The Function F Is Given By F(x)=0.2x41.0x36.6x2 15.4x1.99. For How Many Positive Values Of B Does Limxbf(x)=6

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Introduction to the Problem

Understanding the behavior of functions as they approach specific points is a fundamental aspect of calculus. The question posed involves analyzing a piecewise or potentially complex function, F(x), defined by an algebraic expression, and determining at which positive values of B the limit of F(x) as x approaches B equals 6. This problem intertwines the concepts of limits, function behavior, and algebraic manipulation.

The function in question appears to be a combination of multiple terms involving x, with some coefficients and exponents. The challenge lies in understanding the structure of F(x), simplifying or analyzing it to find the points where the limit equals 6, and then counting the number of positive B values satisfying this condition.

Deciphering the Function Expression

Understanding the Given Expression

The provided function is written as:

F(x) = 0.2x41.0x36.6x2 15.4x1.99

At first glance, this appears to be a concatenation of coefficients and variables, possibly missing operators or delimiters. To interpret this correctly, we need to clarify the structure.

Potential interpretations:

- The function could be a sum of terms, such as:

$$F(x) = 0.2x + 41.0x + 36.6x^2 + 15.4x^{1.99}$$

- Alternatively, it might be a product or a more complex expression.

Given typical mathematical notation and context, the most reasonable interpretation is that the function is a sum of several terms with coefficients and powers of x.

Reconstructing the Function

Based on standard notation and the given string, the most logical form of the function is:

$$F(x) = 0.2x + 41.0x + 36.6x^2 + 15.4x^{1.99}$$

This form makes sense because:

- The coefficients are clear (0.2, 41.0, 36.6, 15.4).
- The variables are powers of x.
- The exponents are either integers or decimal approximations.

Note: The initial coefficient "0.2x" and "41.0x" could be combined, but we will treat them separately unless they are intended to be summed.

Simplified Form of the Function

Given the plausible structure, the simplified form of F(x) is:

$$F(x) = (0.2 + 41.0) x + 36.6 x^2 + 15.4 x^{1.99}$$

Calculating the sum of the linear coefficients:

$$0.2 + 41.0 = 41.2$$

Thus,

$$F(x) = 41.2 x + 36.6 x^2 + 15.4 x^{1.99}$$

Analyzing the Limit Condition: $\lim_{x \to b} F(x) = 6$

The core question is: For how many positive values of B does the limit of F(x) as x approaches B equal 6?

This involves solving the equation:

$$\lim_{x \to b} F(x) = 6$$

Since F(x) is composed of polynomial and fractional power terms, and these are continuous functions for $x > 0$, the limit as x approaches B from either side is simply F(B), assuming F is continuous at B.

Therefore, the problem reduces to finding the positive B such that:

$$F(B) = 6$$

Solving the Equation F(B) = 6

Given the simplified form:

$$F(B) = 41.2 B + 36.6 B^2 + 15.4 B^{1.99} = 6$$

Our goal is to find positive B satisfying this equation.

Rewriting the Equation

Expressed as:

$$36.6 B^2 + 41.2 B + 15.4 B^{1.99} - 6 = 0$$

This is a transcendental equation involving polynomial and fractional powers.

Analytical Approach

- Because of the fractional exponent 1.99, the equation is not purely polynomial.
- However, for positive B, $B^{1.99}$ is well-defined.
- The equation can be viewed as a function:

$G(B) = 36.6 B^2 + 41.2 B + 15.4 B^{1.99} - 6$

- We seek positive roots of $G(B) = 0$.

Behavior of G(B) and Existence of Solutions

Evaluating G(B) at Selected Points

To understand how many solutions exist, analyze G(B) at some key points:

- At $B = 0+$

$G(0+) \approx 0 + 0 + 0 - 6 = -6$

- At $B = 1$

$G(1) = 36.6\ 1 + 41.2\ 1 + 15.4\ 1^{1.99} - 6$

$= 36.6 + 41.2 + 15.4 - 6 = 87.2$

- At $B = 0.5$

Calculate:

$B^{1.99} \approx 0.5^{1.99} \approx e^{1.99 \ln(0.5)} \approx e^{1.99 (-0.6931)} \approx e^{-1.378} \approx 0.251$

Then,

$G(0.5) \approx 36.6(0.25) + 41.2(0.5) + 15.4(0.251) - 6$

$= 9.15 + 20.6 + 3.87 - 6 \approx 27.62$

- At $B = 0.1$

$B^{1.99} \approx e^{1.99 \ln(0.1)} \approx e^{1.99 (-2.3026)} \approx e^{-4.585} \approx 0.0102$

$G(0.1) \approx 36.60.01 + 41.20.1 + 15.40.0102 - 6$

$= 0.366 + 4.12 + 0.157 - 6 \approx -1.357$

Summary of evaluations:

B	G(B)

0.1	-1.357
0.5	27.62
1	87.2

From this, $G(B)$ crosses zero between $B=0.1$ and $B=0.5$ because:

- $G(0.1) \approx -1.36$ (negative)
- $G(0.5) \approx 27.62$ (positive)

By the Intermediate Value Theorem, there exists a B in $(0.1, 0.5)$ such that $G(B) = 0$.

Similarly, at $B=0$, $G(0) = -6$ (negative), and at $B=0.1$, $G(0.1) \approx -1.36$ (still negative), so no crossing occurs between 0 and 0.1.

At larger B:

- $G(1) \approx 87.2$ (positive)

Since $G(0.5) \approx 27.62$, and at $B=2$:

Calculate $G(2)$:

- $B^{1.99} \approx e^{1.99 \ln(2)} \approx e^{1.990.6931} \approx e^{1.378} \approx 3.97$

Then,

$G(2) = 36.64 + 41.22 + 15.43.97 - 6$

$= 146.4 + 82.4 + 61.2 - 6 \approx 283$

$G(2)$ is positive, indicating no zero crossing between 1 and 2.

At $B=0.25$:

- $B^{1.99} \approx e^{1.99 \ln(0.25)} \approx e^{1.99 (-1.386)} \approx e^{-2.757} \approx 0.063$

$G(0.25)$:

$36.60.0625 + 41.20.25 + 15.40.063 - 6$

$= 2.29 + 10.3 + 0.97 - 6 \approx 8.56$

Positive, so no root between 0.1 and 0.5, since $G(0.1)$ negative, $G(0.25)$ positive, indicating a crossing between 0.1 and 0.25.

Similarly, between 0.05 and 0.1:

- At $B=0.05$:

B^{1.99} ≈ e^{1.99 ln(0.

Frequently Asked Questions

What is the function F(x) given in the problem?

The function is $F(x) = 0.2x^4 + 1.0x^3 + 6.6x^2 + 15.4x + 1.99$.

What is the limit we are asked to analyze in the problem?

We are asked to find the number of positive values of B for which $\lim_{x\to\infty} Bf(x) = 6$.

How does the limit $\lim_{x\to\infty} Bf(x)$ relate to the behavior of f(x) as x approaches infinity?

Since f(x) is a polynomial of degree 4 with a positive leading coefficient, f(x) approaches infinity as x approaches infinity, so the limit depends on the value of B and the growth of f(x).

What is the general form of the limit $\lim_{x\to\infty} Bf(x)$?

The limit is either infinite or zero depending on B; specifically, $\lim_{x\to\infty} Bf(x) = B \lim_{x\to\infty} f(x)$.

How do we find for which positive B the limit equals 6?

We set $B \lim_{x\to\infty} f(x) = 6$, but since $\lim_{x\to\infty} f(x) = \infty$, this limit is finite only if $B = 0$, which contradicts B being positive. Therefore, we need to consider an alternative interpretation.

Is the limit $\lim_{x\to\infty} Bf(x)$ ever equal to 6 for positive B?

No, because as x approaches infinity, f(x) goes to infinity for a polynomial with positive leading coefficient, so Bf(x) also approaches infinity unless $B=0$, which isn't positive. Therefore, the limit cannot be 6 for positive B.

Could the problem be asking about the limit as x approaches some finite point instead of infinity?

No, the problem explicitly states the limit as x approaches infinity. Since f(x) tends to infinity, the only way for Bf(x) to approach a finite value (like 6) is if $B=0$, which is not positive.

What is the conclusion about the number of positive B values such that $\lim_{x\to\infty} Bf(x) = 6$?

There are no positive values of B for which the limit $\lim_{x\to\infty} Bf(x)$ equals 6, because $f(x)\to\infty$ as $x\to\infty$ and $B>0$, so $Bf(x)\to\infty$, not 6.

Additional Resources

Mathematics

Understanding the Function F and the Limit Problem: A Detailed Exploration

When tackling a complex problem involving a function like
$$F(x) = 0.2x^{4.1} + 0.6x^{3.6} + 0.2x^{2.15} + 0.4x^{1.99},$$
and analyzing the behavior of its limit as $x \to b$, especially in relation to the parameter b, it’s crucial to methodically dissect each component. This article aims to guide you through an in-depth analysis of the question: For how many positive values of b does $\lim_{x \to b} F(x) = 6$?

This problem combines elements of calculus, function analysis, and algebraic reasoning. We will explore the structure of $F(x)$, examine the nature of limits, and determine the number of positive b-values satisfying the given limit condition.

Dissecting the Function F: Structure and Behavior

The Composition of F

The function $F(x)$ is composed of four terms, each involving a power of x multiplied by a coefficient:

$$
F(x) = 0.2x^{4.1} + 0.6x^{3.6} + 0.2x^{2.15} + 0.4x^{1.99}.
$$

Let's analyze these components:

- Exponent Range: The exponents are all positive and real, with values ranging from approximately 1.99 to 4.1.
- Coefficients: All coefficients are positive, which affects the monotonicity and limits of the function.

Behavior at $x \to 0^+$

As $x \to 0^+$:

$$
x^{k} \to 0, \quad \text{for } k > 0,
$$
so

$$
F(x) \to 0.
$$

Thus, near zero, the function approaches zero, ensuring $F(x)$ is continuous and increasing from 0 at the origin.

Behavior at $x \to +\infty$

As $x \to +\infty$:

$$
x^{k} \to +\infty,
$$
for positive k. Since all terms are positive and increasing with x, $F(x) \to +\infty$. Therefore, F is strictly increasing on $(0, +\infty)$.

Monotonicity and Continuity

Given the structure, $F(x)$ is continuous for all $x > 0$. The sum of increasing functions with positive coefficients suggests that F is strictly increasing on $(0, \infty)$.

Exploring the Limit Condition: $\lim_{x \to b} F(x) = 6$

The significance of the limit

The problem asks: For how many positive values of b does the limit $\lim_{x \to b} F(x) = 6$?

Since F is continuous and strictly increasing, the limit of $F(x)$ as $x \to b$ from the left and right equals $F(b)$, provided F is defined at b. The question implicitly considers the possibility that F might have discontinuities or that the limit may be approached from different directions if F is not continuous at b.

However, given the structure, F is continuous everywhere on $(0, \infty)$, so:

$$
\lim_{x \to b} F(x) = F(b).
$$

Therefore, the condition simplifies to:

$$
F(b) = 6.
$$

The core question reduces to: How many positive values of b satisfy $F(b) = 6$?

Solving $F(b) = 6$: A Step-by-Step Approach

Step 1: Understanding the equation

We are to find the number of positive solutions $b > 0$ such that:

$$
0.2b^{4.1} + 0.6b^{3.6} + 0.2b^{2.15} + 0.4b^{1.99} = 6.
$$

Step 2: Behavior at the endpoints

- At $b \to 0^+$:

$$
F(b) \to 0,
$$
which is less than 6.

- At $b \to +\infty$:

$$
F(b) \to +\infty,
$$
which exceeds 6.

Given F is continuous and strictly increasing, the Intermediate Value Theorem guarantees exactly one solution for $F(b) = 6$ for each crossing of the value 6.

Step 3: Monotonicity guarantees unique solutions

Since F is strictly increasing on $(0, \infty)$, the equation $F(b) = 6$ can have at most one positive solution.

To confirm the existence of this solution, we check the function at some specific points to see if the value crosses 6.

Numerical Approximation and Confirmation

Evaluating $F(b)$ at key points:

- At $b = 1$:

$$
F(1) = 0.2(1)^{4.1} + 0.6(1)^{3.6} + 0.2(1)^{2.15} + 0.4(1)^{1.99} = 0.2 + 0.6 + 0.2 + 0.4 = 1.4,
$$

which is less than 6.

- At $b = 2$:

Calculate each term:

$$
0.2 \times 2^{4.1} \approx 0.2 \times 2^{4} \times 2^{0.1} \approx 0.2 \times 16 \times 1.07 \approx 0.2 \times 17.12 \approx 3.424,
$$
$$
0.6 \times 2^{3.6} \approx 0.6 \times 2^{3} \times 2^{0.6} \approx 0.6 \times 8 \times 1.52 \approx 0.6 \times 12.16 \approx 7.296,
$$
$$
0.2 \times 2^{2.15} \approx 0.2 \times 2^{2} \times 2^{0.15} \approx 0.2 \times 4 \times 1.11 \approx 0.2 \times 4.44 \approx 0.888,
$$
$$
0.4 \times 2^{1.99} \approx 0.4 \times 2^{2} \times 2^{-0.01} \approx 0.4 \times 4 \times 0.99 \approx 0.4 \times 3.96 \approx 1.584.
$$

Adding these:

$$
F(2) \approx 3.424 + 7.296 + 0.888 + 1.584 = 13.192,
$$

which exceeds 6.

Since $F(1) \approx 1.4 < 6$ and $F(2) \approx 13.192 > 6$, and F is strictly increasing, there exists a unique $b \in (1, 2)$ such that $F(b) = 6$.

Final Conclusions: Number of Positive b with $F(b) = 6$

Based on the analysis:

- The function F is continuous and strictly increasing on $(0, \infty)$.
- It passes from 0 at $b \to 0^+$ to $+\infty$ as $b \to +\infty$.
- It crosses the value 6 exactly once, at some positive $b \in (1, 2)$.

Thus, there is exactly one positive value of b for which

$$
\lim_{x \to b} F(x) = 6,
$$

which, due to the continuity, is equivalent to $F(b) = 6$.

Summary and Final Answer

Aspect	Explanation
Function $F(x)$	Sum of positive power functions with increasing behavior
Limit analysis	The limit $\lim_{x \to b} F(x)$ equals $F(b)$ due to continuity
Number of solutions	Exactly one positive b satisfying $F(b) = 6$
Conclusion	There is exactly 1 positive value of b such that $\lim_{x \to b} F(x) = 6$.

Final Words

This problem elegantly combines the properties of power functions, monotonicity, and limits to arrive at a concise conclusion. The key

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