The Dimensions Of This Figure Are Changed So That The New Surface Area Is Exactly 1/4 What It Was Originally.

The Dimensions Of This Figure Are Changed So That The New Surface Area Is Exactly 1/4 What It Was Originally.

Understanding how changes in the dimensions of a geometric figure affect its surface area is a fundamental concept in geometry, calculus, and various engineering applications. Whether you're studying the properties of three-dimensional objects or solving real-world problems involving scaling and design, grasping the relationship between dimension alterations and surface area is crucial. In this article, we will explore the mathematical principles behind modifying the dimensions of a figure to achieve a specific surface area change, particularly reducing it to one-quarter of the original. We will delve into the concepts of scale factors, surface area formulas, and the geometric intuition behind these transformations.

Understanding Surface Area and Its Dependence on Dimensions

What Is Surface Area?

Surface area refers to the total area covered by the outer surfaces of a three-dimensional figure. It is measured in square units (e.g., square meters, square centimeters). Surface area calculations vary depending on the shape, but the general principle involves summing the areas of all the faces or surfaces.

How Does Changing Dimensions Affect Surface Area?

When the dimensions of a figure are scaled uniformly—meaning all length measurements are multiplied by the same factor—the surface area is affected in a predictable way. Specifically, the surface area scales with the square of the scale factor.

- If the original figure has a surface area $A_{original}$, and each linear dimension is scaled by a factor k, the new surface area A_{new} becomes:

$$
A_{new} = k^2 \times A_{original}
$$

This relationship is fundamental in understanding how to manipulate dimensions to achieve a desired surface area.

Mathematical Relationship Between Dimensions and Surface Area

Scale Factor and Surface Area

Suppose you have an original figure with dimensions characterized by some length L, and you scale all dimensions by a factor k. Then:

- Original Surface Area: $A_{original}$
- Scaled Surface Area: $A_{new} = k^2 \times A_{original}$

This quadratic relationship means that doubling the size of a figure (i.e., $k=2$) increases the surface area by a factor of 4, while halving it ($k=0.5$) reduces the surface area to a quarter.

Applying the Relationship to Reduce Surface Area to One-Quarter

Given the goal of reducing the surface area to exactly one-quarter of its original value, we set:

$$
A_{new} = \frac{1}{4} A_{original}
$$

Using the scale factor relationship:

$$
k^2 \times A_{original} = \frac{1}{4} A_{original}
$$

Dividing both sides by $A_{original}$:

$$
k^2 = \frac{1}{4}
$$

Taking the square root:

$$
k = \pm \frac{1}{2}
$$

Since a negative scale factor doesn't make physical sense in this context, the relevant solution is:

$$
k = \frac{1}{2}
$$

This indicates that to reduce the surface area to one-quarter of the original, each linear dimension must be scaled by a factor of 1/2.

Practical Implications and Examples

Example 1: Scaling a Cube

Suppose you have a cube with side length L and surface area:

$$
A_{original} = 6L^2
$$

To make the new cube have one-quarter the surface area:

- Determine the new side length:

$$
L_{new} = k \times L = \frac{1}{2} \times L
$$

- Calculate the new surface area:

$$
A_{new} = 6 \times \left(\frac{L}{2}\right)^2 = 6 \times \frac{L^2}{4} = \frac{6L^2}{4} = \frac{3L^2}{2}
$$

- Compare with original:

$$
\frac{A_{new}}{A_{original}} = \frac{\frac{3L^2}{2}}{6L^2} = \frac{3/2}{6} = \frac{3}{2} \times \frac{1}{6} = \frac{1}{4}
$$

Thus, halving all dimensions results in the surface area being exactly one-quarter of the original.

Example 2: Scaling a Sphere

For a sphere of radius r:

$$
A_{original} = 4\pi r^2
$$

- To reduce surface area to one-quarter:

$$
r_{new} = \frac{1}{2} r
$$

- New surface area:

$$
A_{new} = 4\pi \left(\frac{r}{2}\right)^2 = 4\pi \frac{r^2}{4} = \pi r^2
$$

- Original surface area:

$$
4\pi r^2
$$

- Comparison:

$$
\frac{A_{new}}{A_{original}} = \frac{\pi r^2}{4\pi r^2} = \frac{1}{4}
$$

Again, halving the radius reduces the surface area to one-quarter.

Geometric Intuition and Visual Understanding

Understanding these transformations visually can deepen comprehension. Imagine a cube or sphere shrinking uniformly. Since surface area depends on the square of linear dimensions, halving the size reduces the surface area to a quarter, similar to how the area of a square scales with side length squared.

- Scaling all dimensions by k: The surface area scales by k^2.
- To achieve a quarter surface area: Scale dimensions by $k = 1/2$.

This geometric principle holds for all similar figures, emphasizing the importance of linear scaling in controlling surface area.

Applications and Real-World Significance

Understanding how to manipulate the dimensions of figures to achieve specific surface area goals has many practical applications:

- Manufacturing and Design: Optimizing material usage by reducing surface area while maintaining volume.
- Heat Transfer: Controlling surface area to manage thermal properties of objects.
- Biology: Understanding how scaling affects the surface area-to-volume ratio in organisms.
- Engineering: Designing components where surface area impacts reactions or interactions, such as catalysts or sensors.

Summary and Key Takeaways

- When all dimensions of a figure are scaled uniformly by a factor k, the surface area scales by k^2.
- To reduce the surface area to exactly one-quarter of its original, scale all linear dimensions by a factor of $1/2$.
- This principle applies universally across similar geometric figures, including cubes, spheres, cylinders, and pyramids.
- Visualizing these transformations helps in understanding their practical applications across various fields.

Conclusion

Changing the dimensions of a figure to alter its surface area involves understanding the quadratic relationship between linear scaling and surface area. Specifically, reducing the surface area to one-quarter of its original value requires halving all linear dimensions. This fundamental principle is crucial in many scientific, engineering, and mathematical contexts, enabling precise control over the physical properties of objects through simple geometric transformations. Whether designing efficient structures, studying biological systems, or solving complex problems, mastering the relationship between dimensions and surface area is a valuable skill that bridges theory and real-world applications.

Frequently Asked Questions

If the surface area of a figure is reduced to one-fourth of its original, how do its dimensions change?

The linear dimensions are scaled down by a factor of 1/2 since surface area scales with the square of the linear dimensions.

What is the scale factor for the dimensions when the surface area is reduced to 1/4?

The scale factor for the dimensions is 1/2.

If a cube's surface area is originally 96 square units, what is the surface area after the dimensions are scaled appropriately?

The new surface area is 24 square units, since it's one-fourth of 96.

How does changing the dimensions affect the volume of the figure when the surface area is scaled to a quarter?

The volume scales by the cube of the linear scale factor, which in this case is (1/2)^3 = 1/8, so the volume becomes one-eighth of the original.

Can you determine the new surface area if the original is 200 square units and the surface area is reduced to 1/4?

Yes, the new surface area is 50 square units.

What is the relationship between the change in surface area and the change in linear dimensions?

Surface area changes proportionally to the square of the linear dimensions; reducing surface area to 1/4 means linear dimensions are scaled by 1/2.

If the original figure is a sphere with a surface area of 12.56 square units, what is its new surface area after the dimensions are changed?

The new surface area is 3.14 square units, since it's one-fourth of the original.

What mathematical principle explains how the surface area scales when the dimensions are changed?

The principle is that surface area scales with the square of the linear scale factor.

If the original surface area is doubled, what should be the scale factor for the dimensions to achieve that?

The linear dimensions should be scaled by the square root of 2 (approximately 1.414).

Why does reducing the surface area to a quarter also mean the dimensions are halved?

Because surface area is proportional to the square of the linear dimensions, so to reduce surface area by a factor of 4, the linear dimensions must be scaled by 1/2.

Additional Resources

The Dimensions Of This Figure Are Changed So That The New Surface Area Is Exactly 1/4 What It Was Originally

Understanding how altering a figure's dimensions impacts its surface area is a fundamental concept in geometry, with practical applications spanning engineering, architecture, manufacturing, and scientific research. When the problem states that the dimensions are changed so that the new surface area becomes exactly one-quarter of the original, it invites an exploration of the relationships between linear scaling and surface area. This article delves into this intriguing transformation, unpacking the mathematical principles involved, analyzing the effects of scaling, and illustrating the implications through detailed examples.

Fundamental Concepts: Surface Area and Scale Factors

What Is Surface Area?

Surface area is a measure of the total area that the surface of a three-dimensional object occupies. It is expressed in square units (e.g., square meters, square centimeters). For simple geometric figures such as cubes, spheres, cylinders, and pyramids, surface area can be calculated using well-established formulas that depend on their dimensions.

Linear Scaling and Its Impact

When a figure's dimensions are scaled uniformly—meaning all linear measurements are multiplied by a common scale factor—the surface area does not change proportionally to the linear factor. Instead, it scales with the square of the linear factor. This relationship is central to understanding the problem at hand.

Mathematically:

Let:
- k be the scale factor applied to each linear dimension.
- A be the original surface area.
- A' be the new surface area after scaling.

Then:
$$
A' = k^2 \times A
$$

This quadratic relationship underscores why small changes in dimensions can significantly impact surface area.

Setting Up the Problem: The Surface Area Reduction

Given:
- Original surface area: A
- New surface area: $\frac{1}{4} A$

We need to find:
- The scale factor k such that when all linear dimensions are multiplied by k, the surface area becomes exactly one-quarter of its original value.

Formulating the equation:

$$
k^2 \times A = \frac{1}{4} A
$$

Dividing both sides by A:

$$
k^2 = \frac{1}{4}
$$

Taking the square root:

$$
k = \pm \frac{1}{2}
$$

Since scale factors are positive (we're talking about physical dimensions), we consider only:

$$
k = \frac{1}{2}
$$

Conclusion:
To reduce the surface area to one-quarter of its original, each linear dimension must be scaled by a factor of 1/2.

Implications of Scaling Down: Geometric and Practical Insights

Linear Dimensions Halved

The straightforward outcome is that all linear measurements—lengths, widths, heights—are decreased by half. This uniform reduction ensures the surface area shrinks appropriately.

Example:
- Original cube with side length s.
- Original surface area: $A = 6s^2$.
- New cube with side length $s' = \frac{s}{2}$.
- New surface area: $A' = 6(s/2)^2 = 6 \times \frac{s^2}{4} = \frac{6s^2}{4} = \frac{A}{4}$.

This example confirms the mathematical derivation with a concrete figure.

Volume Considerations

While the surface area diminishes to a quarter, the volume behaves differently. Volume scales with the cube of the linear scale factor:

$$
V' = k^3 \times V
$$

For $k = 1/2$:

$$
V' = \left(\frac{1}{2}\right)^3 \times V = \frac{1}{8} V
$$

Implication:
- When the surface area is reduced to 25%, the volume decreases to just 12.5% of the original.
- This disproportionate change underscores how scaling affects different geometric properties differently.

Analytical Exploration: Beyond Basic Shapes

Complex Geometric Figures

While the initial scenario can be exemplified with simple shapes like cubes or spheres, more complex figures involve additional considerations:
- Composite solids: Changing dimensions can involve multiple scale factors if different parts are scaled differently.
- Non-uniform scaling: If the figure is scaled differently along different axes (anisotropic scaling), the surface area change depends on the specific scaling factors for each dimension.

For example:
Suppose a rectangular prism with dimensions l, w, h, scaled by factors k_l, k_w, k_h:

$$
A' = 2(k_l w h + l k_w h + l w k_h)
$$

The surface area change depends on the combination of these factors, making the problem more complex. However, in the context of the initial problem, uniform scaling suffices.

Implications for Real-World Applications

Understanding how surface area scales is critical in:
- Engineering: Designing heat exchangers or cooling systems where surface area impacts heat transfer.
- Biology: Understanding how animals of different sizes have surface area-to-volume ratios affecting thermoregulation.
- Manufacturing: Scaling prototypes while maintaining surface properties.

Additional Considerations and Limitations

Limitations of Uniform Scaling Assumption

The derivation assumes perfect uniform scaling—an idealization. Real-world objects might undergo non-uniform transformations due to manufacturing constraints, leading to different surface area changes.

Material and Structural Constraints

Reducing dimensions by half may not always be feasible due to material strength, structural integrity, or functional requirements.

Scaling in Non-Linear Geometries

For non-linear geometries or shapes with curved surfaces, the relationship between scaling and surface area can be more complex, involving calculus and differential geometry.

Conclusion: The Power of Scale Factors in Geometric Transformations

This exploration illustrates a fundamental principle in geometry: when a figure's linear dimensions are scaled by a factor \(k\), its surface area scales by \(k^2\), and its volume by \(k^3\). Specifically, to reduce a figure's surface area to exactly one-quarter of its original, all linear dimensions must be scaled by a factor of 1/2. This proportional change results in a significant decrease in volume, emphasizing the importance of understanding how different properties of a shape respond to scaling.

From simple cubes to complex biological forms, the principles outlined here underpin many practical applications. Whether designing more efficient heat exchangers, understanding biological adaptations, or scaling prototypes in manufacturing, grasping the relationship between dimension changes and surface area is vital. The mathematical clarity provided by this analysis ensures that engineers, scientists, and designers can predict and manipulate the properties of their objects with precision.

In essence, the problem exemplifies the elegant interconnectedness of geometry and real-world application, demonstrating how a simple proportional change can have profound implications across disciplines.

The Dimensions Of This Figure Are Changed So That The New Surface Area Is Exactly 1 4 What It Was Originally

The Dimensions Of This Figure Are Changed So That The New Surface Area Is Exactly 1 4 What It Was Originally

Related Articles

- Scatter Diagrams And High-Low Cost Estimation From April 1 Through October 31, Will County Highway Department
- Before Changing Lanes, Identify A Safe Gap, Check Your Mirrors And Blind Spots And __________ Your Intentions
- In One Page, Summarize The Article Parental Involvement: Rhetoric Of Inclusion In An Environment Of Exclusion

Back to Home