Given F Of X Is Equal To The Quantity 4x Minus 7 End Quantity Divided By The Quantity 8x Plus 8 End Quantity,

Given F Of X Is Equal To The Quantity 4x Minus 7 End Quantity Divided By The Quantity 8x Plus 8 End Quantity, understanding this function is fundamental for students and professionals working in calculus, algebra, and related mathematical fields. This rational function provides insights into how the numerator and denominator interact, influencing the overall behavior of the function across different values of x. In this comprehensive guide, we will explore the various aspects of this function, including its definition, domain, range, intercepts, asymptotes, and how to analyze its graph. Whether you're a student preparing for exams or a professional seeking to deepen your understanding, this article will serve as an invaluable resource.

Understanding the Function F(x) = (4x - 7) / (8x + 8)

Definition and Basic Properties

The given function is a rational function, which is a ratio of two polynomials. Specifically:

- Numerator: 4x - 7
- Denominator: 8x + 8

This structure implies that the function's behavior depends on the values of x that make the denominator zero, as division by zero is undefined. The function is continuous everywhere except where the denominator is zero.

Significance of Rational Functions

Rational functions are crucial in modeling real-world phenomena such as rates, ratios, and proportions. They exhibit unique characteristics like asymptotes and discontinuities, making their analysis essential for understanding their behavior.

Determining the Domain of F(x)

Where is F(x) Defined?

Since division by zero is undefined, the domain of F(x) includes all real numbers x except those that satisfy the denominator:

1. Set denominator equal to zero: $8x + 8 = 0$
2. Solve for x: $8x = -8 \rightarrow x = -1$

Therefore, the domain of F(x) is:

Domain: $\{x \in \mathbb{R} \mid x \neq -1\}$

This means the function is defined for all real numbers except $x = -1$.

Finding Intercepts of the Function

Y-Intercept

The y-intercept occurs where $x = 0$. To find it:

1. Substitute $x = 0$ into F(x):

   $F(0) = (40 - 7) / (80 + 8) = (-7) / (8)$

2. Calculate the value:

   $F(0) = -7/8$

Y-intercept: $(0, -7/8)$

X-Intercept

The x-intercept occurs where $F(x) = 0$, i.e., when the numerator is zero (since division by zero is undefined):

1. Set numerator equal to zero:

   $4x - 7 = 0$

2. Solve for x:

   $4x = 7 \rightarrow x = 7/4$

X-intercept: $(7/4, 0)$

Asymptotic Behavior of F(x)

Vertical Asymptote

Vertical asymptotes occur at points where the denominator is zero but the numerator isn't zero, causing the function to approach infinity or negative infinity.

- Since the denominator is zero at $x = -1$, and the numerator at $x = -1$:

Numerator at $x = -1$: $4(-1) - 7 = -4 - 7 = -11 \neq 0$

- Therefore, there is a vertical asymptote at:

$x = -1$

Horizontal Asymptote

Horizontal asymptotes describe the behavior of the function as x approaches infinity or negative infinity.

- Since both numerator and denominator are degree 1 polynomials, the horizontal asymptote is determined by the ratio of the leading coefficients:

Limit as $x \to \pm\infty$ of $F(x)$:

$$F(x) \approx (4x) / (8x) = 1/2$$

- Horizontal asymptote: $y = 1/2$

Analyzing the Graph of F(x)

Behavior Near Asymptotes

Understanding how the function behaves near the asymptotes helps in sketching its graph:

- As x approaches -1 from the left, F(x) tends to $-\infty$ or $+\infty$ depending on the direction.
- As x approaches -1 from the right, F(x) tends to the opposite infinity.
- As x approaches $\pm\infty$, F(x) approaches $y = 1/2$.

Sign of the Function in Different Intervals

- Interval $(-\infty, -1)$:
- Pick a test point, e.g., $x = -2$:

$$F(-2) = (4(-2) - 7) / (8(-2) + 8) = (-8 - 7) / (-16 + 8) = (-15) / (-8) = 15/8 > 0$$

- Interval $(-1, \infty)$:
- Pick a test point, e.g., $x = 0$:

$$F(0) = -7/8 < 0$$

- Summary:
- For $x < -1$, $F(x) > 0$
- For $x > -1$, $F(x) < 0$

Calculating the Derivative for Critical Points and Increasing/Decreasing Intervals

Derivative of F(x)

To analyze the increasing or decreasing nature of F(x), compute its derivative using the quotient rule:

Given $F(x) = u(x)/v(x)$, where:

- $u(x) = 4x - 7$
- $v(x) = 8x + 8$

The quotient rule states:

$$F'(x) = [v(x) u'(x) - u(x) v'(x)] / [v(x)]^2$$

Calculate derivatives:

$$u'(x) = 4$$
$$v'(x) = 8$$

Plug in:

$$F'(x) = [(8x + 8)4 - (4x - 7)8] / (8x + 8)^2$$

Simplify numerator:

$$(8x + 8)4 = 32x + 32$$
$$(4x - 7)8 = 32x - 56$$

Subtract:

$32x + 32 - (32x - 56) = 32x + 32 - 32x + 56 = 88$

Thus:

$F'(x) = 88 / (8x + 8)^2$

Since the denominator is always positive except at $x = -1$ (where $F(x)$ is undefined), and numerator is positive (88), the derivative is positive everywhere in the domain.

Conclusion:

- $F'(x) > 0$ for all $x \neq -1$
- $F(x)$ is strictly increasing on its entire domain.

Key Points Summary

- The domain of $F(x)$: all real numbers except $x = -1$
- Y-intercept at $(0, -7/8)$
- X-intercept at $(7/4, 0)$
- Vertical asymptote at $x = -1$
- Horizontal asymptote at $y = 1/2$
- The function is increasing across its entire domain
- Behavior near asymptotes: approaches infinity or negative infinity near $x = -1$

Applications and Real-World Contexts

Modeling with Rational Functions

Rational functions like F(x) = (4x - 7) / (8x + 8) are useful in various fields:

1. **Economics:** Modeling cost-revenue relationships where the rate of change varies with quantities.
2. **Physics:** Describing phenomena such as velocity or acceleration that depend on ratios of variables.
3. **Biology:** Population models where growth rates depend on ratios of resources or other factors.

Solving Practical Problems

Understanding the intercepts, asymptotes, and behavior of F(x) facilitates solving real-world problems involving ratios, such as:

- Calculating breakpoints where the system behavior changes dramatically (near asymptotes).
- Predicting the limits of the system as variables

Frequently Asked Questions

What is the given function F(x)?

F(x) = (4x - 7) / (8x + 8).

How can we simplify the function F(x)?

We can factor the denominator as 8(x + 1), so F(x) = (4x - 7) / [8(x + 1)].

What are the domain restrictions for F(x)?

The denominator cannot be zero, so $8x + 8 \neq 0$, which implies $x \neq -1$.

How do you find the vertical asymptote of F(x)?

Set the denominator equal to zero: $8x + 8 = 0$, so $x = -1$ is the vertical asymptote.

What is the horizontal asymptote of F(x)?

Since the degrees of numerator and denominator are the same, the horizontal asymptote is the ratio of the leading coefficients: $4/8 = 1/2$.

How do you evaluate F(x) at a specific point, say x = 2?

Substitute $x = 2$ into the function: $F(2) = (42 - 7) / (82 + 8) = (8 - 7) / (16 + 8) = 1 / 24$.

Can F(x) be simplified further?

No, since numerator and denominator do not share common factors, the function is already in simplest form.

What is the end behavior of F(x) as x approaches infinity?

As $x \to \infty$, $F(x) \to (4x)/(8x) = 1/2$, consistent with the horizontal asymptote.

How does F(x) behave near x = -1?

As x approaches -1 from the left or right, the function approaches infinity or negative infinity due to the vertical asymptote at $x = -1$.

How can this function be used in real-world applications?

Functions like F(x) can model ratios in engineering, economics, or physics where relationships involve proportional or inverse relationships between variables.

Additional Resources

Understanding the Function: An In-Depth Analysis of $F(x) = \frac{4x - 7}{8x + 8}$

When exploring the realm of algebra and functions, one of the fundamental concepts students and professionals alike encounter is the rational function. A particularly interesting example is Given F of x is equal to the quantity 4x minus 7 divided by the quantity 8x plus 8, which can be neatly expressed as:

$$F(x) = \frac{4x - 7}{8x + 8}$$

This function encapsulates several key ideas in algebra, including understanding domains, asymptotes, intercepts, and the behavior of the function as variables approach specific values. Whether you're a student seeking clarity or a professional analyzing the properties of this function, this comprehensive guide will walk you through every important aspect of $F(x)$.

Understanding the Structure of $F(x)$

At its core, $F(x)$ is a rational function, meaning it is a ratio of two polynomials. Specifically:

- Numerator: $4x - 7$
- Denominator: $8x + 8$

This structure has several implications:

- The function is undefined where the denominator equals zero.
- The behavior at those points often involves vertical asymptotes.
- The overall shape and intercepts depend on the degrees and coefficients of the numerator and denominator.

Analyzing the Domain of $F(x)$

The domain of a function is the set of all real numbers for which the function is defined. For rational functions, the primary restriction comes from the denominator being zero:

$$
8x + 8 \neq 0
$$
$$
8x \neq -8
$$
$$
x \neq -1
$$

Therefore, the domain of $F(x)$ is:

$$
\boxed{
\text{All real } x \text{ such that } x \neq -1
}
$$

This exclusion at $x = -1$ leads us to identify potential vertical asymptotes, which we will explore next.

Vertical Asymptotes and Their Significance

Vertical asymptotes occur at values of x where the function tends to infinity or negative infinity, typically where the denominator approaches zero, and the numerator doesn't simultaneously zero out to produce a finite limit.

Since the denominator is zero at $x = -1$, we examine the behavior of $F(x)$ near this point:

- Approach from the left ($x \to -1^-$):

For values just less than -1 (say, $x = -1.0001$):

$$
8x + 8 \approx 8(-1.0001) + 8 = -8.0008 + 8 = -0.0008
$$

The numerator at $x = -1.0001$:

$$
4(-1.0001) - 7 = -4.0004 - 7 = -11.0004
$$

So,

$$
F(x) \approx \frac{-11.0004}{-0.0008} \approx 13,750
$$

As $x \to -1^-$, $F(x) \to +\infty$.

- Approach from the right ($x \to -1^+$):

For $x = -0.9999$:

$$
8(-0.9999) + 8 = -7.9992 + 8 = 0.0008
$$

Numerator:

$$
4(-0.9999) - 7 = -3.9996 - 7 = -10.9996
$$

So,

$$
F(x) \approx \frac{-10.9996}{0.0008} \approx -13,750
$$

As $x \to -1^+$, $F(x) \to -\infty$.

Conclusion: There is a vertical asymptote at $x = -1$, with the function tending towards $+\infty$ from the left and $-\infty$ from the right. This indicates a significant discontinuity at this point.

Finding the Horizontal or Oblique Asymptotes

As \(x \to \pm \infty \), the behavior of rational functions is often dominated by the leading terms of numerator and denominator. Here:

\[
F(x) \sim \frac{4x}{8x} = \frac{4}{8} = \frac{1}{2}
\]

Therefore, the function has a horizontal asymptote at:

\[
\boxed{
y = \frac{1}{2}
}
\]

Implication:

- As \(x \to \infty \), \(F(x) \to \frac{1}{2} \).
- As \(x \to -\infty \), \(F(x) \to \frac{1}{2} \).

Finding the Intercepts of \(F(x) \)

Intercepts are points where the graph crosses the axes. To find these:

1. Y-intercept

Set \(x = 0 \):

\[
F(0) = \frac{4(0) - 7}{8(0) + 8} = \frac{-7}{8} = -\frac{7}{8}
\]

Y-intercept: \(\left(0, -\frac{7}{8}\right)\)

2. X-intercept

Set \(F(x) = 0 \):

\[
\frac{4x - 7}{8x + 8} = 0
\]
\[

$$
4x - 7 = 0
$$

$$
x = \frac{7}{4} = 1.75
$$

Check that denominator is not zero at this point:

$$
8 \times 1.75 + 8 = 14 + 8 = 22 \neq 0
$$

X-intercept: $\left(\frac{7}{4}, 0\right)$

Analyzing the Behavior of $F(x)$ and Critical Points

To understand the shape of the graph, analyze the critical points where the function reaches local maxima or minima, which involves derivatives.

Step 1: Find $F'(x)$

Given:

$$
F(x) = \frac{4x - 7}{8x + 8}
$$

Use the quotient rule:

$$
F'(x) = \frac{(4)(8x + 8) - (4x - 7)(8)}{(8x + 8)^2}
$$

Simplify numerator:

$$
4(8x + 8) - 8(4x - 7) = 32x + 32 - (32x - 56) = 32x + 32 - 32x + 56 = 88
$$

Thus:

$$
F'(x) = \frac{88}{(8x + 8)^2}
$$

Since the denominator is always positive (except where undefined), and numerator is positive:

$$
F'(x) > 0 \quad \text{for all } x \neq -1
$$

Implication:

- The function is strictly increasing on its entire domain.

Summary of Function Behavior

Aspect	Details
Domain	$x \neq -1$
Vertical asymptote	$x = -1$
Horizontal asymptote	$y = \frac{1}{2}$
Intercepts	Y-intercept at $(0, -7/8)$; X-intercept at $(7/4, 0)$
Monotonicity	Strictly increasing for all $x \neq -1$
Behavior at infinity	$F(x) \to 1/2$ as $x \to \pm \infty$

Graphical Interpretation and Sketching Tips

Understanding the function's graph involves visualizing the key features:

- The graph approaches the horizontal asymptote $y = 1/2$ from below as $x \to -\infty$ and from above as $x \to \infty$, given the monotonic increase.
- At $x = -1$, the graph exhibits a vertical asymptote, with the function tending toward $+\infty$ on the left and $-\infty$ on the right.
- The intercepts provide points to plot:
- Y-intercept at $(0, -7/8)$
- X-intercept at $(7/4, 0)$

Plotting tips:

- Mark the asymptotes clearly.
- Plot the intercepts.
- Use the

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