Line JK Passes Through Points J(-4,-5) And K(-6,3). If The Equation Of The Line Is Written In Slope-intercept

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Understanding how to find the equation of a line passing through two points is a fundamental skill in coordinate geometry. When given specific points such as J(-4, -5) and K(-6, 3), mathematicians and students alike can determine the precise linear equation that describes the line passing through these coordinates. Writing this equation in slope-intercept form (y = mx + b) not only provides a clear understanding of the line's slope and y-intercept but also enhances the ability to graph and analyze the line's behavior within the coordinate plane.

In this article, we will explore the process of deriving the equation of the line passing through points J and K in slope-intercept form. We will delve into the concepts of slope calculation, y-intercept determination, and the practical significance of these calculations. Whether you're a student preparing for exams or a math enthusiast seeking a deeper understanding, this comprehensive guide aims to clarify each step with detailed explanations and examples.

Understanding the Slope-Intercept Form

What Is the Slope-Intercept Form?

The slope-intercept form of a line's equation is one of the most commonly used formats in algebra and coordinate geometry. It is expressed as:

- **y = mx + b**

where:
- **m** is the slope of the line, representing its steepness.
- **b** is the y-intercept, the point where the line crosses the y-axis.

This form is particularly useful because it directly shows how y relates to x, making graphing straightforward and intuitive.

Why Use the Slope-Intercept Form?

Using the slope-intercept form allows for quick visualization and understanding of the line's characteristics. It simplifies the process of graphing, as you can start plotting from the y-intercept and use the slope to determine subsequent points. Additionally, it makes it easy to compare different lines and understand their relationships.

Calculating the Slope of Line JK

What Is Slope?

The slope of a line indicates how much y changes for a unit change in x. It is calculated as the ratio of the vertical change (rise) to the horizontal change (run) between two points on the line.

Formula for Slope

Given two points, (x_1, y_1) and (x_2, y_2), the slope (m) is calculated as:

- **$m = (y_2 - y_1) / (x_2 - x_1)$**

This formula measures how steep the line is.

Applying to Points J and K

For points J(-4, -5) and K(-6, 3):

- $x_1 = -4$, $y_1 = -5$
- $x_2 = -6$, $y_2 = 3$

Calculating the slope:

```
m = (3 - (-5)) / (-6 - (-4))
= (3 + 5) / (-6 + 4)
= 8 / (-2)
= -4
```

The slope of line JK is -4, indicating that for each unit increase in x, y decreases by 4 units.

Deriving the Equation in Slope-Intercept Form

Finding the Y-Intercept (b)

Once the slope (m) is known, the next step is to find the y-intercept, b. To do this, substitute the slope and the coordinates of one of the points into the slope-intercept form y = mx + b and solve for b.

Choosing point J(-4, -5):

```
-5 = (-4)   (-4) + b
-5 = 16 + b
b = -5 - 16
b = -21
```

Thus, the y-intercept is -21.

Writing the Full Equation

Putting it all together:

1. Slope (m) = -4

2. Y-intercept (b) = -21

   The equation of line JK in slope-intercept form is:

   - **y = -4x - 21**

   This equation fully describes the line passing through points J and K.

Verifying the Equation

Check With the Other Point K(-6, 3)

To confirm the correctness of the derived equation, substitute point K into y = -4x -

21:

```
y = -4  (-6) - 21
= 24 - 21
= 3
```

Since the result matches the y-coordinate of point K, the equation is verified.

Practical Applications and Significance

Graphing the Line

Knowing the slope and y-intercept allows you to easily graph the line on the coordinate plane:

- Start at the y-intercept (0, -21).
- Use the slope (-4) to find additional points: move 1 unit right (increase x by 1), y decreases by 4 units (so y decreases by 4 from the intercept point).

This process helps in visualizing the line accurately.

Real-world Contexts

Understanding the equation of a line passing through specific points has practical implications, including:

- Predicting trends in data analysis.
- Modeling real-world situations such as speed, cost, or growth rates.
- Solving geometric problems involving lines and points.

Summary

To summarize:

- The line passing through points J(-4, -5) and K(-6, 3) has a slope of -4.
- The line's equation in slope-intercept form is $y = -4x - 21$.
- This equation can be used to graph the line, analyze its behavior, and apply it in various mathematical and real-world contexts.

Understanding how to derive this equation enhances one’s ability to work with linear relationships and deepen comprehension of coordinate geometry principles. Whether for academic purposes, professional applications, or personal curiosity, mastering these steps is a valuable skill in the mathematical toolkit.

Frequently Asked Questions

What is the slope of the line passing through points J(-4,-5) and K(-6,3)?

The slope is $(3 - (-5)) / (-6 - (-4)) = (8) / (-2) = -4$.

How do you find the y-intercept of the line passing through points J and K?

First, find the slope (-4), then use one point (e.g., J(-4,-5)) and apply $y = mx + b$ to solve for b: $-5 = (-4)(-4) + b$, so $b = -5 - 16 = -21$.

What is the equation of the line passing through points J and K in slope-intercept form?

The equation is $y = -4x - 21$.

How do you verify that the line equation y = -4x - 21 passes through points J and K?

Substitute the coordinates of each point into the equation: For J(-4,-5), $y = -4(-4) - 21 = 16 - 21 = -5$ (matches). For K(-6,3), $y = -4(-6) - 21 = 24 - 21 = 3$ (matches).

What is the significance of the slope in the line passing through points J and K?

The slope (-4) indicates the line decreases by 4 units in y for each 1 unit increase in x.

Can the line passing through J(-4,-5) and K(-6,3) be written in other forms besides slope-intercept?

Yes, it can also be written in point-slope form or standard form, such as y + 5 = -4(x + 4).

What is the length of the segment connecting points J and K?

Using the distance formula: $\sqrt{(-6 + 4)^2 + (3 + 5)^2} = \sqrt{(-2)^2 + 8^2} = \sqrt{4 + 64} = \sqrt{68} \approx 8.25$ units.

Why is it important to write the line equation in slope-intercept form?

Because it clearly shows the slope and y-intercept, making it easier to graph the line and understand its behavior.

Additional Resources

Understanding the Equation of Line JK Passing Through Points J(-4, -5) and K(-6, 3): A Comprehensive Analysis

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Introduction to the Line Passing Through Two Points

When exploring the properties of a straight line in coordinate geometry, one fundamental task is to determine the equation of the line given two distinct points. This process involves calculating the slope, understanding the line's characteristics, and expressing it in a specific form—most notably, the slope-intercept form.

In this case, the line passes through the points J(-4, -5) and K(-6, 3). Our goal is to derive its equation in slope-intercept form, which is generally expressed as:

$$
y = mx + b
$$

where:
- m is the slope of the line,
- b is the y-intercept (the point where the line crosses the y-axis).

This analysis will delve into each step of deriving this equation, examining the significance of each component, and exploring related concepts such as the slope, intercepts, and possible applications.

Step 1: Calculating the Slope (m) of Line JK

The first crucial step in determining the line's equation is computing its slope. The slope measures how steep the line is and is defined as the ratio of the change in y-coordinates to the change in x-coordinates between the two points.

Slope formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying to points J(-4, -5) and K(-6, 3):

- $(x_1, y_1) = (-4, -5)$
- $(x_2, y_2) = (-6, 3)$

Calculations:

$$
m = \frac{3 - (-5)}{-6 - (-4)} = \frac{3 + 5}{-6 + 4} = \frac{8}{-2} = -4
$$

Interpretation:

- The slope $m = -4$ indicates that for every unit increase in x, the value of y decreases by 4 units.

- The negative slope suggests a line descending from left to right.

Step 2: Deriving the Equation in Slope-Intercept Form

Now that the slope m is known, the next step is to find the equation of the line in the form $y = mx + b$.

Method:

- Choose either of the two points (preferably one with simpler coordinates) to substitute into the slope-intercept form.
- Solve for b (the y-intercept).

Using point J(-4, -5):

$$
-5 = (-4)(-4) + b
$$

$$
-5 = 16 + b
$$

$$
b = -5 - 16 = -21
$$

Resulting equation:

$$
\boxed{y = -4x - 21}
$$

This is the slope-intercept form of the line passing through points J and K.

Step 3: Verifying the Equation with the Other Point

It's good practice to verify the derived equation with the second point K(-6, 3):

$$
y = -4x - 21
$$

Substituting $x = -6$:

$$
y = -4(-6) - 21 = 24 - 21 = 3
$$

Since the calculated y matches the point K's y-coordinate, the equation is verified and accurate.

Analyzing the Characteristics of the Line

Once the line's equation is established, it's insightful to examine its various features and implications.

1. Slope (m)

- As calculated, $m = -4$.
- The slope indicates a negative linear relationship, with the line decreasing steeply.

2. Y-Intercept (b)

- The y-intercept is -21.
- The point where the line crosses the y-axis occurs at (0, -21).

3. X-Intercept

- To find where the line crosses the x-axis ($y = 0$), set $y = 0$:

$$
0 = -4x - 21
$$

$$
4x = -21
$$

$$
x = -\frac{21}{4} = -5.25
$$

- The x-intercept is at $\left(-\frac{21}{4}, 0\right)$ or approximately (-5.25, 0).

4. Graphical Representation

- The line slopes downward from left to right.
- It crosses the y-axis at (0, -21).
- It crosses the x-axis at approximately (-5.25, 0).

Visualizing this line helps in understanding its position relative to the coordinate axes and other geometric figures.

Step 4: Exploring the Line's Properties in Context

Beyond the basic parameters, understanding the line's properties can unveil deeper insights.

1. Parallel and Perpendicular Lines

- Parallel lines: Lines with the same slope (m). For this line, any line with $m = -4$ is parallel.
- Perpendicular lines: Lines with slopes that are negative reciprocals, i.e., $m_1 \times m_2 = -1$. Since our slope is -4, a perpendicular line would have a slope of $\frac{1}{4}$.

2. Distance from a Point to the Line

- The perpendicular distance from a point (x_0, y_0) to the line $y = mx + b$ is:

$$
d = \frac{|mx_0 - y_0 + b|}{\sqrt{m^2 + 1}}
$$

- This formula is useful in applications like shortest path calculations, collision detection, and more.

3. Applications in Real-World Contexts

- Lines expressed in slope-intercept form are crucial for modeling linear relationships in economics, physics, engineering, and social sciences.
- For example, in economics, this line could represent a cost function or supply/demand curve if appropriately contextualized.

Step 5: Broader Mathematical Implications and Variations

Understanding this specific line opens doors to exploring related concepts:

1. Alternative Forms of a Line

- Standard form: $Ax + By + C = 0$
- Point-slope form: $y - y_1 = m(x - x_1)$

Using the point-slope form with point J(-4, -5):

$$
y - (-5) = -4(x - (-4)) \Rightarrow y + 5 = -4(x + 4)
$$

$$
y + 5 = -4x - 16
$$

$$
y = -4x - 21
$$

which confirms our earlier result.

2. Line Equations in Different Quadrants

- Since the y-intercept is negative, the line crosses the y-axis below the origin.
- The x-intercept at (-5.25) indicates the point where the line crosses into the third quadrant (both x and y negative).

3. Intersection Points with Other Lines or Curves

- Given equations of other lines, intersections can be computed by solving simultaneous equations.
- Such intersections are pivotal in optimization problems, geometric constructions, and coordinate-based problem solving.

Conclusion and Final Remarks

The process of deriving the equation of the line passing through points J(-4, -5) and K(-6, 3) demonstrates fundamental principles of coordinate geometry:

- Calculating the slope provides insight into the line's steepness and direction.
- Substituting a known point into the slope-intercept form allows for direct determination of the y-intercept.
- Verification with the second point ensures the accuracy of the derived equation.
- Understanding intercepts, slope, and the line's position relative to axes enriches comprehension of its geometric properties.

In essence, the line JK in slope-intercept form is:

$$
\boxed{y = -4x - 21}
$$

This equation encapsulates the line's entire geometric information, enabling visualization, analysis, and application in various mathematical contexts.

Final thoughts:

Mastering the derivation of line equations from points enhances problem-solving skills in algebra and geometry. It lays the foundation for more advanced topics like linear inequalities, systems of equations, and analytic geometry. Whether applied to real-world data analysis or theoretical mathematics, understanding how to articulate a line's equation thoroughly and accurately is an essential competency for students and professionals alike.

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