Rewrite The Rational Expression As An Equivalent Rational Expression With The Given Denominator.2t+3/27t+15=

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Understanding how to manipulate rational expressions is a fundamental skill in algebra that helps in simplifying, solving equations, and analyzing functions. In particular, rewriting a rational expression with a different denominator is a common task that involves factors, multiplication, and division of polynomials. This article provides a comprehensive guide on how to rewrite the rational expression $\frac{2t+3}{27t+15}$ as an equivalent expression with a given denominator.

Understanding Rational Expressions

What Is a Rational Expression?

A rational expression is a fraction where both the numerator and the denominator are polynomials. For example:

$$
\frac{2t + 3}{27t + 15}
$$

Here, $2t + 3$ is the numerator polynomial, and $27t + 15$ is the denominator polynomial.

Why Rewrite Rational Expressions?

Rewriting rational expressions with different denominators is often necessary for:

- Adding or subtracting fractions
- Simplifying complex expressions
- Solving equations involving rational expressions
- Comparing rational expressions

Step-by-Step Approach to Rewriting the Expression

Suppose the problem asks to rewrite $\frac{2t+3}{27t+15}$ with a specific given denominator, say d. The process involves several key steps:

Step 1: Factor Both Numerator and Denominator if Possible

Factoring simplifies the expression and helps identify common factors.

Factor the denominator:

$$
27t + 15 = 3(9t + 5)
$$

Factor the numerator:

\[

2t + 3

\]

Since \(2t + 3\) does not factor further over the integers, the expression simplifies to:

\[

\frac{2t + 3}{3(9t + 5)}

\]

Step 2: Determine the Given Denominator

Assuming the problem provides a specific denominator \(D\) (e.g., \(D = 3(9t + 5)\) or another polynomial), we need to rewrite the original fraction with that denominator.

Example: Let's say the given denominator is \(d = 3(9t + 5)\), which is the current denominator. In this case, the expression is already expressed with the given denominator.

Alternatively: If the given denominator is different, for example, \(d = 27t + 45\), then:

\[

27t + 45 = 9(3t + 5)

\]

Note that this is different from the original denominator, so rewriting involves adjusting the numerator accordingly.

Step 3: Find the Necessary Multiplier

To rewrite the original fraction with a new denominator d, determine the factor you need to multiply numerator and denominator by.

$$
\text{Multiplier} = \frac{d}{\text{Original Denominator}}
$$

Example: If the new denominator is $d = 45t + 15$, then:

$$
d = 15(3t + 1)
$$

Compare this with the original denominator:

$$
27t + 15 = 3(9t + 5)
$$

Suppose we want to rewrite the original expression with denominator $d = 45t + 15$:

$$
45t + 15 = 15(3t + 1)
$$

Calculate the multiplier:

$$
\text{Multiplier} = \frac{45t + 15}{27t + 15}
$$

Express numerator and denominator:

$$
\frac{15(3t + 1)}{3(9t + 5)} = \frac{15(3t + 1)}{3(9t + 5)}
$$

Simplify:

$$
\frac{5(3t + 1)}{9t + 5}
$$

Note: To get the multiplier, ensure the denominator divides evenly into the new denominator.

Step 4: Multiply Numerator and Denominator by the Multiplier

Once the multiplier M is identified, multiply both the numerator and the denominator of the original rational expression:

$$
\frac{2t + 3}{27t + 15} \times \frac{\text{Multiplier}}{\text{Multiplier}} = \frac{(2t + 3) \times M}{(27t + 15) \times M}
$$

This process maintains the value of the expression but rewrites it with the new denominator.

Example: Rewriting $\frac{2t+3}{27t+15}$ with a Given Denominator

Let's work through a specific example for clarity.

Given: Rewrite $\frac{2t+3}{27t+15}$ with the denominator $d = 45t + 15$.

Step 1: Factor the original denominator:

$$
27t + 15 = 3(9t + 5)
$$

Step 2: Factor the new denominator:

$$
45t + 15 = 15(3t + 1)
$$

Step 3: Find the multiplier:

$$
M = \frac{45t + 15}{27t + 15} = \frac{15(3t + 1)}{3(9t + 5)} = \frac{5(3t + 1)}{9t + 5}
$$

Step 4: Rewrite the original expression:

$$
\frac{2t + 3}{27t + 15} = \frac{2t + 3}{3(9t + 5)}
$$

Multiply numerator and denominator by M:

$$
= \frac{2t + 3}{3(9t + 5)} \times \frac{5(3t + 1)}{9t + 5}
$$

Express numerator:

$$
(2t + 3) \times 5(3t + 1)
$$

Express denominator:

$$
3(9t + 5) \times (9t + 5)
$$

Calculate numerator:

$$
(2t + 3) \times 5(3t + 1) = 5(2t + 3)(3t + 1)
$$

Calculate denominator:

$$
3(9t + 5)^2
$$

Now, expand numerator:

$$
5 \times [(2t)(3t) + (2t)(1) + 3(3t) + 3(1)] = 5 \times [6t^2 + 2t + 9t + 3] = 5 \times (6t^2 + 11t + 3)
$$

Therefore:

$$
\boxed{
\frac{5(6t^2 + 11t + 3)}{3(9t + 5)^2}
}
$$

This is the equivalent rational expression of $\frac{2t+3}{27t+15}$ with the denominator $45t + 15$.

Additional Tips for Rewriting Rational Expressions

Common Mistakes to Avoid

- Not factoring polynomials completely, which may hide common factors.

- Incorrectly calculating the multiplier, leading to an incorrect equivalent expression.
- Overlooking the need to simplify the final expression after multiplication.
- Ignoring the domain restrictions that may arise after rewriting.

Strategies for Efficient Rewriting

1. Always factor polynomials to identify common factors.
2. Express both denominators in factored form to find the least common denominator (LCD).
3. Use the LCD to determine the multiplier for rewriting the expression.
4. Ensure the multiplication is applied to both numerator and denominator equally.
5. Simplify the resulting expression whenever possible.

Applications of Rewriting Rational Expressions

Rewriting rational expressions is not just an academic exercise; it has practical applications in various areas:

- **Algebraic Simplification:** Simplifies complex fractions for easier manipulation.

- **Solving Rational Equations:** Facilitates finding common denominators to combine fractions.

- **Calculus:** Used in integration and differentiation involving rational functions.

- **Real-World Modeling:** Helps in constructing and analyzing models in physics, engineering, economics, and other fields.

Conclusion

Rewriting a rational expression with a given denominator is a vital skill in algebra that enhances understanding and simplifies complex expressions. The process involves factoring polynomials, determining the appropriate multiplier, and ensuring that both numerator and denominator are multiplied by the same factor to preserve the value of the expression. By mastering these steps, students and professionals can manipulate rational expressions more effectively, enabling them to solve equations, perform algebraic operations, and analyze functions with confidence.

Remember, always verify your rewritten expressions for correctness and domain restrictions

Frequently Asked Questions

How do you rewrite the rational expression (2t + 3) / (27t + 15)

with a different denominator?

To rewrite the expression with a different denominator, factor both numerator and denominator if possible, then find an equivalent expression that has the desired denominator, often by multiplying numerator and denominator by the necessary factor.

What is the common factor in the denominator 27t + 15?

The common factor in 27t + 15 is 3, since 27t + 15 = 3(9t + 5).

How can I simplify (2t + 3) / (27t + 15) before rewriting?

First, factor the denominator: 27t + 15 = 3(9t + 5). Check if the numerator can be factored or simplified, but since 2t + 3 has no common factors with 9t + 5, the expression remains as is.

What is an example of rewriting (2t + 3) / (27t + 15) with denominator 9t + 5?

Since the original denominator is 3(9t + 5), you can rewrite the expression as (2t + 3) / (3(9t + 5)). To have denominator 9t + 5, multiply numerator and denominator by 1/3: [(2t + 3) (1/3)] / [(3(9t + 5)) (1/3)] = ((2t + 3)/3) / (9t + 5).

How do you find the equivalent rational expression with a specified denominator?

Identify what the new denominator should be, then multiply numerator and denominator of the original expression by a factor that makes the denominator equal to the desired one, ensuring the value of the expression remains unchanged.

Can you factor the numerator 2t + 3 to match the denominator's

factors?

No, 2t + 3 is already in simplest form and does not share common factors with 9t + 5, so it cannot be factored to match the denominator's factors directly.

What is the final form of (2t + 3) / (27t + 15) with denominator 9t + 5?

The expression can be rewritten as [(2t + 3)/3] / (9t + 5), which simplifies to ((2t + 3) / 3) divided by (9t + 5).

Why is it important to understand how to rewrite rational expressions with different denominators?

Rewriting rational expressions with common or specific denominators is essential for adding, subtracting, comparing, or simplifying complex fractions in algebra and calculus.

Additional Resources

Rewrite The Rational Expression As An Equivalent Rational Expression With The Given Denominator

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Introduction

In the realm of algebra, rational expressions serve as fundamental tools that facilitate the manipulation of ratios involving polynomials. Their applications span various disciplines, including engineering, physics, and computer science, where they model real-world phenomena and simplify complex equations. A core skill within algebra involves rewriting these expressions to have a specified denominator, a process that demands both conceptual understanding and procedural proficiency.

Today, we delve into the specific task of rewriting the rational expression $\frac{2t + 3}{27t + 15}$ as an equivalent expression with a given denominator—an endeavor that exemplifies the core principles of rational expression manipulation. This investigation not only clarifies the step-by-step methodology but also explores the underlying algebraic structures, common pitfalls, and the significance of such transformations.

The Significance of Rational Expression Rewriting

Rewriting rational expressions with a specified denominator is more than an academic exercise; it is a fundamental skill that underpins operations such as addition, subtraction, and comparison of rational expressions. It enables algebraists to combine fractions seamlessly, find common denominators, and simplify complex equations.

Specifically, in applications such as calculus and algebraic modeling, standardizing denominators allows for easier integration, differentiation, and solution of equations. The process involves understanding how to manipulate numerator and denominator factors, leverage properties of fractions, and perform algebraic equivalence transformations—all of which are crucial for higher-level mathematical reasoning.

The Expression in Question: $\frac{2t + 3}{27t + 15}$

The initial rational expression:

$$
\frac{2t + 3}{27t + 15}
$$

presents an opportunity to explore rewriting with a different denominator. Before proceeding, it is instructive to analyze the structure of numerator and denominator:

- The numerator, $2t + 3$, is a linear polynomial.
- The denominator, $27t + 15$, is also linear, and notably, both coefficients share a common factor.

Step 1: Simplify the Original Expression

A common initial step involves simplifying the original fraction, if possible.

- Identify common factors in numerator and denominator:

Factor the denominator:

$$
27t + 15 = 3 \times (9t + 5)
$$

Similarly, examine the numerator:

$$
2t + 3
$$

- Since numerator and denominator do not share common factors directly, the fraction is in simplest form, but factoring out the common factor in the denominator simplifies the process.

Thus, the original expression can be rewritten as:

$$
\frac{2t + 3}{3(9t + 5)}
$$

Step 2: Recognize the Goal and the Given Denominator

Suppose the problem specifies a particular denominator, say D. For the purpose of illustration, let’s assume the given denominator is:

$$
D = 3(9t + 5) = 27t + 15
$$

which matches the denominator of the original expression. Alternatively, if the problem provides a different denominator, such as $36t + 20$, the process would involve adjusting accordingly.

For demonstration, assume the target denominator is:

$$
D = 36t + 20
$$

This specific choice will help illustrate the general method of rewriting the rational expression.

Step 3: Find the Adjustment Factor

To rewrite $\frac{2t + 3}{27t + 15}$ as an equivalent expression with denominator $36t + 20$, we need to determine a value k such that:

$$
\frac{2t + 3}{27t + 15} = \frac{A}{36t + 20}
$$

where A is an algebraic numerator function to be found.

Since these are equivalent expressions, cross-multiplied, they satisfy:

$$
(2t + 3) \times (36t + 20) = A \times (27t + 15)
$$

Our goal is to find A.

Step 4: Express the Equivalent Numerator

Rearranging the above, the equivalent expression will be:

$$
\frac{A}{36t + 20} = \frac{2t + 3}{27t + 15}
$$

which implies:

$$
A = \frac{(2t + 3) \times (36t + 20)}{27t + 15}
$$

This calculation involves polynomial multiplication and division, which we will explore systematically.

Step 5: Polynomial Multiplication

First, expand the numerator:

$$
(2t + 3)(36t + 20)
$$

Applying distributive property:

$$
2t \times 36t + 2t \times 20 + 3 \times 36t + 3 \times 20
$$

Calculate each term:

- $2t \times 36t = 72t^2$
- $2t \times 20 = 40t$
- $3 \times 36t = 108t$
- $3 \times 20 = 60$

Sum these:

$$
72t^2 + 40t + 108t + 60 = 72t^2 + 148t + 60
$$

Now, our numerator becomes:

$$
A = \frac{72t^2 + 148t + 60}{27t + 15}
$$

Step 6: Polynomial Division

To write A as a polynomial (or rational function) with no remainder, divide numerator by denominator:

$$
\frac{72t^2 + 148t + 60}{27t + 15}
$$

Perform polynomial long division or synthetic division.

Factor the denominator:

$$
27t + 15 = 3(9t + 5)
$$

Similarly, factor numerator if possible:

- Numerator: $72t^2 + 148t + 60$

Check for common factors:

- All coefficients divisible by 4:

$$
4 \times (18t^2 + 37t + 15)
$$

Now, divide numerator and denominator by 3:

$$
\frac{4 \times (18t^2 + 37t + 15)}{3 \times (9t + 5)}
$$

which simplifies to:

$$
\frac{4}{3} \times \frac{18t^2 + 37t + 15}{9t + 5}
$$

Focus on the division:

$$
\frac{18t^2 + 37t + 15}{9t + 5}
$$

Perform polynomial division:

- Divide $18t^2$ by $9t$: quotient $2t$

Multiply divisor:

$$

(9t + 5) \times 2t = 18t^2 + 10t

\]

Subtract:

\[

(18t^2 + 37t + 15) - (18t^2 + 10t) = 0 + 27t + 15

\]

Next, divide \(27t\) by \(9t\): quotient 3

Multiply:

\[

(9t + 5) \times 3 = 27t + 15

\]

Subtract:

\[

(27t + 15) - (27t + 15) = 0

\]

Thus, division yields:

\[

18t^2 + 37t + 15 = (9t + 5)(2t + 3)

\]

Therefore,

$$
\frac{18t^2 + 37t + 15}{9t + 5} = 2t + 3
$$

Putting it all together:

$$
A = \frac{4}{3} \times (2t + 3) = \frac{4(2t + 3)}{3}
$$

Final Equivalent Expression

The original expression $\frac{2t + 3}{27t + 15}$, rewritten with the denominator $36t + 20$, is:

$$
\boxed{
\frac{\frac{4(2t + 3)}{3}}{36t + 20} = \frac{4(2t + 3)}{3(36t + 20)}
}
$$

Alternatively, simplifying numerator:

$$
\frac{8t + 12}{3(36t + 20)}
$$

which can be left as the final rewritten form.

General Methodology for Rewriting Rational Expressions with a Given Denominator

The specific example illustrates a systematic approach applicable to similar problems:

1. Identify the original numerator and denominator.
2. Factor numerator and denominator to simplify the expression, if possible.
3. Determine the target denominator provided by the problem.
4. Express the equivalent fraction as $\frac{A}{D}$, where A is to be found.
5. Set up an equality and cross-multiplied equations to relate the numerators.
6. Calculate the necessary adjustment factor by polynomial multiplication and division.
7. Simplify the resulting numerator to obtain the equivalent expression.

Common Pitfalls and

Rewrite The Rational Expression As An Equivalent Rational Expression With The Given Denominator 2t 3 27t 15

Rewrite The Rational Expression As An Equivalent Rational Expression With The Given Denominator 2t 3 27t 15

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