The Dimensions Of This Figure Are Changed So That The New Surface Area Is Exactly 1/3 What It Was Originally.

The Dimensions Of This Figure Are Changed So That The New Surface Area Is Exactly 1/3 What It Was Originally. This statement introduces a fascinating problem in geometry and scaling: how changing the dimensions of a three-dimensional figure affects its surface area. Understanding this relationship is fundamental in fields ranging from engineering and architecture to basic mathematics. In this article, we will explore the principles behind scaling figures, analyze how surface area changes with size modifications, and work through a detailed example to illustrate these concepts.

Understanding the Relationship Between Dimensions and Surface Area

Scaling in Geometry

When we modify the size of a figure, we often do so by applying a scale factor to its dimensions. For instance, if the original figure has length L, width W, and height H, then after scaling by a factor k, the new dimensions become:

- $L' = kL$
- $W' = kW$
- $H' = kH$

This uniform scaling affects all linear measurements proportionally. The key question is: how does this scaling influence surface area?

Surface Area and Scale Factors

Surface area (SA) depends on the square of the linear dimensions. For a given figure, if all its dimensions are scaled by a factor k, then:

$$
\text{New Surface Area} = k^2 \times \text{Original Surface Area}
$$

This quadratic relationship means that even small changes in linear dimensions can significantly impact surface area. For example, doubling the size ($k=2$) increases the surface area by a factor of four ($2^2=4$).

Mathematical Formulation of the Problem

Given this background, consider an initial figure with surface area S. The problem states that after changing its dimensions, the surface area becomes exactly one-third of its original:

$$
S' = \frac{1}{3}S
$$

Since surface area scales with the square of the scale factor:

$$
k^2 \times S = \frac{1}{3}S
$$

Dividing both sides by S:

$$
k^2 = \frac{1}{3}
$$

Taking the square root:

$$
k = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} \approx 0.577
$$

This result indicates that to reduce the surface area to one-third, the linear dimensions must be scaled by a factor of approximately 0.577, or about 57.7% of their original size.

Implications of Scaling on the Figure's Dimensions

Linear Dimensions

As derived, the linear dimensions are scaled down by a factor of $1/\sqrt{3}$. This uniform reduction impacts all measurements—lengths, widths, heights—equally, assuming the figure is scaled proportionally.

Volume Changes

While the surface area decreases to one-third, the volume's change is different because volume scales with the cube of the linear dimensions. The new volume V' relates to the original volume V by:

$$
V' = k^3 \times V
$$

\]

Plugging in $k = 1/\sqrt{3}$:

$$
V' = \left(\frac{1}{\sqrt{3}} \right)^3 V = \frac{1}{(\sqrt{3})^3} V = \frac{1}{3 \sqrt{3}} V \approx \frac{1}{5.196} V
$$

Thus, the volume decreases by approximately a factor of 5.2, indicating a significant reduction in the amount of material or space the figure occupies.

Practical Examples and Applications

Example 1: Scaling a Cube

Suppose we have a cube with an original surface area of 150 square units. To reduce its surface area to one-third:

- Calculate the scale factor:

$$
k = \frac{1}{\sqrt{3}} \approx 0.577
$$

- New dimensions:

If the original side length is L, then the new side length $L' = 0.577L$.

- New surface area:

$$
S' = 0.577^2 \times 150 \approx 0.333 \times 150 = 50
$$

which confirms the surface area is one-third of the original.

Application in Engineering and Design

Understanding how surface area scales is crucial in designing objects like heat exchangers, where surface area impacts efficiency, or in packaging, where size reduction affects material usage. Scaling down a component to reduce material costs must be balanced against volume and surface area changes to ensure functionality.

Additional Considerations and Limitations

Non-Uniform Scaling

The previous analysis assumes uniform scaling in all directions. Non-uniform scaling (changing one dimension more than others) complicates the relationship, requiring individual calculations for each dimension's effect on surface area.

Material and Structural Constraints

In real-world applications, physical constraints may limit how much a figure can be scaled. Material properties, structural integrity, and manufacturing limits can restrict the extent of size modifications.

Other Geometric Properties

While this discussion focused on surface area, volume and other properties like moments of inertia also change with scaling, often in different ways, which must be considered in practical scenarios.

Summary and Conclusion

Scaling a figure to alter its surface area involves understanding the fundamental relationships between linear dimensions and surface properties. In this specific case, reducing the surface area to one-third of its original value requires scaling all dimensions by a factor of \(1/\sqrt{3}\). This results in a proportional reduction in linear measurements, a more significant decrease in volume, and a substantial impact on the figure's physical and functional characteristics. Mastery of these scaling principles is essential for effective design, optimization, and analysis across various scientific and engineering disciplines.

Frequently Asked Questions

What is the main concept behind changing the dimensions of a figure to reduce its surface area to one-third of the original?

The main concept involves scaling the dimensions of the figure uniformly so that its surface area decreases proportionally to one-third, typically by applying a scale factor related to the cube root of the ratio.

If the surface area of a figure is reduced to one-third of its original, by what factor are the linear dimensions scaled?

The linear dimensions are scaled by the cube root of 1/3, which is approximately 0.693, meaning each dimension is multiplied by this factor.

Does changing the dimensions to achieve a new surface area of one-third affect the volume of the figure?

Yes, since volume scales with the cube of the linear dimensions, reducing the surface area to one-third generally results in the volume being scaled by the cube of the linear scale factor, which is (1/3)^{1/3}.

How can you determine the new dimensions if the original surface area is known?

You multiply each original dimension by the cube root of the ratio of the new surface area to the original, which in this case is the cube root of 1/3.

Is it always possible to change the dimensions of any figure to exactly reduce the surface area to one-third?

It depends on the shape of the figure; for similar figures, uniform scaling can achieve this, but for irregular shapes, the process may be more complex or not possible to do uniformly.

What is the significance of understanding how surface area and dimensions relate in geometric transformations?

Understanding this relationship helps in designing structures, optimizing material use, and solving problems in fields like engineering, architecture, and manufacturing where surface area impacts cost and functionality.

Can you provide an example of how to calculate the new dimensions if the original surface area is known?

Certainly! If the original surface area is A and you want the new surface area to be A/3, then the linear scale factor is (1/3)^{1/2} ≈ 0.577. Multiply each original dimension by this factor to find the new dimensions.

Additional Resources

Transformations in Geometric Dimensions: Achieving a One-Third Surface Area Reduction

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Introduction

In the world of geometry, understanding how changes in a figure’s dimensions affect its surface area is fundamental. Whether in manufacturing, design, or scientific modeling, the ability to manipulate shapes to meet specific surface area criteria is a valuable skill. Today, we delve into a fascinating problem: "The dimensions of this figure are changed so that the new surface area is exactly one-third of what it was originally." This scenario not only tests our grasp of proportional relationships but also offers insight into the practical applications of scale transformations.

This article aims to provide an in-depth, comprehensive analysis of this problem, combining mathematical theory with practical considerations. Whether you're a student, educator, engineer, or enthusiast, understanding these principles can enhance your ability to approach similar challenges with confidence.

Understanding the Basics: Surface Area and Scale Factors

Before analyzing the specific problem, it's essential to revisit some foundational concepts.

Surface Area and Its Dependence on Dimensions

Surface area (SA) measures the extent of a figure's outer surface. For simple geometric shapes like cubes, spheres, cylinders, and prisms, surface area calculations are straightforward, involving formulas dependent on their dimensions—length, width, height, radius, etc.

For example:

- Cube: $SA = 6a^2$, where a is the length of a side.
- Sphere: $SA = 4\pi r^2$, where r is the radius.
- Cylinder: $SA = 2\pi r(h + r)$.

In each case, the surface area depends on the dimensions raised to specific powers, which determines how surface area scales when dimensions are changed.

The Concept of Scale Factors

When a figure undergoes a uniform scaling transformation—meaning all linear dimensions are multiplied by the same factor k—its surface area and volume change predictably:

- Linear dimensions: multiplied by k.
- Surface area: multiplied by k^2. (because area depends on squared dimensions)
- Volume: multiplied by k^3. (volume depends on cubed dimensions)

This fundamental principle allows us to analyze how altering dimensions affects surface area, which is central to solving the problem at hand.

Problem Statement: Reducing Surface Area to One-Third

Given an original figure with surface area $SA_{original}$, the task is to find the change in its dimensions—specifically, the scale factor k—such that the new surface area SA_{new} equals exactly one-third of $SA_{original}$:

$$
SA_{new} = \frac{1}{3} SA_{original}
$$

Assuming the transformation is uniform across all dimensions, the relationship between the original and new surface areas simplifies to:

$$
SA_{new} = k^2 \times SA_{original}
$$

From which it follows:

$$
k^2 \times SA_{original} = \frac{1}{3} SA_{original}
$$

Dividing both sides by $SA_{original}$:

$$
k^2 = \frac{1}{3}
$$

Taking the square root:

$$
k = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} \approx 0.577
$$

This result indicates that to reduce the surface area to one-third, all linear dimensions must be scaled by approximately 0.577, or about 57.7% of their original lengths.

Practical Implications of the Scale Factor

This calculation is elegant in its simplicity but carries significant practical implications.

1. Uniform vs. Non-Uniform Scaling

- Uniform scaling (all dimensions scaled equally) directly applies to the above calculation.
- Non-uniform scaling (different dimensions scaled by different factors) complicates the analysis,

requiring separate considerations for each dimension based on their contribution to surface area.

For most practical applications, uniform scaling is preferred or assumed when the problem specifies a simple change in surface area.

2. Effects on Volume

While surface area decreases to one-third, the volume changes even more dramatically:

$$
V_{new} = k^3 \times V_{original} = \left(\frac{1}{\sqrt{3}} \right)^3 \times V_{original} = \frac{1}{3 \sqrt{3}} \times V_{original}
$$

Approximately:

$$
V_{new} \approx \frac{1}{5.196} \times V_{original}
$$

This indicates that the volume diminishes to about 19.3% of its original size, which might be critical in applications like packaging, manufacturing, or design where volume capacity is a concern.

Case Studies: Applying the Concept to Different Shapes

The theoretical derivation applies universally, but the specific shape influences how these principles manifest.

Case 1: Cube

- Original Surface Area: $SA = 6a^2$
- After scaling by k: $SA_{new} = 6 (ka)^2 = 6a^2 k^2$

Since $k^2 = 1/3$, the new surface area becomes:

$$
SA_{new} = 6a^2 \times \frac{1}{3} = 2a^2
$$

which is one-third of the original $6a^2$.

- Original volume: $V = a^3$
- New volume: $V_{new} = (ka)^3 = a^3 \times \left(\frac{1}{\sqrt{3}} \right)^3 \approx 0.192 V$

This example underscores the dramatic reduction in volume accompanying the surface area reduction.

Advanced Considerations: Non-Uniform Scaling and Real-World Applications

While uniform scaling simplifies the mathematics, real-world scenarios often involve non-uniform transformations, especially in manufacturing or architectural design.

Non-Uniform Scaling

Suppose a shape is scaled differently along different axes:

- Lengths along the x-axis are scaled by k_x.
- Along the y-axis by k_y.
- Along the z-axis by k_z.

The new surface area becomes a sum of scaled contributions, often requiring detailed analysis based on the shape's surface formulas.

For example, for a rectangular prism:

$$
SA_{new} = 2(k_x k_y a b + k_y k_z b c + k_x k_z a c)
$$

In such cases, to achieve the surface area reduction to exactly one-third, specific relationships between k_x, k_y, k_z must be established, often involving solving systems of equations.

Practical Applications

- Manufacturing: Reducing surface area to lower coating costs or improve aerodynamics.
- Packaging: Shrinking dimensions to save space while maintaining surface properties.
- Material Science: Adjusting shape dimensions to optimize surface interactions, such as catalytic surfaces.

In all these contexts, understanding how to manipulate dimensions to meet surface area targets is crucial.

Limitations and Considerations

While the mathematical model provides a clear solution, real-world factors could influence the outcome:

- Material constraints: Structural integrity might prevent uniform scaling.
- Design constraints: Functional or aesthetic requirements may limit dimension adjustments.
- Manufacturing tolerances: Can't always achieve perfect scaling, especially in complex shapes.

Hence, the derived scale factor should be viewed as a theoretical guideline, adaptable to practical limitations.

Final Thoughts: Mastering Geometric Transformations for Precision Design

The principle that scaling all dimensions by $1/\sqrt{3}$ reduces the surface area to one-third is a powerful tool in the designer's or engineer's toolkit. It highlights the profound impact of linear dimensions on surface properties and emphasizes the importance of proportional reasoning.

By mastering these concepts, professionals can make informed decisions about size adjustments, optimize resource use, and enhance the efficiency of their designs or processes.

Summary

- To reduce the surface area of a figure to exactly one-third of its original value through uniform scaling:

$$
\boxed{
k = \frac{1}{\sqrt{3}} \approx 0.577
}
$$

- All linear dimensions must be scaled by approximately 57.7% of their original length.
- This results in a significant decrease in volume, about 19.3% of the original volume.
- The principles extend across various shapes, with specific formulas adapting accordingly.
- Practical applications should consider real-world constraints, non-uniform scaling options, and material properties.

Understanding and applying these scaling principles enable precise control over geometric properties, essential for innovation and efficiency in multiple fields.

Disclaimer: This analysis assumes uniform scaling and ideal geometric shapes. Real-world objects may require more complex calculations considering irregularities and non-uniform transformations.

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