The Perimeter Of A Rectangle Is 54 Feet Describe The Possible Lengths Of A Side If The Area Of The Rectangle

The Perimeter Of A Rectangle Is 54 Feet: Describe The Possible Lengths Of A Side If The Area Of The Rectangle

The perimeter of a rectangle is 54 feet. Describe the possible lengths of a side if the area of the rectangle is also known or to be determined. This problem involves understanding the relationship between the perimeter, length, and width of a rectangle, as well as how the area is affected by these dimensions. To explore this, we need to analyze the formulas involved and see how they interrelate to find various possible side lengths that satisfy the given perimeter and potential area constraints.

Understanding the Basic Properties of a Rectangle

Perimeter of a Rectangle

The perimeter (P) of a rectangle is the total length around the shape, calculated as:

- **P = 2(length + width)**

Given that the perimeter is 54 feet, we can set up an equation:

$2(l + w) = 54$

Dividing both sides by 2:

$l + w = 27$

This fundamental relationship indicates that the sum of the length (l) and the width (w) of the rectangle must always be 27 feet.

Area of a Rectangle

The area (A) of a rectangle is given by:

- **A = length × width**

The area depends on the specific values of the length and width, which are constrained by the perimeter condition. Our goal is to analyze how the area varies as these dimensions change, given that the sum of length and width remains 27 feet.

Expressing the Dimensions in Terms of One Variable

Deriving Expressions for Length and Width

From the perimeter relationship, we have:

$l + w = 27$

We can express one variable in terms of the other:

- **Width in terms of Length:** $w = 27 - l$
- **Length in terms of Width:** $l = 27 - w$

This simplifies our analysis, as we only need to consider one variable when exploring different possible side lengths and areas.

Constraints on the Dimensions

Since length and width are physically meaningful dimensions of a rectangle, they must be positive:

- $l > 0$
- $w > 0$

Given that $w = 27 - l$, the constraints imply:

- $l > 0$

- $27 - l > 0$

  which simplifies to:

  $l < 27$

Similarly, for width:

- $w > 0$
- $27 - w > 0$

  which simplifies to:

  $w < 27$

Thus, the possible lengths and widths are in the interval $(0, 27)$.

Exploring the Possible Areas Based on Dimensions

Formula for Area in Terms of One Variable

Using $w = 27 - l$, the area becomes:

$$A = l \times (27 - l) = 27l - l^2$$

This is a quadratic function of l, representing a parabola opening downward with respect to l.

Maximum Area of the Rectangle

The quadratic function $A(l) = -l^2 + 27l$ reaches its maximum at the vertex of the parabola. The vertex of a quadratic function $ax^2 + bx + c$ occurs at $l = -b/(2a)$. In this case:

$$a = -1, b = 27$$

So, the maximum area occurs at:

$$l = -27/(2 \quad -1) = 27/2 = 13.5$$

Correspondingly, the width is:

$w = 27 - 13.5 = 13.5$

Thus, the maximum area for the rectangle with perimeter 54 feet is achieved when the length and width are both 13.5 feet, resulting in:

$A_{max} = 13.5 \times 13.5 = 182.25$ square feet

Range of Possible Areas

As l varies from just above 0 to just below 27, the area $A(l) = 27l - l^2$ varies accordingly:

- At l approaching 0 from above:

  $A \approx 0$

- At l approaching 27 from below:

  $A \approx 0$

- Maximum at $l = 13.5$:

  $A = 182.25$ square feet

Therefore, the area of the rectangle can range from values very close to zero up to 182.25 square feet, depending on the specific side lengths chosen.

Possible Lengths of a Side Given Different Areas

Case 1: Specific Area Values

If the area of the rectangle is specified, we can determine the corresponding side lengths. Suppose the area A is given; then, using the relationship:

$A = l(27 - l)$

Rearranged as a quadratic in l:

$$l(27 - l) = A$$
$$\Rightarrow 27l - l^2 = A$$
$$\Rightarrow l^2 - 27l + A = 0$$

This quadratic can be solved for l:

$$l = [27 \pm \sqrt{(27)^2 - 4 \times 1 \times A}] / 2$$

For real solutions, the discriminant must be non-negative:

$$(27)^2 - 4A \geq 0$$
$$\Rightarrow 729 - 4A \geq 0$$
$$\Rightarrow A \leq 182.25$$

This aligns with our earlier maximum area calculation. For each valid area less than or equal to 182.25, there are two possible length solutions, corresponding to the two sides of the parabola.

Example: Area of 90 Square Feet

Using the quadratic formula:

$$l = [27 \pm \sqrt{729 - 4 \times 90}] / 2$$
$$= [27 \pm \sqrt{729 - 360}] / 2$$
$$= [27 \pm \sqrt{369}] / 2$$

Calculating $\sqrt{369} \approx 19.2$:

$$l \approx [27 \pm 19.2] / 2$$

Thus:

- $l \approx (27 + 19.2) / 2 \approx 46.2 / 2 \approx 23.1$
- $l \approx (27 - 19.2) / 2 \approx 7.8 / 2 \approx 3.9$

Corresponding widths are:

- $w \approx 27 - 23.1 \approx 3.9$
- $w \approx 27 - 3.9 \approx 23.1$

Both pairs are valid, showing the different possible side length combinations for an area of 90 square feet.

Case 2: When the Area Is Not Known

If the area is not specified, the possible side lengths depend on the range of areas between 0 and 182.25 square feet. In this scenario, any pair of positive lengths (l,w) satisfying:

- $l + w = 27$
- $0 < l < 27$
- $0 < w = 27 - l < 27$

are valid, and the area can range from nearly zero to the maximum of 182.25 square feet.

Summary of Possible Side Lengths and Areas

- The sum of the length and width must always be 27 feet.
- Both length and width are positive and less than 27 feet.
- The area varies between near 0 and 182.25 square feet, reaching maximum at a square with sides of 13.5 feet

Frequently Asked Questions

What are the possible side lengths of a rectangle with a perimeter of 54 feet?

The possible side lengths are pairs of positive numbers where twice the sum equals 54, so if one side is x, the other side is (27 - x). Both sides must be positive, so x must be between 0 and 27 feet.

How does the perimeter relate to the side lengths of a rectangle?

The perimeter of a rectangle is calculated as 2 times the sum of its length and width (Perimeter = 2(l + w)). Given the perimeter, this equation helps determine possible side lengths.

What is the maximum possible area of the rectangle with a perimeter of 54 feet?

The maximum area occurs when the rectangle is a square, with each side being 27/2 = 13.5 feet. The maximum area is 13.5 ft × 13.5 ft = 182.25 square feet.

Can the sides of the rectangle be equal? If so, what are the side lengths?

Yes, the sides can be equal, forming a square. In this case, each side would be 54 ÷ 4 = 13.5 feet, resulting in a square with an area of 182.25 square feet.

How do you find the area of a rectangle given its perimeter and one side length?

Once you know one side length, you can find the other side using the perimeter formula: $2(l + w) = 54$. Then, multiply the two sides to find the area: Area = $l \times w$.

What are the possible areas of the rectangle with different side lengths summing to 27 feet?

Since the sides are (x) and $(27 - x)$, the area is $A = x(27 - x) = 27x - x^2$. The maximum area is at $x = 13.5$ feet, giving an area of 182.25 square feet. Other possible areas are less than this, depending on the side lengths chosen.

Additional Resources

The Perimeter Of A Rectangle Is 54 Feet Describe The Possible Lengths Of A Side If The Area Of The Rectangle

Understanding the relationship between a rectangle’s perimeter and its side lengths is fundamental in geometry. When given a specific perimeter—such as 54 feet—the question naturally arises: What are the potential lengths of a side, especially when considering the area of the rectangle? This inquiry not only involves straightforward calculations but also invites a deeper exploration of how dimensions interact within geometric constraints. In this comprehensive analysis, we will dissect the problem, analyze possible solutions, and interpret the implications of various side lengths, all while maintaining clarity and precision.

Understanding the Fundamentals: Perimeter and Area of a Rectangle

Before delving into specific calculations, it’s essential to clarify the core concepts involved:

What is the Perimeter of a Rectangle?

The perimeter (P) of a rectangle is the total length around it, calculated as:
$$ P = 2 \times (length + width) $$
Given $P = 54$ feet, we have:
$$ 2 \times (L + W) = 54 \implies L + W = 27 $$
This simple relation links the two sides, L and W, and constrains their possible values.

What is the Area of a Rectangle?

The area (A) of a rectangle is the measure of the surface it covers, calculated as:
$$ A = L \times W $$
Knowing the perimeter constrains the sum of the sides but does not directly fix the area. Therefore, for a given perimeter, the area can vary depending on the specific side lengths.

The Problem: Connecting Perimeter, Side Lengths, and Area

The core question is:

Given a perimeter of 54 feet, what are the possible lengths of one side if the area of the rectangle varies?

This involves understanding the relationship between side lengths and the resulting area. Since the perimeter constrains the sum of the sides but does not specify the area, multiple rectangles with different dimensions can satisfy the perimeter condition, but their areas will differ accordingly.

Mathematical Exploration of Possible Side Lengths

To analyze the problem systematically, it's helpful to express the dimensions algebraically.

Expressing One Side in Terms of the Other

As established:
$$ L + W = 27 $$
Suppose we designate L as one side, then:
$$ W = 27 - L $$

The area becomes:
$$ A = L \times W = L \times (27 - L) $$
which simplifies to:
$$ A(L) = 27L - L^2 $$

This quadratic function describes how the area varies depending on the chosen side length L.

Constraints on Side Lengths

Since side lengths are positive:
$$ L > 0 $$
$$ W = 27 - L > 0 \implies L < 27 $$
Thus, L can vary in the interval:
$$ 0 < L < 27 $$

Within this interval, the area function reaches its maximum at the vertex of the parabola:
$$ A(L) = -L^2 + 27L $$

The vertex occurs at:
$$ L = -\frac{b}{2a} = -\frac{27}{2 \times -1} = \frac{27}{2} = 13.5 $$

At $L = 13.5$:
$$ W = 27 - 13.5 = 13.5 $$
and the maximum area is:
$$ A_{max} = 13.5 \times 13.5 = 182.25 \text{ square feet} $$

This symmetric property indicates that the rectangle with the maximum area for a fixed perimeter is a square with sides of 13.5 feet.

Possible Side Lengths and Corresponding Areas

The key insight is that for any side length L in the interval $(0, 27)$, the other side W is determined, and the area can be calculated:

$$ A(L) = 27L - L^2 $$

This quadratic relationship indicates that:

- The minimal area approaches zero as $L \to 0^+$ or $L \to 27^-$.
- The maximum area is at $L = 13.5$, with an area of 182.25 sq. ft.

Implication: The possible areas for rectangles with a perimeter of 54 feet range from just above 0 to a maximum of 182.25 square feet, depending on the specific side lengths.

Sample Calculations for Specific Side Lengths

Let’s consider some specific L values:

- Case 1: $L = 10$ ft
$$ W = 17 $$
$$ A = 10 \times 17 = 170 \text{ sq. ft} $$

- Case 2: $L = 13.5$ ft (maximum area)
$$ W = 13.5 $$
$$ A = 182.25 \text{ sq. ft} $$

- Case 3: $L = 20$ ft
$$W = 7$$
$$A = 20 \times 7 = 140 \text{ sq. ft}$$

- Case 4: $L \to 0^+$
$$W \to 27^-$$
$$A \to 0^+$$

This illustrates how the area varies with side lengths, emphasizing the importance of the specific length chosen.

Implications of Varying Side Lengths on Area and Practicality

Understanding the mathematical possibilities is essential, but real-world applications often impose additional constraints. For example:

- Physical Limitations: If the rectangle is a garden or a building, minimum and maximum side lengths may be dictated by practical considerations.

- Design Constraints: Architects or engineers may prefer certain aspect ratios.

- Maximizing Area: If the goal is to maximize the area within a fixed perimeter, the optimal solution is a square with sides of 13.5 feet.

From a mathematical standpoint, the range of possible side lengths is continuous within $(0, 27)$, and the corresponding areas fill the interval $(0, 182.25]$.

Visualizing the Relationship: Graphical Insight

Plotting the area as a function of L offers valuable intuition:

- The parabola opens downward, with a vertex at $L=13.5$.
- As L approaches 0 or 27, the area approaches zero.
- The maximum area occurs at the midpoint, confirming the square shape

as the optimal rectangle.

This visualization reinforces the principle that, for a fixed perimeter, a square maximizes area, a well-known geometric fact.

Concluding Remarks: Practical and Theoretical Significance

The exploration of the possible side lengths of a rectangle with a fixed perimeter reveals key insights:

- Mathematically, side lengths are constrained between just above 0 and just below 27 feet, with the maximum area achieved when the rectangle is a square.
- Practically, the choice of side lengths depends on specific needs—whether maximizing area or adhering to design constraints.
- Theoretical importance lies in understanding the inverse relationship between side lengths and area, highlighting how fixing one geometric property (perimeter) influences the range of others.

In summary, given a perimeter of 54 feet, the possible lengths of a side span a continuous interval from just above 0 to just below 27 feet, with the corresponding areas ranging from near zero up to approximately 182.25 square feet. This analysis exemplifies the interconnectedness of geometric properties and showcases the importance of algebraic and graphical methods in solving real-world mathematical problems.

In essence, the problem underscores a fundamental principle in geometry: for a given perimeter, the shape that maximizes area is a square, and the possible dimensions form a broad spectrum dictated by the simple linear relationship between sides. Whether for academic pursuits or practical design, understanding these relationships equips individuals with a deeper appreciation of geometric harmony and optimization.

The Perimeter Of A Rectangle Is 54 Feet Describe The Possible Lengths Of A Side If The Area Of The Rectangle

The Perimeter Of A Rectangle Is 54 Feet Describe The Possible Lengths Of

A Side If The Area Of The Rectangle

Related Articles

- WILL GIVE BRAINLIEST TO FIRST ANSWERThe Graph Represents The Journey Of A Bus From The Bus Stop To Different
- The Company Is Currently Selling 6,500 Units Per Month. Fixed Expenses Are $184,000 Per Month. The Marketing
- A Business Organization Needs To Make Up A 8 Member Fund-raising Committee. The Organization Has 6 Accounting

Back to Home