If P=(-3,-2) And G = (1,6) Are The Endpoints Of The Diameter Of A Circle, Find The Equation Of The Circle.The

If P=(-3,-2) And G = (1,6) Are The Endpoints Of The Diameter Of A Circle, Find The Equation Of The Circle.The problem presented involves finding the equation of a circle when given the endpoints of its diameter. This is a common question in geometry and algebra, often encountered in coordinate geometry courses. Understanding how to derive the circle's equation from given points requires knowledge of the circle's properties, the midpoint formula, the distance formula, and the standard form of a circle's equation. In this article, we will walk through the entire process step-by-step, providing detailed explanations to ensure a comprehensive understanding.

Understanding the Problem

Before diving into calculations, it’s essential to clarify what the problem asks:

- Given:
- Endpoint P of the diameter: P=(-3, -2)
- Endpoint G of the diameter: G=(1, 6)
- Find:
- The equation of the circle for which P and G are endpoints of its diameter.

The key points to note:

- The endpoints of a diameter lie at the ends of the longest chord passing through the center.
- The center of the circle is the midpoint of the diameter.
- The radius is half the length of the diameter.

Step 1: Find the Midpoint of the Diameter (Center of the Circle)

Since the endpoints P and G define the diameter, the center of the circle is the midpoint of these two points.

Midpoint Formula

The midpoint $M(x_m, y_m)$ of a segment with endpoints (x_1, y_1) and (x_2, y_2) is given by:

$$
x_m = \frac{x_1 + x_2}{2}
$$

$$
y_m = \frac{y_1 + y_2}{2}
$$

Calculations

Applying the formula:

$$
x_m = \frac{-3 + 1}{2} = \frac{-2}{2} = -1
$$

$$
y_m = \frac{-2 + 6}{2} = \frac{4}{2} = 2
$$

Therefore, the center of the circle is at:

$$
C(-1, 2)
$$

Step 2: Find the Radius of the Circle

The radius is the distance from the center to either endpoint of the diameter. Since we already have the center $C(-1, 2)$ and one endpoint $P(-3, -2)$, we can calculate the radius using the distance formula.

Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) is:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Calculations

Applying the formula between $C(-1, 2)$ and $P(-3, -2)$:

$$
d = \sqrt{(-3 - (-1))^2 + (-2 - 2)^2} = \sqrt{(-3 + 1)^2 + (-4)^2} = \sqrt{(-2)^2 + 16} = \sqrt{4 + 16} = \sqrt{20}
$$

Simplify:

$$
d = \sqrt{20} = 2 \sqrt{5}
$$

Alternatively, we can verify with point $G(1, 6)$:

$$
d = \sqrt{(1 - (-1))^2 + (6 - 2)^2} = \sqrt{(2)^2 + (4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2 \sqrt{5}
$$

Both calculations confirm that the radius r:

$$
r = 2 \sqrt{5}
$$

Step 3: Write the Equation of the Circle

The standard form of a circle's equation with center (h, k) and radius r is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Substituting the known values:

- $h = -1$
- $k = 2$
- $r = 2 \sqrt{5}$

Calculate r^2:

$$
r^2 = (2 \sqrt{5})^2 = 4 \times 5 = 20
$$

Therefore, the equation of the circle is:

$$
(x + 1)^2 + (y - 2)^2 = 20
$$

Additional Considerations and Tips

Verifying the Solution

- Confirm that the endpoints P and G satisfy the equation:

$$
(x + 1)^2 + (y - 2)^2 = 20
$$

Check for P(-3, -2):

$$
(-3 + 1)^2 + (-2 - 2)^2 = (-2)^2 + (-4)^2 = 4 + 16 = 20
$$

Check for G(1, 6):

$$
(1 + 1)^2 + (6 - 2)^2 = (2)^2 + (4)^2 = 4 + 16 = 20
$$

Both points satisfy the equation, confirming correctness.

Visual Representation

Plotting the circle centered at $(-1, 2)$ with radius $2 \sqrt{5}$ provides a visual understanding of the problem. Visuals help in grasping the geometric relationships.

Real-life Applications

Understanding how to derive the circle's equation from endpoints of a diameter has practical applications in fields such as:

- Engineering design
- Navigation systems
- Computer graphics
- Astronomy

Summary

To summarize the process of finding the equation of a circle given the endpoints of its diameter:

1. Find the center by calculating the midpoint of the diameter endpoints.
2. Calculate the radius by finding the distance from the center to either endpoint.
3. Write the standard form of the circle's equation using the center and radius squared.

Applying these steps to points $P(-3, -2)$ and $G(1, 6)$, we obtained:

$$
\boxed{
\text{Equation of the circle:} \quad (x + 1)^2 + (y - 2)^2 = 20
}
$$

This comprehensive approach ensures clarity and accuracy when solving similar problems involving circles in coordinate geometry.

Additional Resources

For further study and practice, consider exploring:

- Coordinate Geometry textbooks
- Online tutorials on circles and their equations
- Geometry problem sets involving diameters and centers
- Interactive graphing tools to visualize circles and their properties

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Meta Description: Learn how to find the equation of a circle given the endpoints of its diameter with step-by-step guidance, formulas, and examples. Perfect for students studying coordinate geometry.

Frequently Asked Questions

What is the first step in finding the equation of the circle with endpoints P(-3,-2) and G(1,6)?

The first step is to find the center of the circle, which is the midpoint of the endpoints P and G.

How do you calculate the midpoint of the diameter with endpoints P(-3,-2) and G(1,6)?

The midpoint is found by averaging the x-coordinates and y-coordinates: ((-3 + 1)/2, (-2 + 6)/2) = (-1, 2).

What is the center of the circle given the endpoints P(-3,-2) and G(1,6)?

The center of the circle is at (-1, 2), which is the midpoint of the diameter.

How do you find the radius of the circle from the endpoints of its diameter?

The radius is half the length of the diameter, so first calculate the distance between P and G, then divide by 2.

What is the formula to find the distance between two points P(-3,-2) and G(1,6)?

Distance = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$, so $\sqrt{[(1 - (-3))^2 + (6 - (-2))^2]} = \sqrt{[(4)^2 + (8)^2]} = \sqrt{[16 + 64]} = \sqrt{80}$.

Calculate the length of the diameter of the circle with endpoints P(-3,-2) and G(1,6).

The length of the diameter is $\sqrt{80}$, which simplifies to $4\sqrt{5}$.

What is the radius of the circle with the given endpoints?

The radius is half the diameter length, so $r = (4\sqrt{5})/2 = 2\sqrt{5}$.

What is the standard form equation of the circle with center (-1, 2) and radius $2\sqrt{5}$?

The equation is $(x + 1)^2 + (y - 2)^2 = (2\sqrt{5})^2$, which simplifies to $(x + 1)^2 + (y - 2)^2 = 20$.

What is the final equation of the circle with diameter endpoints P(-3,-2) and G(1,6)?

The equation of the circle is $(x + 1)^2 + (y - 2)^2 = 20$.

Additional Resources

Understanding the Equation of a Circle Given Its Diameter Endpoints

When dealing with circles in coordinate geometry, one of the core concepts involves understanding how to determine the equation of a circle when given certain key pieces of information. A common problem is finding the equation of a circle when its diameter's endpoints are known. This involves multiple steps: calculating the center, finding the radius, and formulating the standard or general equation of the circle.

In this detailed guide, we'll explore the problem: Given the endpoints of a diameter of a circle at P = (-3, -2) and G = (1, 6), find the equation of the circle.

Foundational Concepts

Before diving into the calculations, it’s essential to understand the fundamental properties and formulas involved:

- Circle's Center: The center of the circle is the midpoint of the diameter.
- Radius: The radius is half the length of the diameter.
- Equation of a Circle: The standard form is
\[
(x - h)^2 + (y - k)^2 = r^2
\]
where \((h, k)\) is the center, and \(r\) is the radius.

Step 1: Identifying the Given Data

The problem provides the endpoints of the diameter:
- Point P: \((-3, -2)\)
- Point G: \((1, 6)\)

These points are crucial because:
- They define the diameter directly.
- They allow us to calculate the circle’s center and radius.

Step 2: Calculating the Center of the Circle

Since the diameter's endpoints are known, the first step is to find the midpoint of these endpoints, which is the center of the circle.

Midpoint Formula

The midpoint \(M = (h, k)\) of a segment with endpoints \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is:

\[

$$
h = \frac{x_1 + x_2}{2}
$$

$$
k = \frac{y_1 + y_2}{2}
$$

Applying the Midpoint Formula

For points $P(-3, -2)$ and $G(1, 6)$:

$$
h = \frac{-3 + 1}{2} = \frac{-2}{2} = -1
$$

$$
k = \frac{-2 + 6}{2} = \frac{4}{2} = 2
$$

Therefore, the center of the circle is at $(-1, 2)$.

Step 3: Calculating the Radius

The radius r can be found by calculating the distance between the center and either endpoint of the diameter.

Distance Formula

Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, the distance d is:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Since the diameter is between points P and G, and we already know the endpoints, we can directly compute the diameter length, then divide by 2 to get the radius.

Calculating the Diameter Length

$$
d_{PG} = \sqrt{(1 - (-3))^2 + (6 - (-2))^2}
$$
$$
= \sqrt{(1 + 3)^2 + (6 + 2)^2}
$$
$$
= \sqrt{4^2 + 8^2}
$$
$$
= \sqrt{16 + 64} = \sqrt{80}
$$

Simplify $\sqrt{80}$:

$$
\sqrt{80} = \sqrt{16 \times 5} = 4 \sqrt{5}
$$

Hence, the diameter length is $4 \sqrt{5}$.

Calculating the Radius

$$
r = \frac{d_{PG}}{2} = \frac{4 \sqrt{5}}{2} = 2 \sqrt{5}
$$

The radius of the circle is $2 \sqrt{5}$.

Step 4: Formulating the Equation of the Circle

Now that we have the center $(-1, 2)$ and the radius $2 \sqrt{5}$, we can write the standard form of the circle's equation:

\[

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Substituting the known values:

$$
(x - (-1))^2 + (y - 2)^2 = (2 \sqrt{5})^2
$$

Simplify:

$$
(x + 1)^2 + (y - 2)^2 = 4 \times 5 = 20
$$

Final Equation of the Circle:

$$
\boxed{
(x + 1)^2 + (y - 2)^2 = 20
}
$$

This is the standard form of the circle's equation given the endpoints of its diameter.

Additional Considerations and Variations

While the above process is straightforward for this specific problem, several additional aspects and variations are worth considering:

1. Converting to the General Form

The general form of the circle's equation is:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

Expanding the standard form:

$$
(x + 1)^2 + (y - 2)^2 = 20
$$

$$
x^2 + 2x + 1 + y^2 - 4y + 4 = 20
$$

Combine like terms:

$$
x^2 + y^2 + 2x - 4y + (1 + 4) = 20
$$
$$
x^2 + y^2 + 2x - 4y + 5 = 20
$$

Bring all to one side:

$$
x^2 + y^2 + 2x - 4y + 5 - 20 = 0
$$
$$
x^2 + y^2 + 2x - 4y - 15 = 0
$$

General form:

$$
x^2 + y^2 + 2x - 4y - 15 = 0
$$

---

2. Verifying the Equation

It’s always good practice to verify that the equation corresponds to the given diameter endpoints:

- Plug in $P(-3, -2)$:

$$
(-3 + 1)^2 + (-2 - 2)^2 = (-2)^2 + (-4)^2 = 4 + 16 = 20
$$

- Plug in $G(1,6)$:

$$
(1 + 1)^2 + (6 - 2)^2 = (2)^2 + (4)^2 = 4 + 16 = 20
$$

Both points satisfy the circle equation, confirming correctness.

Mathematical Significance and Practical Applications

Understanding how to derive the equation of a circle from diameter endpoints is fundamental in various fields:

- Engineering: Designing circular components or analyzing circular motion.
- Computer Graphics: Rendering circles when given endpoints or defining regions.
- Navigation and Geospatial Analysis: Calculating coverage areas or zones of influence.
- Physics: Analyzing trajectories or fields with circular symmetry.

The ability to translate geometric data into algebraic equations allows for precise calculations, simulations, and problem-solving across disciplines.

Summary of Key Steps

1. Identify the endpoints of the diameter and label them clearly.
2. Calculate the midpoint to determine the circle's center.
3. Compute the distance between endpoints to find the diameter length.
4. Divide the diameter by 2 to get the radius.
5. Formulate the equation in standard form using the center and radius.
6. Optional: Convert the equation into general form for further analysis.

Conclusion

The process of finding the equation of a circle from its diameter endpoints exemplifies the elegant interplay between geometry and algebra. Given points $P(-3, -2)$ and $G(1, 6)$, we've methodically determined the center at $(-1, 2)$, the radius $2 \sqrt{5}$, and ultimately the equation:

$$
\boxed{
(x + 1)^2 + (y - 2)^2 = 20
}
$$

This comprehensive approach illustrates the power of coordinate geometry in solving real-world problems, and mastering this methodology enhances one's ability to analyze and interpret circular data in various scientific and engineering contexts.

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