The Pulley Is Light And Frictionless. Find The Mass M1 M 1 , Given That M2 M 2 (5.50 Kg) Is Moving Downwards

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In the realm of classical mechanics, understanding the dynamics of pulleys and masses is fundamental to solving numerous physics problems. When dealing with systems involving two masses connected over an ideal (light and frictionless) pulley, the key lies in applying Newton's laws of motion carefully to determine unknown quantities. This article will guide you through the process of finding the mass m_1 when you are given the mass $m_2 = 5.50\, \mathrm{kg}$, which is moving downward. We will explore the problem step-by-step, introduce relevant physics principles, and provide detailed solutions to enhance your understanding.

Understanding the System: Pulley and Connected Masses

In typical pulley problems, two masses are connected via a string passing over a pulley. The pulley is considered light and frictionless, meaning:

- The pulley's mass is negligible; it does not affect the system's inertia.
- No energy is lost to friction; the tension throughout the string is uniform.

Suppose the system involves:

- Mass $m_2 = 5.50\, \mathrm{kg}$, moving downward.
- Mass m_1, which we need to find.
- Both masses are connected by an inextensible string over the pulley.

Depending on the scenario, the masses could be moving with different accelerations, but for simplicity, let's assume the system is accelerating uniformly, and m_2 is moving downward with acceleration a.

Fundamental Principles and Assumptions

Before diving into calculations, it is crucial to understand the assumptions and physics principles:

- Newton's Second Law: $F = m a$, where F is the net force, m is the mass, and a is acceleration.
- Tension in the String: Since the pulley is frictionless and massless, the tension T is the same on both sides.
- Direction of Motion: Given m_2 is moving downward, m_1 will be moving upward if the system is accelerating.

Setting Up the Equations of Motion

Let's define the variables:

- m_1: the unknown mass (kg).
- $m_2 = 5.50\, \mathrm{kg}$: known mass.
- a: acceleration of the masses (m/s^2).
- T: tension in the string (N).
- $g = 9.81\, \mathrm{m/s^2}$: acceleration due to gravity.

For mass m_2 (moving downward):

$$
m_2 g - T = m_2 a
$$

For mass m_1 (moving upward):

$$
T - m_1 g = m_1 a
$$

Assuming the system accelerates uniformly, these two equations can be solved simultaneously to find m_1.

Deriving the Expression for m_1

Adding the two equations:

$$
(m_2 g - T) + (T - m_1 g) = m_2 a + m_1 a
$$

Simplifies to:

$$
m_2 g - m_1 g = (m_2 + m_1) a
$$

Rearranged as:

$$
(m_2 - m_1) g = (m_2 + m_1) a
$$

From this, solve for a:

$$
a = \frac{(m_2 - m_1) g}{m_2 + m_1}
$$

Now, from the equation for m_2:

$$

$$
T = M_2 g - M_2 a
$$

Similarly, substituting a:

$$
T = M_2 g - M_2 \times \frac{(M_2 - M_1) g}{M_2 + M_1}
$$

But to find M_1, it's often more straightforward to express M_1 in terms of known quantities and the acceleration.

Finding M_1 Given M_2, g, and Acceleration a

Since the problem states that M_2 is moving downward, but does not specify the acceleration, we might consider a case where the acceleration is specified or can be observed.

Case 1: M_2 moves downward with acceleration a, and the acceleration is known.

Suppose the acceleration is known or measured, then:

$$
M_1 = \frac{M_2 g - M_2 a}{g + a}
$$

Case 2: M_2 is moving downward at constant velocity (i.e., $a = 0$).

In that scenario, the system is in equilibrium:

$$
M_2 g = T
$$
$$
T = M_1 g
$$
$$
\Rightarrow M_1 = M_2
$$

But if the problem specifies downward movement with acceleration, then the general approach applies.

Example Calculation: Numerical Solution

Let's assume the mass $M_2 = 5.50\, \mathrm{kg}$ is accelerating downward with $a = 2\, \mathrm{m/s^2}$. Find M_1.

Using the earlier derived formula:

$$
a = \frac{(M_2 - M_1) g}{M_2 + M_1}

$$
$$

Rearranged to solve for M_1:

$$
a (M_2 + M_1) = (M_2 - M_1) g
$$
$$
a M_2 + a M_1 = g M_2 - g M_1
$$
$$
a M_1 + g M_1 = g M_2 - a M_2
$$
$$
M_1 (a + g) = M_2 (g - a)
$$
$$
M_1 = \frac{M_2 (g - a)}{a + g}
$$

Plugging in the numbers:

$$
M_1 = \frac{5.50\, (9.81 - 2)}{2 + 9.81} = \frac{5.50 \times 7.81}{11.81}
$$

Calculating numerator:

$$
5.50 \times 7.81 \approx 42.955
$$

Calculating denominator:

$$
11.81
$$

Therefore:

$$
M_1 \approx \frac{42.955}{11.81} \approx 3.64\, \mathrm{kg}
$$

Result: The mass M_1 is approximately 3.64 kg when $M_2 = 5.50\, \mathrm{kg}$ accelerates downward at $2\, \mathrm{m/s^2}$.

Impact of System Conditions on M_1

The actual value of M_1 depends on the acceleration of M_2:

- If M_2 moves faster downward, M_1 will be smaller.
- If M_2 moves slower or is stationary, M_1 adjusts accordingly.
- For equilibrium (no acceleration), $M_1 = M_2$.

Additional Considerations and Real-World Factors

While the idealized model assumes a light and frictionless pulley, real systems may involve:

- Friction in the pulley: leads to energy loss and alters tension.
- Massive pulleys: contribute to the system's inertia, requiring more complex calculations.
- String mass and elasticity: may affect tension and acceleration.
- Air resistance: minimal but can be considered in high-precision experiments.

Understanding these factors helps in designing experiments and interpreting real-world data accurately.

Summary and Key Takeaways

- The problem involves applying Newton's laws to a two-mass pulley system.
- The key equations relate tension, mass, gravity, and acceleration.
- To find M_1, knowledge of the acceleration of M_2 is essential.
- Assuming an acceleration, the formula:

$$
M_1 = \frac{M_2 (g - a)}{a + g}
$$

enables calculation of the unknown mass.

- Real systems may deviate from ideal assumptions, so consider system-specific factors.

Conclusion

Determining the mass M_1 in a pulley system where M_2 is moving downward involves analyzing the forces acting on each mass and applying Newton's second law. When the pulley is light and frictionless, the tension is uniform, streamlining the calculations. By knowing the acceleration of M_2, one can accurately compute M_1 using derived formulas. This understanding is fundamental for physics students and engineers

Frequently Asked Questions

In a pulley system where the pulley is light and frictionless, how is the acceleration of masses related if M2 (5.50 kg) moves downward?

The acceleration of both masses is the same in magnitude but opposite in direction; M2 accelerates downward while M1 moves upward with the same acceleration.

Given that M2 is moving downward with acceleration 'a', how can we determine the mass M1 in a pulley system?

Using Newton's second law, set up equations for both masses, then solve for M1 based on the known values of M2 and the acceleration 'a'.

What assumptions are made when the pulley is considered light and frictionless in such problems?

The pulley's mass and rotational inertia are negligible, and there is no friction in the axle or between the pulley and the string, making tension uniform throughout the string.

If M2 is 5.50 kg and moving downward with acceleration 'a', how do you find the acceleration 'a' in such a pulley system?

Apply Newton's second law to both masses, set up the equations, and solve simultaneously to find 'a' in terms of M1 and M2.

How does the direction of acceleration affect the calculation of M1 in the pulley system?

The direction indicates which mass is accelerating downward or upward; this sign convention helps determine the correct equations and solve for M1 accordingly.

What is the significance of the pulley being frictionless in calculating M1?

It ensures that all tension in the string is the same on both sides, simplifying the calculation by neglecting any energy loss due to friction.

How can the known mass M2 and its downward acceleration help determine the unknown mass M1?

By applying Newton's second law to both masses and knowing M2's acceleration, you can derive an expression for M1 that satisfies the system's equilibrium and motion equations.

What formulas are typically used to find M1 in a pulley system with known M2 and downward acceleration?

The primary formulas are $T = M2(g - a)$ for M2 and $T = M1(g + a)$ for M1, which can be combined to solve for M1 given M2, g, and a.

How do you interpret the result of M1 once you calculate it for the system where M2 is moving downward?

The value of M1 indicates the mass required to produce the observed acceleration when M2 moves downward; if M1 is larger than M2, the system accelerates accordingly, and vice versa.

Additional Resources

The Pulley Is Light And Frictionless. Find The Mass M1, Given That M2 (5.50 Kg) Is Moving Downwards

In the realm of classical mechanics, the simplicity of idealized systems often provides profound insights into the fundamental principles governing motion. One such fundamental scenario involves a massless, frictionless pulley supporting two masses connected by a string—an elegant setup that illustrates Newtonian motion and the conservation of energy. Today, we delve into a specific problem: determining the unknown mass M_1 when the known mass M_2 (5.50 kg) is observed moving downward. This problem not only tests our understanding of Newton's second law but also exemplifies the application of free-body diagrams, equations of motion, and algebraic manipulation in solving real-world physics puzzles.

Understanding the Physical System

Before jumping into calculations, it is crucial to understand the physical setup and assumptions, as these form the foundation for our analysis.

Components of the System

- Mass M_1: Unknown, positioned on one side of the pulley.
- Mass $M_2 = 5.50\,\text{kg}$: Known, observed to move downward.
- Pulley: Assumed to be light (massless) and frictionless, meaning it doesn't add inertia or energy losses to the system.
- String: Ideal, massless, and inextensible, transmitting tension without stretching.

Key Assumptions

- No frictional forces act at the pulley or in the string.
- The pulley is massless, so its moment of inertia is zero, simplifying rotational dynamics.
- The system is released from rest, or the problem specifies the motion at a particular instant.
- The acceleration of the masses is the same in magnitude but opposite in direction, due to the inextensibility of the string.

Analyzing the Motion: Newton's Laws in Action

To find M_1, we need to analyze the forces acting on each mass, relate them through the system's acceleration, and apply Newton's second law.

Setting Up Coordinates and Directions

- Define downward as positive for M_2.
- Since M_2 is moving downward, M_1 must be moving upward (assuming the masses are connected over the pulley).

Free-Body Diagrams (FBDs)

For each mass, draw an FBD:

- Mass M_2:
- Downward force: gravity $M_2 g$
- Upward force: tension T

- Mass M_1:
- Downward force: gravity $M_1 g$
- Upward force: tension T

Writing the Equations of Motion

Let a be the magnitude of acceleration of both masses.

For M_2:
Since it moves downward:

$$M_2 g - T = M_2 a \quad (1)$$

For M_1:
Since it moves upward:

$$T - M_1 g = M_1 a \quad (2)$$

Deriving the Relationship for M_1

The goal is to solve for M_1 in terms of known quantities and the observed motion.

Step 1: Express Tension T from equation (2):

$$T = M_1 a + M_1 g$$

Step 2: Substitute T into equation (1):

$$M_2 g - (M_1 a + M_1 g) = M_2 a$$

Simplify:

$$M_2 g - M_1 a - M_1 g = M_2 a$$

Rearranged as:

$$M_2 g - M_1 g = M_2 a + M_1 a$$

Factor out terms:

$$ (m_2 - m_1) g = (m_2 + m_1) a $$

Step 3: Express acceleration a:

$$ a = \frac{(m_2 - m_1) g}{m_2 + m_1} $$

This equation relates the acceleration a to the known m_2, gravity g, and the unknown m_1.

Incorporating Observed Data: m_2 Moving Downward

The problem states that m_2 is moving downward, which implies the system's acceleration a is positive in the downward direction. If the system's motion is known at a specific instant or if the acceleration a is given, we can directly solve for m_1.

However, in many typical problems, the acceleration isn't immediately specified; instead, the observed behavior (e.g., m_2 moving downward at a certain speed or acceleration) is used to find m_1.

Case 1: System at Instantaneous Equilibrium

If m_2 moves downward with a constant velocity (i.e., zero acceleration), then:

$$ a = 0 $$

From the earlier equation:

$$ (m_2 - m_1) g = 0 \Rightarrow m_2 = m_1 $$

Given $m_2 = 5.50\,kg$, then:

$$ m_1 = 5.50\,kg $$

This is a special case where the masses are equal, and the system moves at constant velocity with no acceleration.

Case 2: System Accelerating Downward with Known a

Suppose the observed acceleration a of m_2 is known (say, from measurements). Then, rearranged:

$$ m_1 = m_2 - \frac{(m_2 + m_1) a}{g} $$

which can be solved iteratively or algebraically if a is specified.

Alternatively, the problem might provide the value of a directly or the velocity change over time, allowing us to compute a.

Numerical Example: Calculating M_1

Suppose the known mass $M_2 = 5.50\,kg$ is observed to accelerate downward at $a = 1.50\,m/s^2$.

Using the derived formula:

$$a = \frac{(M_2 - M_1) g}{M_2 + M_1}$$

Rearranged to solve for M_1:

$$a (M_2 + M_1) = (M_2 - M_1) g$$

$$a M_2 + a M_1 = M_2 g - M_1 g$$

Bring all M_1 terms to one side:

$$a M_1 + M_1 g = M_2 g - a M_2$$

$$M_1 (a + g) = M_2 (g - a)$$

Finally,

$$M_1 = \frac{M_2 (g - a)}{a + g}$$

Plugging in the values:

- $M_2 = 5.50\,kg$
- $g = 9.81\,m/s^2$
- $a = 1.50\,m/s^2$

Calculate numerator:

$$5.50 \times (9.81 - 1.50) = 5.50 \times 8.31 = 45.705\,kg \cdot m/s^2$$

Calculate denominator:

$$1.50 + 9.81 = 11.31\,m/s^2$$

Compute M_1:

$$M_1 = \frac{45.705}{11.31} \approx 4.04\,kg$$

Result: The unknown mass M_1 is approximately 4.04 kg.

Implications and Real-World Applications

Understanding such systems isn't solely academic; it has practical implications across engineering, transportation, and even biomechanics.

- Engineering Design: Engineers often analyze pulley systems in elevators, cranes, and conveyor belts, where knowing the mass distribution affects safety and efficiency.
- Educational Demonstrations: Physics educators utilize such models to illustrate Newtonian principles vividly.
- Robotics and Automation: Precise calculations of masses and accelerations inform the design of robotic arms and automated systems.

Conclusion: The Power of Simplification and Mathematical Analysis

The problem of determining an unknown mass \(M_1 \) when a known mass \(M_2 \) (5.50 kg) is moving downward showcases the elegance of classical mechanics. By assuming an ideal pulley—light and frictionless—and applying Newton's second law, we derive an expression linking the system's acceleration to the masses involved. Whether the system is at rest, moving at constant velocity, or accelerating, the fundamental principles remain consistent and powerful.

In particular, when

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