The Plane P Contains The Lire L Given By X=1-t, Y= 1+2t, Z=2-3t And The Point 9-1,1,2). A. Find The Egontion

The Plane P Contains The Lire L Given By X=1-t, Y= 1+2t, Z=2-3t And The Point 9-1,1,2). A. Find The Equation

---

Introduction to Planes and Lines in 3D Space

Understanding the geometric relationships between lines and planes in three-dimensional space is fundamental in vector calculus and analytical geometry. In this context, we are given a line \(L \) defined parametrically and a point not necessarily on the line. Our goal is to determine the equation of the plane \(P \) that contains this line \(L \) and passes through the given point \((9, -1, 1) \).

This task involves:

- Recognizing the parametric equations of the line \(L \).
- Determining the direction vector of \(L \).
- Finding a point on \(L \).
- Using the point \((9, -1, 1) \) along with the line's direction vector and a point on \(L \) to establish the plane equation.

Understanding the Given Line \(L \)

Parametric Equations of the Line

The line \(L \) is given by:

\[
\begin{cases}
x = 1 - t \\
y = 1 + 2t \\
z = 2 - 3t
\end{cases}
\]

where \(t \) is a real parameter.

Identifying the Direction Vector of L

From the parametric equations, the direction vector $\vec{d}$ of L can be directly read as the coefficients of t:

$$
\vec{d} = \langle -1, 2, -3 \rangle
$$

This vector indicates the direction in which the line extends in 3D space.

Finding a Point on L

To find a point P_0 on the line, choose any value of t. For simplicity, take $t = 0$:

$$
x = 1 - 0 = 1 \\
y = 1 + 0 = 1 \\
z = 2 - 0 = 2
$$

Thus, a point on L is:

$$
P_0 = (1, 1, 2)
$$

Determining the Plane P

Given Data

- A point $Q = (9, -1, 1)$, through which the plane passes.
- The line L lying entirely in the plane P.

Step 1: Confirm that L is in the plane P

Since P contains the line L, any point on L must satisfy the plane's equation.

Step 2: Find two vectors lying in the plane P

- Vector $\vec{P_0Q}$: from point P_0 on L to point Q:

$$
\vec{P_0Q} = \langle 9 - 1, -1 - 1, 1 - 2 \rangle = \langle 8, -2, -1 \rangle
$$

- Direction vector $\vec{d}$ of L, as previously identified:

$$
\vec{d} = \langle -1, 2, -3 \rangle
$$

Both vectors $\vec{P_0Q}$ and $\vec{d}$ lie in the plane P.

Step 3: Find the normal vector $\vec{n}$ of the plane P

The normal vector $\vec{n}$ of the plane is perpendicular to any vectors lying in the plane. Therefore, $\vec{n}$ can be obtained via the cross product of $\vec{P_0Q}$ and $\vec{d}$:

$$
\vec{n} = \vec{P_0Q} \times \vec{d}
$$

Calculating the cross product:

$$
\vec{n} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
8 & -2 & -1 \\
-1 & 2 & -3
\end{vmatrix}
$$

Expanding:

$$
\vec{n} = \mathbf{i}((-2)(-3) - (-1)(2)) - \mathbf{j}(8(-3) - (-1)(-1)) + \mathbf{k}(8 \times 2 - (-2)(-1))
$$

Calculations:

$$
\begin{aligned}
\mathbf{i} & : (-2)(-3) - (-1)(2) = 6 - (-2) = 6 + 2 = 8 \\
\mathbf{j} & : 8(-3) - (-1)(-1) = -24 - 1 = -25 \\
\end{aligned}
$$

\mathbf{k} & : 8 \times 2 - (-2)(-1) = 16 - 2 = 14
\end{aligned}
\]

Thus,

\[
\boxed{
\vec{n} = \langle 8, -(-25), 14 \rangle = \langle 8, 25, 14 \rangle
}
\]

Note: Remember to change the sign for the \(\mathbf{j} \) component because of the cofactor expansion:

\[
\vec{n} = \langle 8, 25, 14 \rangle
\]

Formulating the Equation of Plane \(P \)

Step 1: Write the general plane equation

The plane passing through point \(Q = (9, -1, 1) \) with normal vector \(\vec{n} = \langle 8, 25, 14 \rangle \) is given by:

\[
8(x - 9) + 25(y + 1) + 14(z - 1) = 0
\]

This simplifies to:

\[
8x - 72 + 25y + 25 + 14z - 14 = 0
\]

Combine like terms:

\[
8x + 25y + 14z - 72 + 25 - 14 = 0
\]

\[
8x + 25y + 14z - 61 = 0
\]

Step 2: Final equation of the plane P

The explicit form of the plane's equation is:

$$
\boxed{
8x + 25y + 14z = 61
}
$$

This is the required equation of the plane P that contains the line L and passes through the point $(9, -1, 1)$.

Summary and Key Takeaways

- The parametric equations of line L: $x=1-t, y=1+2t, z=2-3t$.
- The direction vector of L: $\langle -1, 2, -3 \rangle$.
- A point on L: $(1, 1, 2)$.
- The point through which the plane passes: $(9, -1, 1)$.
- The vector from this point to a point on L: $\langle 8, -2, -1 \rangle$.
- The normal vector of the plane: $\langle 8, 25, 14 \rangle$.
- The equation of the plane P:

$$
8x + 25y + 14z = 61
$$

Conclusion

By analyzing the parametric equations of the line, calculating the direction vector, and employing vector operations such as the cross product, we successfully derived the explicit equation of the plane P. This process exemplifies the importance of vector calculus in solving spatial geometry problems involving lines and planes in three-dimensional space.

Understanding these concepts enhances spatial reasoning and provides tools for tackling complex geometric problems in fields such as engineering, computer graphics, and physics.

Frequently Asked Questions

What is the equation of the plane that contains the line P given by X=1−t, Y=1+2t, Z=2−3t and passes through the point (9, -1, 1)?

To find the plane equation, first find a point on the line, for example at t=0: (1, 1, 2). The plane passes through this point and the point (9, -1, 1). Next, find the direction vector of the line: d = <-1, 2, -3>. Using these points, determine two vectors on the plane: v1 = (1−1, 1−1, 2−2) = (0, 0, 0) (not useful), so better to use the point on the line at t=0: P0=(1,1,2) and the given point Q=(9, -1, 1). Vector QP0 = (8, -2, -1). The direction vector of the line is d = <-1, 2, -3>. The plane's normal vector is the cross product of the vectors QP0 and d: n = QP0 × d = ((−2)(−3)−(−1)(2), (−1)(−1)−(8)(−3), (8)(2)−(−2)(−1)) = (6+2, 1+24, 16−2) = (8, 25, 14). Equation of the plane: 8(x−1) + 25(y−1) + 14(z−2) = 0. Simplified: 8x + 25y + 14z = 8 + 25 + 28 = 61.

How do you determine if the point (9, -1, 1) lies on the plane containing the line P?

Substitute the point into the plane equation 8x + 25y + 14z = 61: 8(9) + 25(-1) + 14(1) = 72 - 25 + 14 = 61. Since the left side equals the right side, the point (9, -1, 1) lies on the plane.

What is the parametric form of the line P given by X=1−t, Y=1+2t, Z=2−3t?

The parametric equations are: X=1−t, Y=1+2t, Z=2−3t, where t is a real parameter that defines points along the line.

How can you find the normal vector to the plane containing line P and passing through (9, -1, 1)?

The normal vector is obtained by calculating the cross product of two vectors lying on the plane: one vector along the line, d=<-1, 2, -3>, and another vector from a point on the line to the given point, such as QP0=(8, -2, -1). The normal vector n = QP0 × d = (8, 25, 14).

What is the significance of the cross product in finding the plane's equation?

The cross product of two vectors lying on the plane yields a vector perpendicular to both, which is the normal vector of the plane. This normal vector is essential for writing the plane's equation.

Can the plane be uniquely determined with the given line and point? Why?

Yes, because the line provides a direction and a point on the line, and the additional point (9, -1, 1) lies on the plane, allowing us to determine a unique plane passing through these points and containing the line.

How do you verify that the point (9, -1, 1) is on the plane once the equation is found?

Substitute the point's coordinates into the plane equation. If the left side equals the right side, the point lies on the plane. In this case, substituting (9, -1, 1) yields 8(9) + 25(-1) + 14(1) = 61, matching the plane's constant term, confirming the point is on the plane.

What is the final equation of the plane containing the line and passing through the point (9, -1, 1)?

The equation of the plane is $8x + 25y + 14z = 61$.

Additional Resources

The Plane P Contains The Lire L Given By X=1–t, Y=1+2t, Z=2–3t And The Point (9–1, 1, 2): A Comprehensive Investigation

---

Introduction

In the realm of analytical geometry, the study of planes and lines forms a foundation for understanding spatial relationships, intersections, and geometric properties. One common problem involves determining the equation of a plane that contains a given line and passes through a specific point. This investigation delves into such a problem: The Plane P Contains The Lire L Given By X=1–t, Y=1+2t, Z=2–3t And The Point (9–1, 1, 2).

This article aims to thoroughly analyze and derive the equation of the plane P, based on the given line L and the point provided. We will explore the geometric principles involved, methodically compute necessary vectors, and elucidate the step-by-step process to arrive at the plane's equation. This investigation not only demonstrates the fundamental techniques of coordinate geometry but also highlights their practical applications in spatial reasoning and problem-solving.

Background and Fundamental Concepts

Understanding Lines and Planes in 3D Space

In three-dimensional Euclidean space, a line can be represented parametrically as:

$$\mathbf{r} = \mathbf{r}_0 + t \mathbf{d}$$

where:

- $\mathbf{r}_0$ is a point on the line
- $\mathbf{d}$ is the direction vector
- t is a parameter

Similarly, a plane can be described by the scalar equation:

$$ a x + b y + c z + d = 0 $$

where $\mathbf{n} = (a, b, c)$ is the normal vector to the plane.

The Relationship Between a Line and a Plane

Given a line L and a point P, to find the plane P that contains L and passes through P, the key steps are:

1. Identify a point on the line (say, $\mathbf{r}_L$)
2. Find the direction vector of the line ($\mathbf{d}_L$)
3. Determine the vector from the point P to a point on the line
4. Find the normal vector to the plane by taking the cross product of the line's direction vector and the vector connecting the point P to a point on the line
5. Formulate the plane's equation using the normal vector and a known point on the plane

---

Step-by-Step Derivation

Step 1: Extract the Given Data

The line L:

Given parametric equations:

$$
\begin{cases}
x = 1 - t \\
y = 1 + 2t \\
z = 2 - 3t
\end{cases}
$$

- Point on the line ($t=0$):

$$
\mathbf{r}_0 = (1, 1, 2)
$$

- Direction vector $\mathbf{d}_L$:

$$
\mathbf{d}_L = \frac{d}{dt}(x, y, z) = (-1, 2, -3)
$$

The given point P:

$$
(9 - 1, 1, 2) \Rightarrow \text{Note: The notation seems to suggest } (9 - 1, 1, 2) = (8, 1, 2)
$$

Clarification: The point is $\mathbf{P} = (8, 1, 2)$.

Step 2: Find a Vector from P to a Point on the Line

Using $\mathbf{r}_0 = (1, 1, 2)$ on the line and $\mathbf{P} = (8, 1, 2)$:

$$
\mathbf{V} = \mathbf{r}_0 - \mathbf{P} = (1 - 8, 1 - 1, 2 - 2) = (-7, 0, 0)
$$

Step 3: Compute the Normal Vector to the Plane

The plane contains the line L, which has direction vector $\mathbf{d}_L = (-1, 2, -3)$. To find the plane's normal vector $\mathbf{n}$, we need a vector perpendicular to both the line's direction and the vector connecting P to the line, i.e.,

$$
\mathbf{n} = \mathbf{d}_L \times \mathbf{V}
$$

Calculate the cross product:

$$
\mathbf{n} = (-1, 2, -3) \times (-7, 0, 0)
$$

Using the standard determinant form:

$$
\mathbf{n} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-1 & 2 & -3 \\
-7 & 0 & 0
\end{vmatrix}
$$

Compute component-wise:

- $n_x = (2)(0) - (-3)(0) = 0 - 0 = 0$
- $n_y = - [(-1)(0) - (-3)(-7)] = - [0 - 21] = -(-21) = 21$
- $n_z = (-1)(0) - (2)(-7) = 0 + 14 = 14$

Thus,

$$
\boxed{
\mathbf{n} = (0, 21, 14)
}
$$

For simplicity, we can factor out 7:

$$
\mathbf{n} = (0, 3, 2)
$$

Step 4: Write the Equation of the Plane

The general form:

$$
a x + b y + c z + d = 0
$$

where $(a, b, c) = \mathbf{n} = (0, 3, 2)$.

Use the point $\mathbf{P} = (8, 1, 2)$ to solve for d:

$$
0 \times 8 + 3 \times 1 + 2 \times 2 + d = 0 \Rightarrow 0 + 3 + 4 + d = 0
$$

$$
d = -7
$$

Final Equation of the Plane:

$$
0 \times x + 3 y + 2 z - 7 = 0
$$

which simplifies to:

$$
\boxed{
3 y + 2 z = 7
}
$$

Geometric Interpretation and Validation

Confirming the Line Lies in the Plane

Substitute the parametric equations into the plane equation:

$$
3(1 + 2t) + 2(2 - 3t) = 7
$$

Compute:

$$
3 + 6t + 4 - 6t = 7
$$

$$
(3 + 4) + (6t - 6t) = 7 \Rightarrow 7 + 0 = 7
$$

which is true for all t, confirming the line L lies entirely within the plane P.

Confirming the Point $P = (8, 1, 2)$

Substitute into the plane equation:

$$
3(1) + 2(2) = 3 + 4 = 7
$$

which satisfies the equation, confirming that P is on the plane.

Additional Insights and Applications

Significance of the Normal Vector

The normal vector $\mathbf{n} = (0, 3, 2)$ indicates the plane's orientation in space. Its components reveal how the plane inclines relative to the coordinate axes, offering insights into the geometric setup.

Potential Extensions

- Finding the intersection of the plane with other geometric entities.
- Analyzing the distance from other points to the plane.
- Exploring the angle between the plane and other planes or lines.

Conclusion

This thorough investigation demonstrates a systematic approach to determining the equation of a plane containing a given line and passing through a specified point. By leveraging vector operations, parametric representations, and geometric reasoning, the problem's solution is both elegant and robust.

The derived plane equation:

$$
\boxed{
3 y + 2 z = 7
}
$$

succinctly captures the spatial relationship between the line, the point, and the plane. Such analytical techniques are fundamental in advanced geometry, computer graphics, engineering design, and various scientific applications, showcasing the intersection of algebraic method and geometric intuition.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Lay, D. C. (2012). Linear Algebra and Its Applications. Pearson.
- Anton, H., Rorres, C. (2013). Elementary Linear Algebra. Wiley.

Note: This investigation underscores the importance of precise notation, systematic calculation

The Plane P Contains The Lire L Given By X 1 T Y 1 2t Z 2 3t And The Point 9 1 1 2 A Find The Egontion

The Plane P Contains The Lire L Given By X 1 T Y 1 2t Z 2 3t And The Point 9 1 1 2 A Find The Egontion

Related Articles

- Assume That Adults Have IQ Scores That Are Normally Distributed With A Mean Of 103.3 And A Standard Deviation
- Rugged Adventures Inc. Wants To Terminate Its Franchise Arrangement With Stillwater Kayaking. Their Contract
- Suppose You Deposit $ 1000 $ 1000 In An Account With An Annual Interest Rate Of 6 % 6 % Compounded Quarterly.

Back to Home