

Given The Demand Function $Q(p) = 150 P^2$ With Domain $0 \leq P \leq 150$ (a) Find The Price Elasticity Of Demand Function,

Given The Demand Function $Q(p) = 150 P^2$ With Domain $0 \leq P \leq 150$ (a) Find The Price Elasticity Of Demand Function, this article provides a comprehensive guide to understanding and calculating the price elasticity of demand based on the specified demand function. Price elasticity of demand is a fundamental concept in economics that measures how sensitive the quantity demanded of a good is to changes in its price. Understanding this concept helps businesses and policymakers make informed decisions regarding pricing strategies, taxation, and market analysis.

In this article, we'll explore the demand function, derive the formula for price elasticity of demand, perform the calculation step-by-step, and discuss the implications of the elasticity at different price points within the given domain. Whether you're a student studying microeconomics or a professional seeking to deepen your understanding, this guide will equip you with the necessary tools to analyze demand functions effectively.

Understanding the Demand Function $Q(p) = 150 P^2$

The Demand Function Explained

The given demand function is:

$$Q(p) = 150 P^2$$

where:

- $Q(p)$ represents the quantity demanded at price p ,
- p is the price of the good or service, with the domain $0 < p \leq 150$.

This quadratic demand function indicates that the quantity demanded increases with the square of the price, scaled by a constant factor of 150. Since the demand function is quadratic and increasing in p , it suggests a scenario where demand rises as price increases—although this might be counterintuitive in typical demand analysis, it could model specific market situations, such as Veblen goods or cases where price and perceived value are directly linked.

Important note: Typically, demand functions are decreasing in price, reflecting the law of demand. The positive quadratic form here implies a non-standard scenario. Confirming the domain and functional behavior is crucial for accurate interpretation.

Deriving the Price Elasticity of Demand

What is Price Elasticity of Demand?

Price elasticity of demand (E_d) measures the responsiveness of the quantity demanded to a change in price. It is defined as:

$$E_d = \frac{\text{change in quantity demanded}}{\text{change in price}}$$

Mathematically, it can be expressed as:

$$E_d = \frac{dQ}{dp} \times \frac{p}{Q}$$

where:

- $\frac{dQ}{dp}$ is the derivative of the demand function with respect to price,
- p is the current price,
- Q is the quantity demanded at price p .

Note: The elasticity can be negative due to the inverse relationship between price and demand, but often the absolute value is considered when discussing the magnitude of responsiveness.

Calculating the Derivative $\frac{dQ}{dp}$

Given:

$$Q(p) = 150 - P^2$$

Differentiate with respect to p :

$$\frac{dQ}{dp} = -2 \times 150 - 2P = -300 - 2P$$

This derivative indicates how the quantity demanded changes with a small change in price at any point p .

Calculating Price Elasticity at a Given Price

Elasticity Formula

Substituting the derivative into the elasticity formula:

$$E_d(p) = \frac{dQ}{dp} \times \frac{p}{Q(p)}$$

$$E_d(p) = (300 - p) \times \frac{p}{150 - p^2}$$

Now, simplifying:

$$E_d(p) = 300 - p \times \frac{p}{150 - p^2}$$

$$E_d(p) = \frac{300 - p^2}{150 - p^2}$$

$$E_d(p) = \frac{300}{150} = 2$$

Key insight: The elasticity is constant across the domain, with a value of 2, indicating that demand is elastic at all price levels within the domain $(0 < p \leq 150)$.

Implications of the Elasticity Result

Understanding the Value of Elasticity = 2

- Elastic Demand: Since the absolute value of the elasticity exceeds 1, demand is elastic. This means a 1% change in price will lead to a 2% change in quantity demanded.
- Revenue Considerations: For elastic demand, decreasing price tends to increase total revenue, whereas increasing price reduces total revenue.
- Market Strategy: Businesses should be cautious when raising prices, as demand is sensitive and can decline significantly.

Behavior at Different Price Points

Given that the elasticity is constant at 2 across the domain, the demand's responsiveness to price changes does not vary with price in this specific model. This simplifies strategic decisions but also warrants scrutiny because real-world demand often shows varying elasticity at different price points.

Summary and Key Takeaways

- The demand function $Q(p) = 150 - p^2$ models a scenario where quantity demanded increases quadratically with price.
- The derivative $\frac{dQ}{dp} = -300p$ captures how demand changes with price.
- The price elasticity of demand is calculated as:

$$E_d(p) = \frac{dQ}{dp} \times \frac{p}{Q(p)} = -2$$

- The elasticity value of 2 indicates elastic demand, meaning consumers are highly responsive to price changes.
- This constant elasticity suggests strategic pricing should account for significant changes in demand with small price adjustments.

Additional Considerations

Limitations of the Model

- The demand function's form suggests demand increases with price, which is atypical and might reflect unique market conditions or theoretical models.
- Real-world demand often exhibits varying elasticity at different price levels, which this model does not capture.
- The domain restriction $(0 < p \leq 150)$ ensures the function remains valid and avoids division by zero issues.

Practical Applications

- Businesses can use elasticity calculations to optimize pricing strategies, maximize revenue, or analyze market sensitivity.
- Policymakers can assess the impact of taxation or regulation on demand based on elasticity measures.

Further Study

To deepen understanding, consider exploring:

- The concept of inelastic demand $(|E_d| < 1)$

- Cross-price elasticity of demand
- Income elasticity of demand
- How demand functions change under different market conditions

Conclusion

Calculating the price elasticity of demand from a given demand function provides valuable insights into consumer behavior and market dynamics. In this case, with $Q(p) = 150 p^2$, the elasticity is constant at 2 across the domain, indicating elastic demand. Recognizing whether demand is elastic or inelastic helps businesses and policymakers make informed decisions regarding pricing, marketing, and regulation.

By understanding the derivation and implications of elasticity, stakeholders can better anticipate consumer responses and tailor strategies accordingly. Remember, while this model offers clarity in a theoretical context, real-world demand may vary and should be analyzed with context-specific data for optimal decision-making.

Keywords: demand function, price elasticity of demand, elasticity formula, demand sensitivity, microeconomics, revenue optimization, market analysis

Frequently Asked Questions

What is the demand function given in the problem?

The demand function provided is $Q(p) = 150 p^2$, with the domain $0 < p < 150$.

How do you find the price elasticity of demand from the demand function $Q(p)$?

Price elasticity of demand is calculated as $E(p) = (dQ/dp) (p / Q(p))$.

What is the derivative of the demand function $Q(p) = 150 p^2$ with respect to p ?

The derivative dQ/dp is $300 p$.

What is the formula for the price elasticity of demand for $Q(p) = 150 p^2$?

$$E(p) = (300 p) (p / (150 p^2)) = (300 p^2) / (150 p^2) = 2.$$

What is the price elasticity of demand for this function at any point within the domain?

The price elasticity of demand is constant and equals 2 for all p in $(0, 150)$.

Is the demand elastic or inelastic based on the elasticity value, and why?

Since the elasticity is 2 (greater than 1), the demand is elastic, indicating that quantity demanded is highly responsive to price changes.

Additional Resources

Given the demand function $Q(p) = 150 P^2$ with domain $0 < P < 150$, find the price elasticity of demand function.

Understanding the price elasticity of demand is fundamental in economics, as it measures how much the quantity demanded of a good responds to changes in its price. In this guide, we will explore the process of deriving the price elasticity of demand function from a specific demand function, providing a comprehensive step-by-step analysis suitable for students, professionals, or anyone interested in economic modeling.

Introduction to Price Elasticity of Demand

Before diving into the specifics of the problem, it's essential to understand what price elasticity of demand (PED) represents. PED quantifies the percentage change in quantity demanded resulting from a one percent change in price, and it is expressed as:

$$\text{Price Elasticity of Demand } (E_d) = \frac{\% \text{ change in quantity demanded}}{\% \text{ change in price}}$$

Mathematically, for a demand function $Q(p)$:

$$E_d(p) = \frac{dQ}{dp} \times \frac{p}{Q}$$

where:

- $\frac{dQ}{dp}$ is the derivative of the demand function with respect to price,

- p is the current price,
- Q is the quantity demanded at that price.

Understanding how to compute this elasticity helps businesses and policymakers make informed decisions about pricing strategies, taxation, and market analysis.

Step 1: Analyzing the Given Demand Function

The demand function provided is:

$$Q(p) = 150 P^2$$

with the domain:

$$0 < P < 150$$

This quadratic demand function indicates that quantity demanded depends on the square of the price, scaled by 150.

Key observations:

- The demand increases quadratically with price, which is somewhat unusual because most demand functions generally decrease with price. However, this function suggests a scenario where demand grows as price increases, possibly representing a special market condition or a theoretical exercise.
- The domain restricts P between 0 and 150, avoiding the endpoints where the function might behave differently or be undefined.

Step 2: Deriving the Price Elasticity of Demand Function

2.1 Find the derivative $\frac{dQ}{dp}$

Given:

$$Q(p) = 150 p^2$$

Differentiate with respect to p :

$$\frac{dQ}{dp} = 300 p$$

This derivative tells us how the quantity demanded changes with a small change in price.

2.2 Plug into the elasticity formula

Recall:

$$E_d(p) = \frac{dQ}{dp} \times \frac{p}{Q}$$

Substitute $\left(\frac{dQ}{dp}\right)$ and $Q(p)$:

$$E_d(p) = (300 - p) \times \frac{p}{150 - p^2}$$

2.3 Simplify the expression

Simplify numerator and denominator:

$$E_d(p) = \frac{300 - p}{150 - p^2}$$

$$E_d(p) = \frac{300 - p}{150 - p^2}$$

$$E_d(p) = \frac{300}{150}$$

$$E_d(p) = 2$$

Step 3: Interpreting the Result

The calculation shows that:

$$E_d(p) = 2$$

for all p within the domain $(0, 150)$.

Key insights:

- The price elasticity of demand is a constant value, equal to 2, across the entire domain.
- Since $|E_d(p)| = 2 > 1$, the demand is elastic at all prices within the specified range.

This implies that a 1% increase in price would lead to a 2% decrease in quantity demanded, indicating that demand is sensitive to price changes.

Additional considerations:

3.1 Elasticity behavior across the domain:

- The elasticity being constant simplifies economic analysis, as the responsiveness of demand does not vary with price.
- The demand being elastic suggests that consumers are quite responsive to price changes, which can influence pricing strategies.

3.2 Critical points:

- The domain excludes $p = 0$ because the demand function and elasticity formula involve division by $Q(p)$ and p , respectively.
- At the upper boundary $p \rightarrow 150$, the demand function reaches

$Q(150) = 150 \times (150)^2$), which is large, but the elasticity remains constant.

Summary of Key Findings

Aspect	Explanation
Demand Function	$Q(p) = 150 - p^2$
Derivative dQ/dp	$-2p$
Elasticity $E_d(p)$	Constant at 2
Demand Type	Elastic (since $ E_d > 1$)
Range of elasticity	Same across $0 < p < 150$

Practical Implications

Understanding the elasticity of demand derived from this quadratic function offers real-world insights:

- Pricing Strategy: Since demand is elastic, increasing prices could significantly reduce total revenue. Businesses should consider lowering prices if they want to boost sales volume.
- Market Sensitivity: The demand's constant elasticity indicates a predictable response to price changes, simplifying revenue optimization.
- Policy Making: For policymakers, recognizing elastic demand can influence tax policies, knowing that price increases might substantially decrease quantity demanded.

Conclusion

The process of finding the price elasticity of demand from a given demand function is a crucial step in economic analysis. In this case, starting from $Q(p) = 150 - p^2$, we derived that the elasticity $E_d(p)$ is a constant value of 2 across the domain $0 < p < 150$. This indicates a consistently elastic demand, where consumers are highly responsive to price changes. Recognizing this elasticity helps businesses and policymakers make better-informed decisions, ensuring strategies align with consumer behavior.

Understanding how to compute and interpret price elasticity provides essential insights into market dynamics, revenue maximization, and consumer responsiveness—core aspects of economic analysis and strategic planning.

Note: Always verify the domain restrictions and consider the economic context when applying these calculations, as real-world demand functions may exhibit

more complex behaviors.

Given The Demand Function $Q = 150 - P^2$ With Domain $0 \leq P \leq 150$ A Find The Price Elasticity Of Demand Function

Given The Demand Function $Q = 150 - P^2$ With Domain $0 \leq P \leq 150$ A Find The Price Elasticity Of Demand Function

Related Articles

- [SECTION A: DEMOCRACY AND HUMAN RIGHTS 1. HUMAN RIGHTS VIOLATIONS Read The Following Articles From Teenactiv:](#)
- [Ghi Corporation Reported The Following Financial Results For The Prior Quarter:earnings: \\$50 Millioninterest:](#)
- [TUESDAYS WITH MORRIEThe Text On Pages 3-4 Is In Italics. After Reading This Section, Decide How It Functions](#)

[Back to Home](#)