Suppose That The Functions F And G Are Defined As Follows. F(x)={(x-3)(x-5)} G(x)=x+9 (a) Find ((f)/(g))(-4)

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Introduction

In the realm of algebra and calculus, understanding how to work with functions, especially when combining or dividing them, is a fundamental skill. When given specific functions, such as F(x) and G(x), you often need to evaluate expressions involving these functions at particular points. This process not only reinforces your understanding of functions but also prepares you for more advanced topics like limits, derivatives, and integrals.

In this article, we will explore how to evaluate the expression $\frac{F}{G}(-4)$ given the functions:

- $F(x) = (x - 3)(x - 5)$
- $G(x) = x + 9$

Our goal is to provide a thorough, step-by-step explanation that covers the necessary concepts, calculations, and potential pitfalls. Whether you're a student learning about functions for the first time or someone looking to deepen your understanding, this guide aims to clarify the process and reinforce key mathematical principles.

Understanding the Given Functions

Before tackling the problem, it's crucial to understand the nature of the functions provided.

The Function F(x)

The function $F(x) = (x - 3)(x - 5)$ is a quadratic function expressed in factored form. This form makes it easy to identify its zeros and understand its behavior.

- Zeros of F(x): The solutions to $F(x) = 0$ occur when either factor is zero:
- $x - 3 = 0 \Rightarrow x = 3$
- $x - 5 = 0 \Rightarrow x = 5$

- Graphical interpretation: The parabola opens upwards (since the coefficient of x^2 is positive) and crosses the x-axis at $x=3$ and $x=5$.

The Function G(x)

The function $G(x) = x + 9$ is a linear function. Its simplicity makes it straightforward to evaluate at any point.

- Zero of G(x): $x + 9 = 0 \Rightarrow x = -9$

- Graphical interpretation: A straight line with a slope of 1 and a y-intercept at 9.

The Objective: Evaluating $\frac{F}{G}(-4)$

Our goal is to evaluate the expression:

$$
\frac{F}{G}(-4) = \frac{F(-4)}{G(-4)}
$$

which implies:

1. Find $F(-4)$.
2. Find $G(-4)$.
3. Divide the two results.

This process involves substitution and basic arithmetic, but understanding each step's significance is essential.

Step-by-Step Solution

Step 1: Calculate $F(-4)$

Given $F(x) = (x - 3)(x - 5)$, substitute $x = -4$:

$$
F(-4) = (-4 - 3)(-4 - 5)
$$

Calculate each factor:

- $-4 - 3 = -7$
- $-4 - 5 = -9$

Now multiply:

$$
F(-4) = (-7) \times (-9) = 63
$$

Result: $\boxed{F(-4) = 63}$

Step 2: Calculate $G(-4)$

Given $G(x) = x + 9$, substitute $x = -4$:

$$
G(-4) = -4 + 9 = 5
$$

Result: $\boxed{G(-4) = 5}$

Step 3: Compute $\frac{F(-4)}{G(-4)}$

Now, divide the two results:

$$
\frac{F(-4)}{G(-4)} = \frac{63}{5}
$$

Expressed as a mixed number or decimal:

- As a decimal: $\frac{63}{5} = 12.6$
- As a mixed number: $12 \frac{3}{5}$

Final Answer:

$$
\boxed{\frac{F}{G}(-4) = \frac{63}{5}}
$$

Additional Insights and Considerations

While the above steps provide a straightforward calculation, let's explore some additional concepts related to the problem to deepen understanding.

Domain and Validity

- Domain of F(x): Since $F(x)$ is a quadratic polynomial, it is defined for all real numbers.
- Domain of G(x): Similarly, a linear function is defined everywhere.
- Division considerations: Since $G(-4) = 5 \neq 0$, division by zero does not occur here. However, if at any point $G(x) = 0$, the division would be undefined, which is critical to check in similar problems.

Significance of Function Evaluation

Evaluating functions at specific points helps understand their behavior, roots, and intersections. It also forms the foundation for more advanced concepts like limits and derivatives.

Application in Real-World Contexts

Functions like these appear in various real-world applications:

- Physics: Modeling linear relationships (G(x)) and quadratic relationships (F(x))
- Economics: Cost functions and revenue models
- Engineering: Signal processing and system responses

Understanding how to evaluate and manipulate functions is essential across numerous disciplines.

Common Mistakes to Avoid

When performing similar calculations, be mindful of the following pitfalls:

- Incorrect substitution: Always double-check your substitution to prevent sign errors.
- Ignoring domain restrictions: Division by zero is undefined, so ensure the denominator is not zero.
- Misinterpretation of notation: $\frac{F}{G}(x)$ means $\frac{F(x)}{G(x)}$, not $F(\frac{1}{G(x)})$.
- Simplification errors: When dividing fractions, simplify carefully to reduce errors.

Summary

To summarize, the process of evaluating $\frac{F}{G}(-4)$ involves:

- Substituting -4 into each function.
- Calculating each value carefully.
- Dividing the results to find the final value.

In this case, the explicit calculations were:

$$
F(-4) = 63 \quad \text{and} \quad G(-4) = 5
$$

Therefore,

$$
\frac{F}{G}(-4) = \frac{63}{5}
$$

which simplifies to a decimal of 12.6 or a mixed number $12 \frac{3}{5}$.

Understanding these steps not only solves the problem at hand but also reinforces foundational skills in algebra, preparing you for more complex function manipulations and calculations in higher mathematics.

Additional Practice Problems

To strengthen your understanding, try evaluating similar expressions with different functions or points:

1. If $F(x) = (x + 2)(x - 4)$ and $G(x) = 2x - 3$, find $\frac{F}{G}(3)$.

2. For $F(x) = x^2 - 7x + 10$ and $G(x) = x - 2$, compute $\frac{F}{G}(4)$.

3. Given $F(x) = (x - 1)^2$ and $G(x) = x + 5$, find $\frac{F}{G}(-6)$.

Practicing these will enhance your ability to manipulate functions and evaluate complex expressions confidently.

Conclusion

Evaluating the quotient of functions at a specific point is a fundamental skill in algebra and calculus. By carefully substituting the given value, performing arithmetic operations accurately, and considering the domain restrictions, you can confidently solve such problems. Remember, attention to detail and understanding the properties of functions are key to mastering these concepts.

Frequently Asked Questions

What is the domain of the functions F and G defined as F(x) = (x - 3)(x - 5) and G(x) = x + 9?

The domain of both functions is all real numbers, since both are polynomial (or linear) functions with no restrictions.

How do you interpret the expression ((f)/(g))(-4)?

It represents the value of the quotient of functions F and G evaluated at x = -4, specifically F(-4) divided by G(-4).

What is the value of F(-4) given F(x) = (x - 3)(x - 5)?

F(-4) = (-4 - 3)(-4 - 5) = (-7)(-9) = 63.

What is the value of G(-4) if G(x) = x + 9?

G(-4) = -4 + 9 = 5.

How do you compute ((f)/(g))(-4) with the given functions?

Calculate F(-4) and G(-4), then divide F(-4) by G(-4).

What is ((f)/(g))(-4) for the functions provided?

It is 63 divided by 5, which equals 12.6.

Are there any restrictions when calculating ((f)/(g))(-4)?

Yes, since G(-4) = 5, which is not zero, the division is valid; division by zero would be undefined.

Additional Resources

Suppose That The Functions F And G Are Defined As Follows. F(x) = (x - 3)(x - 5), G(x) = x + 9.

When working with functions in algebra, particularly in the context of ratios or compositions, understanding how to evaluate expressions such as \(\frac{f}{g}(-4)\) is fundamental. This particular problem requires us to analyze two functions, F and G, and determine the value of their ratio at a specific input. Such problems are common in algebra courses, providing a foundation for more advanced topics like calculus, where functions are often combined and manipulated.

In this guide, we will walk through the process step-by-step, illustrating how to evaluate \(\frac{f}{g}(-4)\) with the given functions. We will discuss the significance of the functions, how to substitute values, and how to interpret the results. Whether you're a student looking to strengthen your understanding or someone brushing up on algebraic skills, this detailed explanation aims to clarify the process thoroughly.

Understanding the Functions F and G

Before diving into calculations, it's essential to understand the definitions of our functions:

- Function F: \(F(x) = (x - 3)(x - 5)\)
- Function G: \(G(x) = x + 9\)

Analyzing F(x)

F is expressed as the product of two binomials. Factoring or expanding this expression can help us understand its behavior:

$$
F(x) = (x - 3)(x - 5)
$$

Expanding F(x):

$$
F(x) = x \times x - 5x - 3x + 15 = x^2 - 8x + 15
$$

This quadratic form reveals that F(x) is a parabola opening upwards with roots at x=3 and x=5.

Analyzing G(x)

G is a simple linear function:

$$
G(x) = x + 9
$$

This indicates a straight line with a y-intercept at 9 and a slope of 1.

Step-by-Step Solution to $\frac{f}{g}(-4)$

Our goal is to evaluate:

$$
\left(\frac{f}{g}\right)(-4)
$$

which is the same as:

$$
\frac{F(-4)}{G(-4)}
$$

Step 1: Find $F(-4)$

Using the expanded form of F(x):

$$
F(x) = x^2 - 8x + 15
$$

Substitute $x = -4$:

$$
F(-4) = (-4)^2 - 8 \times (-4) + 15
$$

Calculate each term:

- $(-4)^2 = 16$
- $-8 \times -4 = 32$
- 15 remains as is

Adding these:

$$
F(-4) = 16 + 32 + 15 = 63
$$

Step 2: Find $G(-4)$

Using the definition of G:

$$
G(x) = x + 9
$$

Substitute $x = -4$:

$$
G(-4) = -4 + 9 = 5
$$

Step 3: Calculate $\frac{F(-4)}{G(-4)}$

Now, divide the two values:

$$
\frac{F(-4)}{G(-4)} = \frac{63}{5}
$$

This fraction cannot be simplified further, so the exact value is $\frac{63}{5}$.

Final Result:

$$
\boxed{\frac{63}{5}}
$$

which is equivalent to 12.6 in decimal form.

Additional Considerations

Domain Restrictions

While the calculation is straightforward, it's important to consider the domain of the ratio $\frac{f}{g}$. Since division by zero is undefined, we must ensure that $G(x) \neq 0$ at $x = -4$. From earlier:

$$
G(-4) = 5 \neq 0
$$

Thus, the ratio is defined at $x = -4$.

Significance of the Result

The ratio $\frac{F(x)}{G(x)}$ at specific points can have various interpretations depending on the context—be it in physics, economics, or pure mathematics. In algebra, it primarily helps us understand how two functions relate to each other at particular values of x.

Extending the Concept

Evaluating at Different Points

You can apply the same steps to evaluate $\frac{f}{g}$ at any other value of x. For example, evaluate at $x = 0$:

- Find $F(0)$ and $G(0)$
- Divide to get the ratio

This approach fosters understanding of how functions behave relative to each other across their domains.

Graphical Interpretation

Plotting F(x) and G(x) provides visual insights:

- F(x): a parabola opening upwards with roots at 3 and 5
- G(x): a line with slope 1 and intercept 9

The ratio $\frac{F(x)}{G(x)}$ can be visualized as the ratio of the parabola's value to that of the line at various points, revealing where the ratio is increasing, decreasing, or approaching particular asymptotes.

Summary

In this guide, we've thoroughly examined how to evaluate $\frac{f}{g}(-4)$ given the functions:

- $F(x) = (x - 3)(x - 5)$
- $G(x) = x + 9$

The key steps involved:

1. Expanding F(x) to $x^2 - 8x + 15$
2. Substituting $x = -4$ into both functions
3. Calculating $F(-4) = 63$
4. Calculating $G(-4) = 5$
5. Dividing to find the ratio: $\frac{63}{5}$

This process illustrates fundamental algebra skills—substitution, expansion, and division—and reinforces understanding of how functions can be combined and evaluated at specific points.

By mastering these techniques, you'll be well-equipped to handle more complex function operations, paving the way for success in higher-level mathematics and related fields.

Remember: Always check for domain restrictions before performing division, and interpret your results within the context of the problem. Happy calculating!

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