For Each Of These Relations On The Set {1, 2, 3, 4}, Decide Whether It Is Reflexive, Whether It Is Symmetric,

For Each Of These Relations On The Set {1, 2, 3, 4}, Decide Whether It Is Reflexive, Whether It Is Symmetric, understanding the properties of relations is fundamental in set theory and discrete mathematics. These properties help classify and analyze how elements within a set relate to each other, providing insights into the structure and behavior of the relations. In this article, we will explore various relations defined on the set {1, 2, 3, 4} and determine whether each relation is reflexive, symmetric, or possesses other notable properties. By doing so, we aim to develop a clear understanding of how to analyze relations systematically and to understand the significance of these properties in mathematical contexts.

Understanding Key Properties of Relations

Before delving into specific relations, it is essential to review the fundamental properties that characterize relations, especially reflexivity and symmetry.

Reflexive Relations

A relation R on a set S is called reflexive if every element is related to itself. Formally:
- For all a in S, (a, a) ∈ R.

This means that the diagonal elements of the relation matrix (if represented as such) are always present.

Symmetric Relations

A relation R on a set S is symmetric if, whenever an element a is related to an element b, then b is also related to a:
- For all a, b in S, if (a, b) ∈ R, then (b, a) ∈ R.

This property indicates a mutual relationship between related pairs.

Analyzing Specific Relations on the Set {1, 2, 3, 4}

Let's consider several relations defined on the set {1, 2, 3, 4} and analyze their properties.

Relation R1: The "Less Than" Relation

- Definition: R1 = { (a, b) | a < b } on {1, 2, 3, 4}.

Evaluation:

- Reflexive?
- For a relation to be reflexive, (a, a) must be in R for all a.
- Since a < a is never true, (a, a) is not in R1 for any a.
- Conclusion: R1 is not reflexive.

- Symmetric?
- Suppose (a, b) ∈ R1, which means a < b.
- Is (b, a) in R1? Since b < a would be needed, which is false if a < b.
- Therefore, the relation does not contain both (a, b) and (b, a) unless a = b, which it does not.
- Conclusion: R1 is not symmetric.

Summary for R1:
- Reflexive? No
- Symmetric? No

Relation R2: The "Equal To" Relation

- Definition: R2 = { (a, b) | a = b }.

Evaluation:

- Reflexive?
- For all a, (a, a) ∈ R2.
- Yes, since every element equals itself.
- Conclusion: R2 is reflexive.

- Symmetric?
- If (a, b) ∈ R2, then a = b.
- It follows that (b, a) = (a, b), so (b, a) ∈ R2.
- Conclusion: R2 is symmetric.

Summary for R2:
- Reflexive? Yes
- Symmetric? Yes

Relation R3: The "Divides" Relation

- Definition: R3 = { (a, b) | a divides b }.

Evaluation:

- Reflexive?

- Every number divides itself, so (a, a) ∈ R3 for all a.
- Conclusion: R3 is reflexive.

- Symmetric?
- Suppose (a, b) ∈ R3, meaning a divides b.
- Is (b, a) in R3? Only if b divides a.
- For example, (2, 4) ∈ R3 (since 2 divides 4), but (4, 2) ∉ R3 (4 does not divide 2).
- Conclusion: R3 is not symmetric.

Summary for R3:
- Reflexive? Yes
- Symmetric? No

Relation R4: The "Is a Friend of" Relation (Assuming Symmetry)

- Definition: R4 = { (a, b) | a is a friend of b }.

Suppose this relation is symmetric by nature (friendship is generally mutual).

Evaluation:

- Reflexive?
- Does everyone consider themselves a friend? Usually, in social contexts, yes.
- Assuming yes: (a, a) ∈ R4 for all a.
- Conclusion: R4 is reflexive.

- Symmetric?
- Friendship is mutual, so if (a, b) ∈ R4, then (b, a) ∈ R4.
- Conclusion: R4 is symmetric.

Summary for R4:
- Reflexive? Yes (assuming self-friendship)
- Symmetric? Yes

Additional Relations and Their Properties

Beyond the basic relations above, other relations can be constructed or analyzed to understand their properties.

Relation R5: The "Greater Than" Relation

- Definition: R5 = { (a, b) | a > b }.

Evaluation:

- Reflexive?
- Since $a > a$ is false, $(a, a) \notin R5$, so not reflexive.
- Symmetric?
- If $a > b$, then $b > a$ is false, so $(b, a) \notin R5$.
- Not symmetric.
- Summary: Not reflexive, not symmetric.

Relation R6: The "Square" Relation

- Definition: $R6 = \{ (a, b) \mid a^2 = b \}$.

Evaluation:

- Reflexive?
- For a to be related to itself, $a^2 = a$.
- Let's check for each element:
- 1: $1^2 = 1 \rightarrow$ yes
- 2: $4 \neq 2 \rightarrow$ no
- 3: $9 \neq 3 \rightarrow$ no
- 4: $16 \neq 4 \rightarrow$ no
- Only (1, 1) belongs.
- Not all elements are related to themselves.
- Conclusion: Not reflexive.

- Symmetric?
- For symmetry, if $(a, b) \in R6$, then $(b, a) \in R6$.
- $(a, b) \in R6$ implies $a^2 = b$.
- For (b, a) to be in R6, $b^2 = a$.
- Combining these:
- $a^2 = b$ and $b^2 = a \rightarrow a^2 = b$ and $(a^2)^2 = a \rightarrow a^4 = a$.
- For $a \in \{1, 2, 3, 4\}$, check which satisfy $a^4 = a$:
- 1: $1^4 = 1 \rightarrow$ yes
- 2: $2^4 = 16 \neq 2 \rightarrow$ no
- 3: $81 \neq 3 \rightarrow$ no
- 4: $256 \neq 4 \rightarrow$ no
- Only a=1 satisfies this.
- So (1, 1) is in R6, but not necessarily the others.
- Thus, the relation is not symmetric.

Summary Table of Relations and Properties

Relation	Is Reflexive?	Is Symmetric?	Comments
R1: Less Than	No	No	Strict inequality
R2: Equal To	Yes	Yes	Identity relation
R3: Divides	Yes	No	Divisibility relation

R4: Friendship	Yes	Yes	Assumed mutuality
R5: Greater Than	No	No	Opposite of R1
R6: Square Relation	No	No	Based on squares

Conclusion

Analyzing relations on a finite set like {1, 2, 3, 4} provides valuable insights into their structure and properties. Recognizing whether a relation is reflexive or symmetric helps classify the relation and understand its behavior. For example, the equality relation is both reflexive and symmetric, typical of equivalence relations. In contrast, the "less than" and "greater than" relations are neither reflexive nor symmetric, but they are transitive, which could be analyzed further.

Understanding these properties is crucial in fields such as computer science, mathematics, and logic, where relations underpin structures like graphs, equivalence classes, and ordering systems. By systematically evaluating each relation's properties, one can

Frequently Asked Questions

Given the relation R = {(1,1), (2,2), (3,3), (4,4)}, is R reflexive on the set {1, 2, 3, 4}?

Yes, R is reflexive because it contains all pairs (a, a) for every element in the set.

Is the relation R = {(1,2), (2,1), (3,4), (4,3)} on the set {1, 2, 3, 4} symmetric?

Yes, R is symmetric because for every (a, b), the pair (b, a) is also in R.

If the relation R = {(1,2), (2,3)} on {1, 2, 3, 4}, is R reflexive?

No, R is not reflexive because it does not contain (1,1), (2,2), (3,3), or (4,4).

On the set {1, 2, 3, 4}, is the relation R = {(1,1), (2,2), (3,3)} symmetric?

Yes, R is symmetric because it only contains pairs where the first and second elements are the same, so symmetry holds trivially.

For the relation R = {(1,2), (2,3), (3,1)} on {1, 2, 3, 4}, is R symmetric?

No, R is not symmetric because it contains (1,2) but not (2,1), nor does it contain the reverse pairs for all elements.

Is the relation R = {(1,1), (2,2), (3,3), (4,4)} on {1, 2, 3, 4} both reflexive and symmetric?

Yes, R is both reflexive and symmetric because it contains all pairs (a, a) and their reverses (which are the same pairs).

Given R = {(1,2), (2,1), (3,4)}, is R symmetric on {1, 2, 3, 4}?

No, R is not symmetric because although it contains (1,2) and (2,1), it does not contain pairs (3,4) and (4,3).

On the set {1, 2, 3, 4}, if R = {(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)}, is R reflexive and symmetric?

Yes, R is both reflexive (contains all (a, a) pairs) and symmetric (for each (a, b), (b, a) is also present).

Additional Resources

Understanding Relations on the Set {1, 2, 3, 4}: Reflexivity, Symmetry, and Beyond

When exploring the fascinating world of relations in mathematics, one fundamental task is to analyze specific relations defined on a set. For the set {1, 2, 3, 4}, understanding whether a relation is reflexive, symmetric, antisymmetric, or transitive is crucial for grasping their properties and implications. These concepts are foundational in areas like order theory, graph theory, and algebra, providing insight into how elements relate to each other within a set.

In this guide, we will delve into various relations on the set {1, 2, 3, 4}—examining their nature and determining whether they satisfy key properties such as reflexivity and symmetry. This comprehensive analysis is designed to help students, educators, and enthusiasts better understand how to analyze relations systematically.

The Basics: Key Properties of Relations

Before analyzing specific relations, let's clarify the main properties we'll focus on:

Reflexive
A relation R on a set S is reflexive if every element relates to itself. Formally:

- For all elements a in S, the pair (a, a) is in R.

Example: The relation "is equal to" on any set is reflexive because every element equals itself.

Symmetric
A relation R on S is symmetric if whenever a relates to b, then b also relates to a:

- For all a, b in S, if (a, b) is in R, then (b, a) is also in R.

Example: "is a sibling of" is symmetric because if a is a sibling of b, then b is a sibling of a.

Antisymmetric
A relation R is antisymmetric if:

- For all a, b in S, if (a, b) and (b, a) are in R, then a = b.

Example: "less than or equal to" (≤) is antisymmetric because if a ≤ b and b ≤ a, then a = b.

Transitive
A relation R is transitive if:

- For all a, b, c in S, if (a, b) and (b, c) are in R, then (a, c) is also in R.

Example: "is an ancestor of" is transitive because if a is an ancestor of b, and b is an ancestor of c, then a is an ancestor of c.

Analyzing Specific Relations on {1, 2, 3, 4}

Now, let's examine various relations defined on the set S = {1, 2, 3, 4}. To do this systematically, we will present the relation, then evaluate whether it is reflexive, symmetric, antisymmetric, and transitive.

Relation 1: R = {(1, 1), (2, 2), (3, 3), (4, 4)}

This relation is the identity relation on the set, containing all pairs where elements relate to themselves.

Is it Reflexive?
- Yes. All elements relate to themselves, as all pairs (a, a) for a in S are in R.

Is it Symmetric?
- Yes. Since the pairs are only pairs of the form (a, a), symmetry holds trivially: if (a, a) is in R, then (a, a) is in R.

Is it Antisymmetric?
- Yes. No pair (a, b) with a ≠ b exists, so the condition holds vacuously.

Is it Transitive?
- Yes. Because the only pairs are of the form (a, a), transitivity is satisfied: if (a, a) and (a, a) are in R, then (a, a) is in R.

Summary: The identity relation on {1, 2, 3, 4} is reflexive, symmetric, antisymmetric, and transitive. It’s an equivalence relation and a partial order (by triviality).

Relation 2: R = {(1, 2), (2, 3), (3, 4)}

This relation links 1 to 2, 2 to 3, and 3 to 4, with no other pairs.

Is it Reflexive?
- No. None of the pairs (1, 1), (2, 2), etc., are in R. Therefore, it is not reflexive.

Is it Symmetric?
- No. For example, (1, 2) is in R, but (2, 1) is not.

Is it Antisymmetric?
- Yes. Since there are no pairs (a, b) and (b, a) with a ≠ b, the antisymmetric property holds vacuously.

Is it Transitive?
- Let's check:
- (1, 2) and (2, 3) are in R. Is (1, 3) in R? No.
- (2, 3) and (3, 4) are in R. Is (2, 4) in R? No.

Since transitivity requires that whenever (a, b) and (b, c) are in R, (a, c) must be in R, and this is not the case here, R is not transitive.

Summary: The relation is not reflexive or symmetric, antisymmetric, but is not transitive.

Relation 3: R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)}

This relation includes the identity pairs and a pair (1, 2) and its reverse.

Is it Reflexive?
- Yes. All elements (a, a) are present.

Is it Symmetric?
- Yes. Since (1, 2) and (2, 1) are both in R, and the identity pairs are symmetric, the relation is symmetric.

Is it Antisymmetric?
- No. Because (1, 2) and (2, 1) are both in R, but 1 ≠ 2, violating antisymmetry.

Is it Transitive?
- Check:
- (1, 2) and (2, 1) are in R, so for transitivity, (1, 1) must be in R — it is.
- (2, 1) and (1, 2) are in R, so (2, 2) must be in R — it is.

No other pairs violate transitivity.

Summary: The relation is reflexive, symmetric, not antisymmetric, and transitive.

Relation 4: R = {(1, 2), (2, 3), (3, 4), (4, 1)}

This relation forms a cycle among the elements.

Is it Reflexive?
- No. None of the pairs (a, a) are in R.

Is it Symmetric?
- No. For example, (1, 2) is in R, but (2, 1) is not.

Is it Antisymmetric?
- Yes. Because there are no pairs (a, b) and (b, a) with a ≠ b simultaneously.

Is it Transitive?
- Check:
- (1, 2) and (2, 3) → (1, 3)? Not in R.
- (2, 3) and (3, 4) → (2, 4)? Not in R.
- (3, 4) and (4, 1) → (3, 1)? Not in R.

Since the required (a, c) pairs are missing, R is not transitive.

Summary: The relation is not reflexive or symmetric, antisymmetric, but not transitive.

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Relation 5: R = {(a, a) | a in {1, 2, 3, 4}} ∪ {(1, 2), (2, 1), (
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For Each Of These Relations On The Set 1 2 3 4 Decide Whether It Is Reflexive Whether It Is Symmetric

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