Given F(x,y) = X^3 - 3x + Xy + Y^2, The Saddle Point Is (_____,_____) And The Local Minimum Is (_____,_____).

Given F(x,y) = X^3 - 3x + Xy + Y^2, The Saddle Point Is (_____,_____) And The Local Minimum Is (_____,_____). Understanding the critical points and the nature of extremas for a multivariable function is a fundamental aspect of calculus and optimization. In this article, we will analyze the function $F(x,y) = x^3 - 3x + xy + y^2$, identify its critical points, and determine which ones correspond to saddle points and local minima. This process involves calculating partial derivatives, solving for critical points, and applying the second derivative test for functions of two variables.

Understanding the Function $F(x,y) = x^3 - 3x + xy + y^2$

Before delving into the calculus, it's important to grasp the structure of the function:

- The term x^3 introduces a cubic behavior that affects the overall shape along the x-axis.
- The linear term $-3x$ shifts the graph along the x-axis.
- The mixed term xy couples the x and y variables, influencing the surface's tilt and saddle-like features.
- The quadratic term y^2 ensures the function tends to infinity as $y \to \pm \infty$, contributing to the convexity in the y-direction.

Understanding these components helps predict where critical points might occur and what their nature could be.

Finding Critical Points of $F(x,y)$

Critical points occur where the gradient of the function is zero; that is, where both partial derivatives vanish simultaneously.

Calculating the Partial Derivatives

The partial derivatives with respect to x and y are:

\[

\frac{\partial F}{\partial x} = 3x^2 - 3 + y
\]

\[
\frac{\partial F}{\partial y} = x + 2y
\]

These derivatives provide the slopes of the tangent plane to the surface at any point \((x, y)\).

Setting the Partial Derivatives to Zero

To find critical points, solve the system:

\[
\begin{cases}
3x^2 - 3 + y = 0 \quad (1) \\
x + 2y = 0 \quad (2)
\end{cases}
\]

From equation (2), express \(y \) in terms of \(x \):

\[
y = -\frac{x}{2}
\]

Substitute this into equation (1):

\[
3x^2 - 3 - \frac{x}{2} = 0
\]

Multiply through by 2 to clear the denominator:

\[
6x^2 - 6 - x = 0
\]

Rearranged as:

\[
6x^2 - x - 6 = 0
\]

This quadratic in \(x \) can be solved using the quadratic formula:

\[
x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 6 \times (-6)}}{2 \times 6}
\]

Calculate the discriminant:

$$
\Delta = 1 + 144 = 145
$$

Therefore,

$$
x = \frac{1 \pm \sqrt{145}}{12}
$$

Corresponding y values are:

$$
y = -\frac{x}{2} = -\frac{1 \pm \sqrt{145}}{24}
$$

Critical points are:

$$
\left(\frac{1 + \sqrt{145}}{12}, -\frac{1 + \sqrt{145}}{24} \right)
$$
and
$$
\left(\frac{1 - \sqrt{145}}{12}, -\frac{1 - \sqrt{145}}{24} \right)
$$

Classifying Critical Points: Saddle Point or Local Minimum

Once critical points are identified, the next step is to determine their nature—whether they are saddle points, local minima, or maxima. This is achieved through the second derivative test for functions of multiple variables.

The Second Derivative Test for Two Variables

Define the following second derivatives:

$$
F_{xx} = \frac{\partial^2 F}{\partial x^2} = 6x
$$
$$
F_{yy} = \frac{\partial^2 F}{\partial y^2} = 2
$$

$$
F_{xy} = F_{yx} = \frac{\partial^2 F}{\partial x \partial y} = 1
$$

Calculate the discriminant D:

$$
D = F_{xx} \times F_{yy} - (F_{xy})^2
$$

Depending on the sign of D and the value of F_{xx}:

- If $D > 0$ and $F_{xx} > 0$, the point is a local minimum.
- If $D > 0$ and $F_{xx} < 0$, the point is a local maximum.
- If $D < 0$, the point is a saddle point.

Evaluating the Critical Points

Let's analyze each critical point separately.

First Critical Point: $\left(\frac{1 + \sqrt{145}}{12}, -\frac{1 + \sqrt{145}}{24} \right)$

Calculate F_{xx}:

$$
F_{xx} = 6x = 6 \times \frac{1 + \sqrt{145}}{12} = \frac{6}{12}(1 + \sqrt{145}) = \frac{1 + \sqrt{145}}{2}
$$

Since $\sqrt{145} \approx 12.04$, then:

$$
F_{xx} \approx \frac{1 + 12.04}{2} = \frac{13.04}{2} \approx 6.52 > 0
$$

Calculate D:

$$
D = F_{xx} \times F_{yy} - (F_{xy})^2 \approx 6.52 \times 2 - 1^2 = 13.04 - 1 = 12.04 > 0
$$

Because $D > 0$ and $F_{xx} > 0$, this critical point is a local minimum.

Second Critical Point: $\left(\frac{1 - \sqrt{145}}{12}, -\frac{1 - \sqrt{145}}{24} \right)$

Calculate F_{xx}:

$$
F_{xx} = 6x = 6 \times \frac{1 - \sqrt{145}}{12} = \frac{6}{12}(1 - \sqrt{145}) = \frac{1 - \sqrt{145}}{2}
$$

Using $\sqrt{145} \approx 12.04$:

$$
F_{xx} \approx \frac{1 - 12.04}{2} = \frac{-11.04}{2} \approx -5.52 < 0
$$

Calculate D:

$$
D = F_{xx} \times F_{yy} - (F_{xy})^2 \approx -5.52 \times 2 - 1 = -11.04 - 1 = -12.04 < 0
$$

Since $D < 0$, this critical point is a saddle point.

Summary of Critical Points and Their Nature

Based on the above analysis, the critical points are:

- Local Minimum: $\left(\frac{1 + \sqrt{145}}{12}, -\frac{1 + \sqrt{145}}{24} \right)$

- Saddle Point: $\left(\frac{1 - \sqrt{145}}{12}, -\frac{1 - \sqrt{145}}{24} \right)$

Final Answer

Replacing the placeholders with the calculated values:

- Given $F(x,y) = x^3 - 3x + xy + y^2$, The Saddle Point Is $\left(\frac{1 - \sqrt{145}}{12}, -\frac{1 - \sqrt{145}}{24} \right)$ And The Local Minimum Is $\left(\frac{1 + \sqrt{145}}{12}, -$

\frac{1 + \sqrt{145}}{24} \right)\).

Conclusion

The analysis of the function \(F(x,y) = x^3 - 3x + xy + y^2 \) illustrates how calculus tools such as partial derivatives, critical point solving, and the second derivative test work together to identify the nature of extremal points. Recognizing saddle points and minima helps in understanding the topology and behavior of multivariable functions, which is essential in optimization problems across mathematics, physics, economics, and engineering.

Additional Tips for An

Frequently Asked Questions

Given the function F(x,y) = x^3 - 3x + xy + y^2, how do you determine the saddle point and local minimum?

First, find the critical points by setting the first derivatives to zero, then analyze the second derivatives to classify each critical point as a saddle point or local minimum.

What are the critical points of F(x,y) = x^3 - 3x + xy + y^2?

By solving the system of equations obtained from setting the partial derivatives to zero, the critical points are at (1, -2) and (-1, 2).

How do you identify the saddle point and local minimum for the given function?

Calculate the Hessian matrix at each critical point and evaluate its definiteness; a saddle point occurs where the Hessian is indefinite, while a positive definite Hessian indicates a local minimum.

For the function F(x,y) = x^3 - 3x + xy + y^2, what are the coordinates of the saddle point and the local minimum?

The saddle point is at (0, 0), and the local minimum is at (1, -2).

Why is the point (1, -2) considered a local minimum for the function F(x,y)?

Because the second derivative test shows the Hessian matrix at (1, -2) is positive definite, indicating a local minimum at that point.

Additional Resources

In-Depth Analysis of the Critical Points of the Function $F(x, y) = x^3 - 3x + xy + y^2$

Understanding the nature of critical points in multivariable functions is fundamental in calculus and optimization. The function $F(x, y) = x^3 - 3x + xy + y^2$ serves as an excellent example for exploring concepts such as saddle points and local minima. In this comprehensive review, we will methodically analyze this function to identify its critical points, classify them, and understand their geometric significance.

Introduction to the Function and Its Significance

The function in question, $F(x, y) = x^3 - 3x + xy + y^2$, is a bivariate polynomial that combines cubic, linear, and quadratic terms. Its structure suggests the potential for complex critical points, including saddle points and local minima, which are essential in understanding the topology of the surface it defines.

Understanding such functions is vital in several fields:
- Mathematics: For studying critical point classification and surface topology.
- Physics: In potential energy surface analysis.
- Economics: For modeling optimization problems with multiple variables.
- Engineering: In control systems and stability analysis.

Finding Critical Points: First-Order Conditions

The first step in analyzing the function is to determine where the function's gradient vanishes. Critical points occur where both partial derivatives are zero.

Calculating the Partial Derivatives

Given:

$$F(x, y) = x^3 - 3x + xy + y^2$$

Compute:

- $\frac{\partial F}{\partial x}$
- $\frac{\partial F}{\partial y}$

Partial derivative with respect to x:

$$\frac{\partial F}{\partial x} = 3x^2 - 3 + y$$

Partial derivative with respect to y:

$$\frac{\partial F}{\partial y} = x + 2y$$

Setting the Derivatives Equal to Zero

Solve the system:

$$
\begin{cases}
3x^2 - 3 + y = 0 \quad \Rightarrow \quad y = -3x^2 + 3
\\
x + 2y = 0
\end{cases}
$$

Substitute y into the second equation:

$$
x + 2(-3x^2 + 3) = 0
$$

$$
x - 6x^2 + 6 = 0
$$

$$
-6x^2 + x + 6 = 0
$$

Multiply through by -1 for simplicity:

$$
6x^2 - x - 6 = 0
$$

Solving the Quadratic Equation for x

Use the quadratic formula:

$$
x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 6 \times (-6)}}{2 \times 6}
$$

$$
x = \frac{1 \pm \sqrt{1 + 144}}{12}
$$

$$
x = \frac{1 \pm \sqrt{145}}{12}
$$

Calculate the approximate numerical values:

- $\sqrt{145} \approx 12.0416$

Thus:

$$
x_1 = \frac{1 + 12.0416}{12} \approx \frac{13.0416}{12} \approx 1.0868
$$

$$
x_2 = \frac{1 - 12.0416}{12} \approx \frac{-11.0416}{12} \approx -0.9201
$$

Corresponding y-values:

$$
y = -3x^2 + 3
$$

Calculate for each x:

For $x \approx 1.0868$:

$$
x^2 \approx (1.0868)^2 \approx 1.1808
$$

$$
y \approx -3 \times 1.1808 + 3 = -3.5424 + 3 = -0.5424
$$

For $x \approx -0.9201$:

$$
x^2 \approx (0.9201)^2 \approx 0.846
$$

$$
y \approx -3 \times 0.846 + 3 = -2.538 + 3 = 0.462
$$

Critical points:

$$
\boxed{
\begin{cases}
\left(1.0868, -0.5424\right) \\
\left(-0.9201, 0.462\right)
\end{cases}
}
$$

Classifying Critical Points: Second-Order Conditions

Once the critical points are identified, the next step is to classify each point as a saddle point, local maximum, or local minimum. This classification involves the second derivatives and the Hessian matrix.

Calculating Second Derivatives

Compute the second derivatives:

- $F_{xx} = \frac{\partial^2 F}{\partial x^2}$
- $F_{yy} = \frac{\partial^2 F}{\partial y^2}$
- $F_{xy} = F_{yx} = \frac{\partial^2 F}{\partial x \partial y}$

Given:

$$
\frac{\partial F}{\partial x} = 3x^2 - 3 + y
$$

$$
\frac{\partial F}{\partial y} = x + 2y
$$

Second derivatives:

$$
F_{xx} = 6x

$$
F_{yy} = 2
$$

$$
F_{xy} = F_{yx} = 1
$$

Hessian Matrix and Discriminant

The Hessian matrix H is:

$$
H = \begin{bmatrix}
F_{xx} & F_{xy} \\
F_{yx} & F_{yy}
\end{bmatrix} = \begin{bmatrix}
6x & 1 \\
1 & 2
\end{bmatrix}
$$

The discriminant D is:

$$
D = F_{xx} \times F_{yy} - (F_{xy})^2 = (6x)(2) - 1^2 = 12x - 1
$$

Classification rules:

- If $D > 0$ and $F_{xx} > 0$, the point is a local minimum.
- If $D > 0$ and $F_{xx} < 0$, the point is a local maximum.
- If $D < 0$, the point is a saddle point.

Analyzing Each Critical Point

Let's evaluate D and F_{xx} at each critical point.

Critical Point 1: $\left(1.0868, -0.5424\right)$

Calculate F_{xx}:
$$
F_{xx} = 6 \times 1.0868 \approx 6.5208
$$
Calculate D:
$$
D = 12 \times 1.0868 - 1 \approx 13.0416 - 1 = 12.0416 > 0
$$

Since $D > 0$ and $F_{xx} > 0$, this point is a local minimum.

Critical Point 2: $\left(-0.9201, 0.462\right)$

Calculate F_{xx}:

$$
F_{xx} = 6 \times (-0.9201) \approx -5.5206
$$

Calculate D:

$$
D = 12 \times (-0.9201) - 1 \approx -11.0412 - 1 = -12.0412 < 0
$$

Since $D < 0$, this point is a saddle point.

Final Classification and Filling in the Blanks

Based on the above analysis:

- The saddle point occurs at $\left(-0.9201, 0.462\right)$.
- The local minimum occurs at $\left(1.0868, -0.5424\right)$.

Thus, the completed statement is:

> Given \(F(x, y) = x^3 - 3x + xy + y^2\), The Saddle Point Is \((-0.9201, 0.462)\) And The Local Minimum Is \((1.0868, -0.5424)\).

Geometric and Intuitive Understanding of These Critical Points

Understanding the nature of these points provides insight into the surface's behavior:

- Saddle Point \((-0.9201, 0.462)\):
At this point, the surface transitions from regions where it curves upward in one direction and downward in another. It resembles a saddle, indicating

[Given F X Y X 3 3x Xy Y 2 The Saddle Point Is And The Local Minimum Is]()

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