).

## What is the final parametric form of the line of intersection for these planes?

After finding a point P on the line and the direction vector v, the line is R(t) = P + t v, with specific values determined through substitution and calculation.

# Additional Resources

Vector Equations of Planes and Lines of Intersection: A Comprehensive Guide

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Introduction

Understanding the intersection of planes in three-dimensional space is a foundational concept in vector calculus and analytical geometry. It allows mathematicians, engineers, and students to visualize complex spatial relationships and solve real-world problems such as determining the line of intersection between two planes—an essential skill in fields like computer graphics, robotics, and physics.

In this detailed exploration, we will derive the vector equation for the line of intersection of two given planes:

- Plane 1: $2y - 7x + 3z = 26$
- Plane 2: $x - 2z = 13$

This process involves a systematic approach to solving simultaneous equations, identifying direction vectors, and parametrizing the line. Our goal is to present an exhaustive, step-by-step methodology to arrive at the vector form of the intersection line, making it accessible for learners and professionals alike.

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## Understanding the Problem

Before diving into calculations, it’s crucial to comprehend what the problem entails. Two planes in three-dimensional space typically intersect along a line unless they are parallel or coincident. Here, the intersection line can be described by a vector equation of the form:

$$
\mathbf{R}(t) = \mathbf{a} + t \mathbf{b}
$$

where:

- $\mathbf{a}$ is a point on the line,
- $\mathbf{b}$ is the direction vector of the line,
- $t$ is a real parameter.

Our task is to determine $\mathbf{a}$ and $\mathbf{b}$ based on the equations of the given planes.

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## Step 1: Rewriting the Plane Equations

Let's restate the given planes explicitly:

1. Plane 1:

$$
2y - 7x + 3z = 26
$$

2. Plane 2:

$$
x - 2z = 13
$$

To facilitate calculations, it’s often easier to express these planes in standard form:

$$
-7x + 2y + 3z = 26 \quad \text{(rearranged Plane 1)}
$$

$$
x - 0y - 2z = 13 \quad \text{(rearranged Plane 2)}
$$

---

Step 2: Expressing Variables

Since we have two equations with three variables, we can choose a parameter (say, $z = t$) and express $x$ and $y$ in terms of this parameter.

From Plane 2:

$$
x - 2z = 13
$$
$$
x = 13 + 2z
$$
$$
x = 13 + 2t
$$

From Plane 1:

$$
-7x + 2y + 3z = 26
$$
Substitute $x$ and $z$:

$$
-7(13 + 2t) + 2y + 3t = 26
$$

Simplify:

$$
-91 - 14t + 2y + 3t = 26
$$
$$
2y = 26 + 91 + 14t - 3t
$$
$$
2y = 117 + 11t
$$
$$
y = \frac{117 + 11t}{2}
$$

Now, the parametric equations for $x, y, z$ are:

$$
x = 13 + 2t
$$

$$
y = \frac{117 + 11t}{2}
$$

$$
z = t
$$

---

Step 3: Deriving the Vector Equation

The parametric form indicates that the line of intersection can be written as:

$$
\mathbf{R}(t) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ \frac{117}{2} \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ \frac{11}{2} \\ 1 \end{bmatrix}
$$

Expressed as a vector equation:

$$
\boxed{
\mathbf{R}(t) = \mathbf{a} + t \mathbf{b}
}
$$

where:

- Point $\mathbf{a}$: $\left(13, \frac{117}{2}, 0\right)$
- Direction vector $\mathbf{b}$: $\left(2, \frac{11}{2}, 1\right)$

This vector form precisely describes the line where the two planes intersect.

---

Step 4: Simplifying the Equation

For clarity and elegance, it’s often preferable to write the direction vector with integer components. Multiply $\mathbf{b}$ by 2:

$$
\mathbf{b} = \left(2, \frac{11}{2}, 1\right) \times 2 = (4, 11, 2)
$$

Similarly, multiply the point $\mathbf{a}$ by 2:

$$
\mathbf{a} = \left(13, \frac{117}{2}, 0\right) \times 2 = (26, 117, 0)
$$

Thus, the simplified vector equation:

$$
\boxed{
\mathbf{R}(t) = \mathbf{a} + t \mathbf{b} = \left(26, 117, 0\right) + t \left(4, 11, 2\right)
}
$$

Alternatively, written explicitly:

$$
\boxed{
\mathbf{R}(t) = \left(26 + 4t, \quad 117 + 11t, \quad 2t \right)
}
$$

---

Understanding the Components of the Line Equation

- Point on the line ($\mathbf{a}$): The vector $\left(26, 117, 0\right)$ is a specific point through which the intersection line passes.
- Direction vector ($\mathbf{b}$): The vector $\left(4, 11, 2\right)$ indicates the orientation of the line in space.

---

Additional Insights

- The direction vector is orthogonal to the normal vectors of the planes. To verify, consider the normal vectors:

$$
\mathbf{n}_1 = \left(-7, 2, 3\right)
$$
$$
\mathbf{n}_2 = \left(1, 0, -2\right)
$$

- The direction vector of the line is the cross product of these normals:

$$
\mathbf{b} = \mathbf{n}_1 \times \mathbf{n}_2
$$

Compute:

$$
\mathbf{b} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-7 & 2 & 3 \\
1 & 0 & -2 \\
\end{vmatrix}
$$

Calculating component-wise:

$$
b_x = (2)(-2) - (3)(0) = -4 - 0 = -4
$$
$$
b_y = -\left((-7)(-2) - (3)(1)\right) = -\left(14 - 3\right) = -11
$$
$$
b_z = (-7)(0) - (2)(1) = 0 - 2 = -2
$$

The cross product yields $\left(-4, -11, -2\right)$, which is parallel to the earlier direction vector $\left(4, 11, 2\right)$ (they differ by a sign).

Thus, our direction vector aligns with the geometric interpretation, confirming the correctness.

---

Summary of the Vector Equation

The line of intersection of the two planes can be succinctly expressed as:

$$
\boxed{
\mathbf{R}(t) = \left(26 + 4t, \quad 117 + 11t, \quad 2t \right), \quad t \in (-\infty, \infty)
}
$$

This parametrized form offers a complete description of the line, allowing for easy plotting, intersection checks, and further geometric analysis.

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Conclusion

Deriving the vector equation of the line of intersection between two planes involves:

```
- Expressing the planes in standard form
- Solving for variables to parametrize the line
- Selecting a parameter to express the points along the line
- Simplifying the resulting equations for clarity

This process not only yields the explicit vector form but also deepens
understanding of the geometric relationships in three-dimensional space.
Whether for academic purposes, engineering applications, or advanced spatial
visualization, mastering this technique is invaluable.

Through this detailed examination, we've demonstrated a comprehensive, step-
by-step approach, ensuring clarity and precision—hallmarks of expert
mathematical analysis.
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