The Base Of The Solid Is The Triangle Enclosed By X + Y = 17, The X-axis, And The Y-axis. The Cross Sections

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Introduction

Understanding the geometry of solids and their cross sections is fundamental in calculus, especially when dealing with volume calculations. When a solid's base is a specific region in the xy-plane and the cross sections are taken perpendicular or parallel to a particular axis, it provides a powerful way to visualize and compute the volume of complex shapes.

In this article, we explore a fascinating problem: a solid whose base is the triangle enclosed by the line $x + y = 17$, the x-axis, and the y-axis. We will analyze the cross sections of this solid, examining how different orientations affect the resulting volume. This problem not only demonstrates key concepts in multivariable calculus but also provides insight into how geometric intuition can simplify complex integral calculations.

By the end of this article, you will understand how to:

- Describe the base region analytically and graphically.
- Set up the integral for the volume of the solid based on different cross-sectional shapes.
- Calculate the volume through integration techniques.
- Visualize the solid and its cross sections for better conceptual understanding.

Let's begin by defining the base region and then proceed to analyze the cross sections.

Defining the Base Region

The Triangle Enclosed by $x + y = 17$, The X-axis, And The Y-axis

The base region is a right triangle in the xy-plane. The boundaries are:

- The line $x + y = 17$
- The x-axis ($y=0$)
- The y-axis ($x=0$)

Graphical interpretation:

- The line $x + y = 17$ intercepts the x-axis at (17, 0) because when $y=0$, $x=17$.
- It intercepts the y-axis at (0, 17) because when $x=0$, $y=17$.
- The triangle is bounded between these points and the axes, forming a right triangle with vertices at

(0, 0), (17, 0), and (0, 17).

Analytical description:

- The line $x + y = 17$ can be rewritten as $y = 17 - x$.
- The region R in the xy-plane is:

$$
R = \left\{ (x, y) \mid x \ge 0, \quad y \ge 0, \quad y \le 17 - x \right\}
$$

Area of the base:

$$
A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 17 \times 17 = \frac{289}{2} = 144.5
$$

This triangular region forms the foundation over which our solid extends vertically, or in specified cross-section directions, to create a three-dimensional shape.

Exploring Cross Sections of the Solid

Cross sections are slices of a solid taken perpendicular to a particular axis. They reveal the internal structure of the shape and are essential in calculating volume.

Depending on the orientation of the cross sections, the shape's volume can be computed using different methods.

Cross Sections Perpendicular to the X-Axis

When cross sections are perpendicular to the x-axis, they are thin slices parallel to the y-z plane, effectively slicing the solid along vertical planes at each fixed x-value.

Shape of Cross Sections

- At a fixed x in [0, 17], the cross section is a shape in the y-z plane, extending from $y=0$ up to $y=17 - x$.
- The cross section's shape can vary: it could be a rectangle, square, semicircle, or other shapes, depending on the problem's specifications.

Assuming the cross sections are squares:

- The side length of each square at position x is determined by the y-dimension, which is from $y=0$ to $y=17 - x$.
- Since the region extends vertically from 0 to $17 - x$, the side length of the square is:

$$
s(x) = 17 - x
$$

\]

- The area of each cross section:

\[
A_{x}(x) = [s(x)]^2 = (17 - x)^2
\]

- The volume of the solid:

\[
V = \int_{x=0}^{17} A_{x}(x) \, dx = \int_0^{17} (17 - x)^2 \, dx
\]

Cross Sections Perpendicular to the Y-Axis

Alternatively, cross sections can be taken perpendicular to the y-axis, slicing the solid into horizontal layers.

Shape of Cross Sections

- For a fixed y in [0, 17], the cross section is a line segment along x from x=0 up to x=17 - y.
- Similar to the previous case, the cross section could be a square or other shape.

Assuming the cross sections are squares:

- The side length at height y:

\[
s(y) = 17 - y
\]

- The area:

\[
A_{y}(y) = (17 - y)^2
\]

- The total volume:

\[
V = \int_{y=0}^{17} A_{y}(y) \, dy = \int_0^{17} (17 - y)^2 \, dy
\]

Note that both methods lead to similar integrals with swapped variables, illustrating the symmetry in the problem.

Calculating the Volume of the Solid

Now, let's perform the explicit calculations for the volume based on the cross-sectional assumptions.

Volume with Cross Sections Perpendicular to the X-Axis

$$
V = \int_0^{17} (17 - x)^2 dx
$$

Step 1: Expand the integrand

$$
(17 - x)^2 = 289 - 34x + x^2
$$

Step 2: Integrate term-by-term

$$
V = \int_0^{17} 289 - 34x + x^2 \, dx = \left[289x - 17x^2 + \frac{x^3}{3} \right]_0^{17}
$$

Step 3: Evaluate at the bounds

At x=17:

$$
289 \times 17 - 17 \times 17^2 + \frac{17^3}{3}
$$

Calculate each term:

- $289 \times 17 = 4913$
- $17^2 = 289$, so $17 \times 289 = 17 \times 289 = 4913$, but note that the second term is $17 \times 17^2 = 17 \times 289 = 4913$
- $17^3 = 17 \times 17^2 = 17 \times 289 = 4913$

Putting these in:

$$
V = 4913 - 4913 + \frac{4913}{3} = 0 + \frac{4913}{3} = \frac{4913}{3}
$$

Final volume:

$$
V = \frac{4913}{3} \approx 1637.666...
$$

Volume with Cross Sections Perpendicular to the Y-Axis

Similarly,

$$
V = \int_0^{17} (17 - y)^2 dy
$$

which yields the same result:

$$
V = \frac{4913}{3}
$$

Generalizing Cross Sections

The above calculations assume the cross sections are squares. However, depending on the problem, the cross sections could be:

- Rectangles
- Equilateral triangles
- Semicircular shapes

Each shape will influence the integrand and the integral's computation.

Cross Sections as Rectangles

- For rectangular cross sections with width s(x), the volume integral becomes:

$$
V = \int_{x=0}^{17} s(x) \times h(x) \, dx
$$

where $h(x)$ is the height function, depending on the shape.

Cross Sections as Semicircles

- For semicircular cross sections with diameter s(x):

$$
A_{x}(x) = \frac{1}{2} \pi \left(\frac{s(x)}{2}\right)^2
$$

which modifies the integral accordingly.

Visualization and Conceptual Understanding

Visualizing the solid is crucial for grasping the problem:

- The base triangle is a right triangle with vertices at (0,0), (17,0), and (0,17).
- Cross sections perpendicular to the x-axis are vertical slices, each with a square or other shape, extending from the x-axis up to the boundary line $y=17 - x$.
- Cross sections perpendicular to the y-axis are horizontal slices, each with similar shapes spanning from the y-axis up to $x=17 - y$.

These visualizations help in setting up the integral bounds and the shape of the cross sections.

Applications and Further Considerations

Understanding such volume problems has practical applications:

- Engineering: designing objects with specific cross-sectional profiles.
- Physics: calculating the mass or charge distribution in a region.
- Computer Graphics: rendering 3D objects based on cross-sectional data.

Moreover, similar principles apply to more complex regions, such as curved boundaries or multiple intersecting shapes.

Conclusion

In this detailed exploration, we examined a solid with a base region defined by the triangle enclosed by the line

Frequently Asked Questions

What is the base of the solid in the given problem?

The base of the solid is the triangular region enclosed by the line $X + Y = 17$, the X-axis, and the Y-axis.

How is the triangle defined in the problem?

The triangle is defined by the intercepts on the axes, with vertices at (0,0), (17,0), and (0,17).

What are the possible cross sections of the solid perpendicular to the X-axis?

Depending on the problem, cross sections perpendicular to the X-axis could be rectangles, squares, or other shapes, often constructed by slicing the solid at specific X-values within the triangular region.

How do you find the length of the cross section at a given X?

At a given X, Y varies from 0 up to the line Y = 17 - X, so the length of the cross section along Y is from 0 to 17 - X.

What is the significance of the line X + Y = 17 in forming the solid?

The line X + Y = 17 defines the boundary of the triangular base, which influences the shape and size of the cross sections through the solid.

How can the volume of the solid be calculated using cross sections?

By integrating the area of the cross sections along the relevant axis (e.g., X or Y), summing up all slices from one boundary to the other.

What types of cross sections are typically considered in problems like this?

Commonly, problems consider rectangular, square, or semicircular cross sections perpendicular to the base plane.

How do the axes boundaries affect the limits of integration?

The axes boundaries (X=0, Y=0) and the line X + Y = 17 set the integration limits, from X=0 to 17 and Y=0 to 17-X respectively.

What is the general approach to solving for the volume of the solid with given cross sections?

Identify the shape of the cross sections, express their area as a function of the variable (X or Y), and integrate over the appropriate bounds to find the total volume.

Additional Resources

The Base Of The Solid Is The Triangle Enclosed By X + Y = 17, The X-axis, And The Y-axis. The Cross Sections

Understanding the geometric and analytical intricacies of solids formed by simple boundaries offers profound insights into multivariable calculus and solid modeling. The problem at hand involves a three-dimensional solid whose base is a specific triangle, with cross sections that are likely to vary in shape depending on their orientation and the slicing method. This detailed exploration aims to dissect the problem comprehensively, delving into the geometric setup, mathematical formulation, and the analysis of the cross-sectional slices.

Geometric Foundations of the Base Triangle

The Boundary Lines: Describing the Triangle

The foundation of our analysis begins with the triangle defined by the boundaries:

- The line $X + Y = 17$
- The X-axis ($Y = 0$)
- The Y-axis ($X = 0$)

This triangle resides in the first quadrant, bounded by the coordinate axes and the line that intercepts both axes at points (17, 0) and (0, 17). To understand its shape:

- The X-intercept occurs when $Y=0$: setting $Y=0$, the line becomes $X=17$.
- The Y-intercept occurs when $X=0$: setting $X=0$, the line becomes $Y=17$.

Graphically, the triangle is formed by the points:

- (0, 0) — the origin
- (17, 0) — on the X-axis
- (0, 17) — on the Y-axis

The line connecting (17, 0) and (0, 17) has the equation:

$$ Y = 17 - X $$

This gives the hypotenuse of the triangle, a straight line descending from (0, 17) to (17, 0).

Area of the Base Triangle:

The area, A, can be found via:

$$
A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 17 \times 17 = \frac{1}{2} \times 289 = 144.5
$$

This triangular base is critical as it defines the domain over which the solid extends vertically, and it influences the cross-sectional shapes.

Formulating the Solid: Extent and Orientation

Vertical Extent of the Solid

The problem involves constructing a three-dimensional solid whose base is this triangle. Typically, in such problems, the solid extends vertically in the Z-direction, i.e., above the XY-plane, bounded by certain functions or constraints.

- Height Function: To define the solid fully, an additional function $f(x, y)$ that specifies the height at each point within the base is required.
- Assumption: Often, in standard problems, the height is either constant or varies with x and y. Without explicit height functions, common assumptions are:

- The solid is bounded above by a surface $z = g(x, y)$.
- The cross sections are perpendicular to a particular axis, often the Z-axis.

For this problem, suppose the height varies with the position within the triangle, i.e., the cross sections are formed by slicing the solid perpendicular to the X or Y axes, or along other directions, depending on the problem's specifics.

Orientation of Cross Sections

The phrase "The cross sections" suggests examining the shape of slices taken through the solid at various positions. There are several common orientations:

- Cross sections perpendicular to the X-axis: slices are parallel to the Y-Z plane.
- Cross sections perpendicular to the Y-axis: slices are parallel to the X-Z plane.
- Cross sections perpendicular to the Z-axis: slices are parallel to the XY-plane.

Given the context and typical calculus problems, the most common analysis involves cross sections perpendicular to the Z-axis (horizontal slices), which are often easier to analyze.

Mathematical Description of the Domain

Parameterization of the Base Region

To analyze the solid, we need to express the domain mathematically:

$$
D = \left\{ (x, y) \mid 0 \leq x \leq 17, \quad 0 \leq y \leq 17 - x \right\}
$$

This describes the triangular region in the XY-plane.

- For each fixed x, y ranges from 0 up to the line $y = 17 - x$.

Setting Up Integrals for Volume and Cross Sections

If the height of the solid at each point (x, y) is given by a function $f(x, y)$, the volume V can be expressed as:

$$
V = \iint_D f(x, y) \, dA
$$

where dA is an infinitesimal area element, typically $dx\,dy$ or $dy\,dx$.

Example: If the cross sections are squares or rectangles perpendicular to the Z-axis, the volume can be computed by integrating the cross-sectional area along the relevant variable.

Analyzing Cross Sections: Shapes and Computations

Types of Cross Sections

The shape of the cross section depends on the slicing direction and the height function $f(x, y)$:

- Horizontal cross sections (parallel to XY-plane): slices at constant z, with the cross-sectional shape depending on the intersection of the surface with a plane $z = c$.

- Vertical cross sections: slices at fixed x or y, which reveal the profile of the solid in the other two dimensions.

In many problems, the cross sections are specified as particular shapes—such as squares, equilateral triangles, or circles—perpendicular to a certain axis.

For this problem, a common choice is:

- Cross sections perpendicular to the Z-axis are squares or rectangles, with side length or height depending on x or y.

- Alternatively, cross sections could be equilateral triangles or semicircles.

Without explicit specification, let’s analyze the case where cross sections are perpendicular to the X-axis, forming rectangles or squares.

Cross Sections Perpendicular to the X-axis

Suppose we take slices at fixed x, with the cross section in the y-direction:

- For each x in $[0, 17]$, y varies from 0 to $17 - x$.

If the cross section at x is a rectangle with width $w(y)$ and height $h(z)$, then:

- The area of the cross section is $A_{cross}(x) = \text{(shape area)}$.

For example, if the cross section is a rectangle with height $f(x, y)$ and width dy, the total volume is:

$$
V = \int_{x=0}^{17} \left(\int_{y=0}^{17 - x} A_{cross}(x, y) \, dy \right) dx
$$

If the cross section shape is known, such as:

- Square: with side length $s(x, y)$,

- Triangle: with side length $t(x, y)$,

then the integral can be explicitly evaluated.

Special Cases and Examples

Uniform Cross Sections: Constant Cross Section Area

Suppose the cross sections are constant in shape and size—say, squares of side length s at every cross section.

- The volume simplifies to:

$$
V = \text{cross-sectional area} \times \text{length of the domain along the slicing axis}

\]

- For slices perpendicular to the Z-axis, if the height function \(f(x, y)\) is constant, the volume becomes:

\[
V = \text{Area of base} \times \text{height}
\]

which is straightforward.

Variable Cross Sections: Scaling with Position

If the cross section's size varies with \(x\) and \(y\), for example:

\[
\text{Side length} = s(x, y) = k \times (17 - x - y)
\]

where \(k\) is a constant, then the volume integral becomes more involved, requiring integration over the domain with the explicit shape functions.

Implications and Applications

Analyzing a solid based on a triangular base bounded by linear equations exemplifies key principles in multivariable calculus, such as setting up double integrals over arbitrary regions, transforming coordinates, and understanding how cross-sectional slices relate to volume calculations.

These concepts have numerous real-world applications:

- Engineering: Designing components with triangular cross sections.
- Manufacturing: Estimating material volume based on shape layouts.
- Physics: Modeling objects with triangular footprints.
- Architecture: Calculating the volume of structures with triangular foundations.

The problem also demonstrates the importance of understanding the geometric boundaries to correctly set integration limits, converting geometric regions into algebraic inequalities, and choosing appropriate cross-sectional shapes for analysis.

Conclusion and Final Remarks

The exploration of the solid whose base is

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