The Curve Y = 2x^28 Is Revolved Occured The X-axis, What Is The Volume Of The Solid Formed By The Revolution?

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Understanding the volume of solids generated by revolving a curve around an axis is a fundamental concept in calculus, especially in the study of solids of revolution. In this article, we will explore the process of calculating the volume of the solid formed when the curve y = 2x^28 is revolved about the x-axis. This involves applying methods such as the disk method and integral calculus to find a precise answer. Whether you're a student preparing for exams or a mathematics enthusiast interested in the geometric interpretation of functions, this comprehensive guide will walk you through each step in detail.

Understanding the Problem

The problem involves a specific curve, y = 2x^28, and the task is to determine the volume of the three-dimensional solid obtained when this curve is revolved around the x-axis. To approach this problem systematically, it is crucial to understand the key components involved:

- The curve: y = 2x^28, a polynomial function with a very high degree.
- The axis of revolution: the x-axis, around which the curve is rotated.
- The interval of interest: generally, for such problems, the interval is from x = a to x = b. If not specified, we often assume from x = 0 to x = some value, or analyze over a specific domain.

In most typical problems involving such functions, the interval is either from 0 to some positive value or from negative to positive values if the function is symmetric.

Note: Since the problem does not specify the limits, we will assume the interval from x = 0 to x = 1 for simplicity and general understanding, but the methodology applies for any interval.

Mathematical Background: Solids of Revolution and the Disk Method

The process of determining the volume of a solid of revolution involves integrating the cross-sectional areas perpendicular to the axis of rotation. The most common methods are:

The Disk Method

When a curve $y = f(x)$ is revolved around the x-axis, the volume V between $x = a$ and $x = b$ is given by:

$$
V = \pi \int_{a}^{b} [f(x)]^2 \, dx
$$

This formula represents summing the volumes of infinitesimally thin disks with radius $f(x)$, where each disk's volume is $\pi [f(x)]^2 dx$.

Application to Our Function

In our case, the function is $y = 2x^{28}$, so the radius of each disk is $y = 2x^{28}$. The volume element becomes:

$$
dV = \pi [2x^{28}]^2 dx = \pi \times 4x^{56} dx
$$

Therefore, the total volume V from $x = 0$ to $x = 1$ is:

$$
V = \pi \int_{0}^{1} 4x^{56} dx
$$

Now, we can proceed to evaluate this integral.

Calculating the Volume Step-by-Step

Step 1: Set Up the Integral

Assuming the interval from 0 to 1, the volume is:

$$
V = 4\pi \int_{0}^{1} x^{56} dx
$$

If the interval differs, simply replace the limits accordingly.

Step 2: Integrate the Power Function

Recall the integral of x^n:

\[

$$
\int x^n dx = \frac{x^{n+1}}{n+1} + C
$$

Applying this, we get:

$$
V = 4\pi \times \left[\frac{x^{57}}{57} \right]_0^{1}
$$

Evaluating at the bounds:

$$
V = 4\pi \times \left(\frac{1^{57}}{57} - \frac{0^{57}}{57} \right) = 4\pi \times \frac{1}{57}
$$

Final Volume:

$$
V = \frac{4\pi}{57}
$$

This is the volume of the solid formed when the curve $y = 2x^{28}$ is revolved around the x-axis from $x = 0$ to $x = 1$.

Generalization for Different Intervals

If you need to compute the volume over a different interval, say from $x = a$ to $x = b$, the integral becomes:

$$
V = \pi \int_{a}^{b} [2x^{28}]^2 dx = 4\pi \int_{a}^{b} x^{56} dx
$$

which evaluates to:

$$
V = 4\pi \times \left[\frac{x^{57}}{57} \right]_a^b = \frac{4\pi}{57} (b^{57} - a^{57})
$$

This formula allows you to compute the volume for any desired bounds.

Understanding the Behavior of the Function and Result

The function $y = 2x^{28}$ is an even function because x^{28} is an even power, and multiplying by 2 does not change that property. This symmetry implies the volume generated when revolving around the x-axis from -b to b can be calculated similarly, considering the symmetry to simplify calculations.

Implication:
If the interval is symmetric around zero, the volume can be doubled:

$$
V_{-b}^{b} = 2 \times V_{0}^{b}
$$

since the contributions from negative x-values mirror those from positive x-values.

Visualizing the Solid of Revolution

Visualizing the solid can help better understand the problem:

- The curve $y = 2x^{28}$ starts at the origin and rises very slowly for small x, but since the power is high, it grows rapidly as x increases.
- When revolved around the x-axis, each point on the curve traces a circular disk with radius $y = 2x^{28}$.
- The resulting solid resembles a highly elongated, symmetric shape with a narrow central region near $x = 0$ and expanding outward as x increases.

Practical Applications and Significance

Calculating volumes of solids of revolution is important in various scientific and engineering fields:

- Manufacturing: Designing objects with rotational symmetry.
- Physics: Calculating moments of inertia or mass distributions.
- Mathematics: Visualizing functions and their associated volumes.
- Architecture: Creating complex structures based on mathematical models.

Understanding how to compute these volumes provides a foundation for more advanced topics like surface area calculations and three-dimensional modeling.

Summary and Key Takeaways

- The volume of the solid formed by revolving $y = 2x^{28}$ around the x-axis over an interval [a, b] is given by:

$$
V = \frac{4\pi}{57} (b^{57} - a^{57})
$$

- The computation hinges on applying the disk method, integrating the squared function over the specified interval.
- Symmetry properties of the function can simplify calculations over symmetric intervals.
- High-degree polynomial functions produce complex but predictable volumetric shapes when revolved around an axis.

Conclusion

Calculating the volume of a solid of revolution, such as that generated by revolving $y = 2x^{28}$ about the x-axis, demonstrates the power of integral calculus in solving three-dimensional geometry problems. By understanding the fundamental principles, setting up the integral correctly, and evaluating it meticulously, you can determine the exact volume for any interval. This process not only enhances mathematical comprehension but also opens the door to numerous applications across sciences and engineering.

Remember: Always clarify the limits of integration before starting calculations, and consider the symmetry of the function for more straightforward solutions. Whether for academic purposes or practical applications, mastering these techniques is essential for advanced mathematical problem-solving.

Frequently Asked Questions

What is the general method to find the volume of a solid formed by revolving a curve around the x-axis?

The typical method is to use the disk or washer method, where the volume is calculated by integrating π times the square of the function over the given interval.

How do you set up the integral to find the volume of the solid formed by revolving y = 2x^28 around the x-axis?

The volume V is given by $V = \int_{a}^{b} \pi [f(x)]^2 \, dx$, where $f(x) = 2x^{28}$, over the interval [a, b].

What is the specific integral expression for the volume when revolving y = 2x^28 around the x-axis from x = 0 to x = 1?

$V = \int_{0}^{1} \pi (2x^{28})^2 \, dx = \int_{0}^{1} \pi 4x^{56} \, dx$.

How do you evaluate the integral to find the volume formed by the revolution of y = 2x^28?

Evaluate the integral as $V = \pi \, 4 \int_{0}^{1} x^{56} \, dx = 4\pi \left[\frac{x^{57}}{57}\right]_{0}^{1} = 4\pi / 57$.

What is the final volume of the solid formed by revolving y = 2x^28 around the x-axis from x = 0 to x = 1?

The volume is $V = 4\pi / 57$ cubic units.

Can this method be applied for any interval or just from 0 to 1?

Yes, the same method applies for any interval [a, b], with the integral adjusted accordingly: $V = \int_{a}^{b} \pi [f(x)]^2 \, dx$.

What is the significance of the power 28 in the function y = 2x^28 regarding the volume calculation?

The power 28 affects the exponent in the integral, resulting in a higher power (x^{56}) after squaring, which influences the shape and size of the solid's volume.

Are there any special considerations when calculating the volume of a solid of revolution for polynomial functions like y = 2x^28?

Yes, because the function involves high powers, ensure the integral converges over the chosen interval and carefully evaluate the resulting polynomial integrals for accuracy.

Additional Resources

The Curve $y = 2x^{28}$ Is Revolved Occurred The X-axis, What Is The Volume Of The Solid Formed By The Revolution?

Understanding the geometric and calculus principles behind the revolution of curves around axes is fundamental to many applications in engineering, physics, and mathematics. The specific case of revolving the curve $y = 2x^{28}$ about the x-axis provides a fascinating example of how to determine the volume of the resulting solid using integral calculus. This article offers a detailed exploration of this problem, highlighting the mathematical techniques involved, the significance of the curve's properties, and the implications of the volume calculation.

Introduction to Solid of Revolution and Its Significance

The concept of creating a three-dimensional object by revolving a two-dimensional curve around an axis is a cornerstone of integral calculus, often referred to as the method of disks or washers. This process is pivotal in various fields such as:

- Manufacturing: Designing objects with rotational symmetry.
- Physics: Calculating moments of inertia or mass distributions.
- Mathematics: Exploring volume, surface area, and related properties of complex shapes.

By revolving the curve $y = 2x^{28}$ about the x-axis, we generate a solid whose volume can be precisely calculated. This process transforms a simple algebraic function into a tangible three-dimensional shape, enabling analysis and applications that require understanding of its size and structure.

Mathematical Foundations of the Problem

Understanding the Curve $y = 2x^{28}$

The given function $y = 2x^{28}$ is a polynomial of a very high degree. Its key properties include:

- Domain: $x \geq 0$ (assuming the standard context for volume calculations, since negative x would produce negative y, which in some contexts might be meaningful, but typically the focus is on $x \geq 0$).
- Range: $y \geq 0$, because x^{28} is non-negative for all real x.

The curve is extremely flat near $x = 0$ due to the high exponent, but as x increases, y grows rapidly because of the 28th power.

Revolution Around the X-Axis

When the curve $y = 2x^{28}$ is revolved around the x-axis, each point on the curve traces out a circle with radius equal to y at that x-value. The volume of the generated solid is computed via the method of disks:

$$
V = \pi \int_{a}^{b} [\text{radius}]^2 dx
$$

In this context:

- Radius: $y = 2x^{28}$
- Range of integration: depends on the interval over which the revolution occurs. For a finite segment [a, b], the volume is finite; for the entire domain $x \geq 0$, the volume may be infinite if the integral diverges.

Determining the Limits of Integration

The problem as posed does not specify explicit bounds, so we analyze both:

- Finite interval [0, c]
- Infinite interval [0, ∞)

1. Finite Interval [0, c]:

Choosing a specific upper limit $c > 0$, the volume becomes:

$$
V = \pi \int_{0}^{c} [2x^{28}]^2 dx = \pi \int_{0}^{c} 4x^{56} dx
$$

2. Infinite Interval [0, ∞):

The volume is:

$$
V = \pi \int_{0}^{\infty} 4x^{56} dx
$$

The convergence of this integral depends on whether the integral of x^{56} over [0, ∞) converges, which it does not because:

$$
\int_{0}^{\infty} x^{n} dx \to \text{diverges for all } n \geq 0
$$

Hence, the volume for the entire positive x-axis is infinite, which is a critical insight.

Calculating the Volume for a Finite Interval

Let's proceed with the calculation over a finite interval [0, c].

Integral Setup

$$
V = \pi \int_{0}^{c} 4x^{56} dx
$$

This simplifies to:

$$
V = 4\pi \int_{0}^{c} x^{56} dx
$$

Using the power rule for integrals:

$$
\int x^{n} dx = \frac{x^{n+1}}{n+1}
$$

Applying this:

$$
V = 4\pi \left[\frac{x^{57}}{57} \right]_0^{c} = \frac{4\pi}{57} c^{57}
$$

Result:

$$
V(c) = \frac{4\pi}{57} c^{57}
$$

This expression allows us to compute the volume of the solid generated by revolving the curve $y = 2x^{28}$ over any finite interval [0, c].

Analysis of the Volume: Behavior and

Implications

Dependence on the Interval Length

The volume grows rapidly with increasing c, proportional to c^{57}. For small c, the volume is negligible, but as c increases, the volume explodes exponentially due to the high power.

Example:
- For c = 1,

$$
V(1) = \frac{4\pi}{57} \approx 0.2209
$$

- For c = 2,

$$
V(2) = \frac{4\pi}{57} \times 2^{57}
$$

This illustrates the enormous growth, emphasizing how the high degree polynomial influences the size of the solid.

Implications for the Infinite Interval

As established, the volume over $[0, \infty)$ diverges to infinity because:

$$
\lim_{c \to \infty} V(c) = \infty
$$

This indicates that the entire revolution of the curve $y = 2x^{28}$ about the x-axis produces an infinitely large solid, which is not physically realizable but mathematically significant.

Mathematical Significance and Applications

The calculation underscores key concepts:

- High-degree polynomials produce rapidly expanding volumes upon revolution, especially over infinite domains.

- The importance of bounds in integral calculus to determine finite versus infinite volumes.
- Application in engineering design: understanding how the shape of a curve influences the volume of a generated object, crucial in manufacturing and material science.

In broader terms, this problem illustrates how calculus bridges the gap between simple algebraic functions and complex three-dimensional geometries, enabling precise volume calculations essential in science and engineering.

Extensions and Further Considerations

1. Surface Area Calculations:

Beyond volume, the surface area of the solid can also be computed, involving integrals with the arc length formula.

2. Other Axis of Revolution:

Revolving around axes other than the x-axis (e.g., y-axis) introduces different integral setups, often involving inverse functions or other techniques.

3. Numerical Methods:

For more complex or less straightforward functions, numerical integration methods like Simpson’s rule or trapezoidal rule are employed to approximate the volume.

4. Physical Realism:

While mathematically intriguing, physical objects with such high exponents are impractical; however, these calculations help in understanding theoretical models and approximations.

Conclusion

The problem of determining the volume of the solid formed by revolving the curve y = 2x^{28} about the x-axis encapsulates fundamental calculus principles and highlights the profound impact of polynomial degrees on geometric properties. For a finite interval, the volume is explicitly calculable, revealing a dependence on the upper bound raised to the 57th power. Over an infinite domain, the volume diverges, emphasizing the importance of bounds in integral calculus.

This exploration not only provides insight into the mathematical techniques involved but also underscores the significance of such calculations in practical applications and theoretical modeling. It demonstrates how high-degree polynomial functions, though seemingly simple, can generate complex and expansive geometric structures when revolved around an axis.

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Author's Note:

This article aims to provide a comprehensive understanding of the mathematical process behind calculating the volume of a solid of revolution for the specific function y = 2x^{28}. The analysis underscores the relevance of integral calculus in translating algebraic functions into geometric objects, a fundamental aspect of mathematical sciences.

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