The Remaining Mass Of 28 Grams Of A Radioactive Element Whose Half Life Is 20 Years Is Given By The Following

The Remaining Mass Of 28 Grams Of A Radioactive Element Whose Half Life Is 20 Years Is Given By The Following

Understanding the decay of radioactive elements is fundamental in fields such as nuclear physics, medicine, archaeology, and environmental science. When dealing with radioactive substances, a common question arises: How much of a radioactive element remains after a certain period? Specifically, if we start with a known initial mass, like 28 grams of a radioactive element, and know its half-life, how can we determine the remaining mass after a given time? This article provides a comprehensive explanation of this process, focusing on a radioactive element with a half-life of 20 years, and guides you through the calculations involved.

Introduction to Radioactive Decay and Half-Life

Radioactive decay is a spontaneous process where unstable atomic nuclei lose energy by emitting radiation. This process results in the transformation of the original element into a different element or isotope over time.

Half-life is a key concept in understanding radioactive decay. It is defined as the time required for half of the radioactive nuclei in a sample to decay. Different elements have different half-lives, ranging from fractions of a second to billions of years.

Key points about half-life:
- It remains constant for a given isotope.
- It is independent of the initial amount of the substance.
- It governs the rate of decay, which follows an exponential pattern.

Mathematical Model of Radioactive Decay

The decay of a radioactive substance can be modeled mathematically using exponential decay formulas. The fundamental expression for the remaining amount of substance after a certain period is:

$$ N(t) = N_0 \times e^{-\lambda t} $$

Where:
- $N(t)$ is the remaining quantity at time t.
- N_0 is the initial quantity.
- λ is the decay constant, specific to each isotope.
- t is the elapsed time.

Alternatively, the decay can be expressed in terms of half-life:

$$ N(t) = N_0 \times \left(\frac{1}{2} \right)^{\frac{t}{T_{1/2}}} $$

Where:
- $T_{1/2}$ is the half-life of the isotope.

This formula is particularly useful because it directly relates the initial amount, the half-life, and the elapsed time.

Calculating Remaining Mass of a Radioactive Element

Given:
- Initial mass $N_0 = 28 \, \text{grams}$
- Half-life $T_{1/2} = 20 \, \text{years}$
- Time elapsed t (which can vary)

The general formula to determine the remaining mass after time t:

$$ N(t) = N_0 \times \left(\frac{1}{2} \right)^{\frac{t}{T_{1/2}}} $$

Step-by-step calculation process:

1. Identify the known variables:
- Initial mass $N_0 = 28$ grams
- Half-life $T_{1/2} = 20$ years
- Time t (specify as needed)

2. Plug into the formula:

$$ N(t) = 28 \times \left(\frac{1}{2} \right)^{\frac{t}{20}} $$

3. Calculate the exponent:
- Divide the elapsed time t by 20 years to find the number of half-lives elapsed.

4. Compute the remaining mass:

- Raise 1/2 to the power of the number of half-lives.
- Multiply by the initial mass.

Examples of Remaining Mass Calculations

Let’s explore specific examples to understand how the remaining mass decreases over time.

Example 1: After 20 Years

Time elapsed: $t = 20$ years

Applying the formula:

$$N(20) = 28 \times \left(\frac{1}{2} \right)^{\frac{20}{20}} = 28 \times \left(\frac{1}{2} \right)^1 = 28 \times 0.5 = 14 \, \text{grams}$$

Result: After 20 years, 14 grams of the radioactive element remain, which aligns with the definition of half-life.

Example 2: After 40 Years

Time elapsed: $t = 40$ years

$$N(40) = 28 \times \left(\frac{1}{2} \right)^{\frac{40}{20}} = 28 \times \left(\frac{1}{2} \right)^2 = 28 \times 0.25 = 7 \, \text{grams}$$

Result: After 40 years, 7 grams remain, which is one-quarter of the original amount, consistent with two half-lives.

Example 3: After 60 Years

Time elapsed: $t = 60$ years

$$N(60) = 28 \times \left(\frac{1}{2} \right)^{\frac{60}{20}} = 28 \times \left(\frac{1}{2} \right)^3 = 28 \times 0.125 = 3.5 \, \text{grams}$$

Result: After 60 years, 3.5 grams remain, which is one-eighth of the initial amount.

Generalized Formula and Its Application

The general form of the decay formula is:

$$ N(t) = N_0 \times 2^{-\frac{t}{T_{1/2}}} $$

Expressed in logarithmic form, this can be used to solve for the time t if the remaining mass is known:

$$ t = T_{1/2} \times \log_2 \left(\frac{N_0}{N(t)} \right) $$

This allows calculation of the elapsed time given initial and remaining masses or vice versa.

Implications and Practical Uses

Understanding how to calculate the remaining mass of a radioactive element has numerous practical applications:

- **Radiocarbon Dating:** Determining the age of archaeological samples based on remaining Carbon-14.
- **Medical Treatments:** Managing radiopharmaceutical dosages over time.
- **Nuclear Waste Management:** Estimating the decay and remaining hazardous material.
- **Environmental Monitoring:** Tracking decay of radioactive pollutants.

In these contexts, precise calculations ensure safety, accuracy, and effective decision-making.

Factors Affecting Radioactive Decay Calculations

While the mathematical model provides an idealized scenario, real-world factors can influence decay measurements:

1. Environmental Conditions

- Temperature, pressure, and chemical environment may slightly affect decay rates, especially in complex systems.

2. Measurement Accuracy

- Precision in initial mass measurement and detection of decay products impacts calculations.

3. Isotope Purity

- Presence of other isotopes or impurities can alter decay dynamics.

Conclusion

Calculating the remaining mass of a radioactive element after a certain period is straightforward when using the exponential decay formula related to half-life. Specifically, for an initial mass of 28 grams and a half-life of 20 years, the remaining mass after any elapsed time t can be directly computed using:

$$N(t) = 28 \times \left(\frac{1}{2} \right)^{\frac{t}{20}}$$

This approach provides essential insights into the decay process, aiding in scientific research, medical applications, environmental safety, and archaeological dating. Understanding these principles enables professionals and students alike to predict and analyze the behavior of radioactive materials accurately.

References:

- Glasstone, S., & Sesonske, A. (1967). Nuclear Reactor Physics. Van Nostrand Reinhold.
- Krane, K. S. (1988). Introductory Nuclear Physics. Wiley.
- Knoll, G. F. (2010). Radiation Detection and Measurement. Wiley.
- U.S. Nuclear Regulatory Commission. (n.d.). Radioactive Decay and Half-Life. [Online resource]

Keywords: radioactive decay, half-life, remaining mass, exponential decay, nuclear physics, decay constant, radioactive isotope, calculations, initial mass, elapsed time

Frequently Asked Questions

What is the remaining mass of a radioactive element after a certain period if the initial mass is 28 grams and the half-life is 20 years?

The remaining mass can be calculated using the formula: remaining mass = initial mass × (1/2)^(time / half-life).

How do you determine the remaining mass of a radioactive isotope after multiple half-lives?

You apply the exponential decay formula, reducing the initial mass by half after each half-life period: remaining mass = initial mass × (1/2)^(number of half-lives).

If 28 grams of a radioactive element with a half-life of 20 years decay for 40 years, what is the remaining mass?

After 40 years, which is 2 half-lives, the remaining mass is 28 × (1/2)^2 = 28 × 1/4 = 7 grams.

What is the significance of the half-life in calculating the remaining mass of a radioactive element?

The half-life indicates the time required for half of the radioactive substance to decay, enabling calculation of the remaining mass after any period.

How can I find the remaining mass of 28 grams after 60 years for a radioactive element with a 20-year half-life?

Calculate the number of half-lives: 60 / 20 = 3. Then, remaining mass = 28 × (1/2)^3 = 28 × 1/8 = 3.5 grams.

Is the decay of the radioactive element linear or exponential?

The decay is exponential, following the half-life rule, meaning the mass halves after each half-life period.

How does the concept of half-life help in radioactive dating techniques?

Half-life allows scientists to determine the age of a sample by measuring remaining radioactive material and calculating how many half-lives have passed since formation.

What is the formula to calculate the remaining mass of a radioactive element after a given time?

Remaining mass = initial mass × (1/2)^(time / half-life).

Additional Resources

The Remaining Mass Of 28 Grams Of A Radioactive Element Whose Half Life Is 20 Years Is Given By The Following

Radioactive decay has long fascinated scientists and laypeople alike, not only because of its fundamental implications for nuclear physics but also because of its practical applications in medicine, archaeology, and energy. One of the most critical aspects of understanding radioactive decay is determining how much of a radioactive substance remains after a certain period. In this article, we delve into the specifics of calculating the remaining mass of a radioactive element, focusing on an initial sample of 28 grams with a half-life of 20 years. By examining the principles and mathematical models involved, we aim to provide a clear, comprehensive picture of how decay works over time.

Understanding Radioactive Decay and Half-Life

What is Radioactive Decay?

Radioactive decay is a spontaneous process by which unstable atomic nuclei lose energy by emitting radiation. This process transforms the original unstable nucleus into a more stable one, often resulting in a different element or isotope. The decay occurs randomly at the level of individual atoms, but statistically, it follows a predictable pattern characterized by a half-life.

Defining Half-Life

The half-life of a radioactive substance is the time required for half of the initial amount of the radioactive isotope to decay. For example, if you start with 28 grams of a particular isotope, after one half-life (20 years in this case), only 14 grams would remain. This concept is central to nuclear physics and has significant implications for practical applications like radiometric dating and nuclear medicine.

Mathematical Foundations of Radioactive Decay

The Decay Law

Radioactive decay follows an exponential law described mathematically as:

$$ N(t) = N_0 \times e^{-\lambda t} $$

Where:

- $N(t)$ = remaining quantity of the substance after time t
- N_0 = initial quantity of the substance
- λ = decay constant (specific to each isotope)
- t = time elapsed

Relationship Between Decay Constant and Half-Life

The decay constant λ relates directly to the half-life $T_{1/2}$:

$$ \lambda = \frac{\ln 2}{T_{1/2}} $$

For a half-life $T_{1/2} = 20$ years:

$$ \lambda = \frac{\ln 2}{20} \approx \frac{0.6931}{20} \approx 0.03466 \, \text{per year} $$

This decay constant indicates the probability per unit time that an individual atom will decay.

Calculating Remaining Mass After a Given Time

Step-by-Step Approach

Given an initial mass $N_0 = 28$ grams and a half-life $T_{1/2} = 20$ years, we can compute the remaining mass after a certain period t:

1. Determine the decay constant λ:

$$ \lambda = \frac{\ln 2}{20} $$

2. Apply the exponential decay formula:

$$ N(t) = N_0 \times e^{-\lambda t} $$

Alternatively, since the half-life is known, the decay can also be expressed as:

$$ N(t) = N_0 \times \left(\frac{1}{2} \right)^{\frac{t}{T_{1/2}}} $$

This form is often more straightforward for half-life calculations.

Example Calculations

Let's evaluate the remaining mass after specific time intervals:

- After 20 years (1 half-life):

$$ N(20) = 28 \times \left(\frac{1}{2} \right)^{20/20} = 28 \times \frac{1}{2} = 14 \, \text{grams} $$

- After 40 years (2 half-lives):

$$ N(40) = 28 \times \left(\frac{1}{2} \right)^{40/20} = 28 \times \left(\frac{1}{2} \right)^2 = 28 \times \frac{1}{4} = 7 \, \text{grams} $$

- After 60 years (3 half-lives):

$$ N(60) = 28 \times \left(\frac{1}{2} \right)^3 = 28 \times \frac{1}{8} = 3.5 \, \text{grams} $$

- After 100 years:

$$ N(100) = 28 \times \left(\frac{1}{2} \right)^{100/20} = 28 \times \left(\frac{1}{2} \right)^5 = 28 \times \frac{1}{32} \approx 0.875 \, \text{grams} $$

This pattern illustrates how the mass diminishes exponentially, approaching zero but never actually reaching it within finite time.

Practical Applications and Implications

Radiometric Dating

One of the key applications of understanding decay and remaining mass is in radiometric dating. By measuring the remaining amount of a radioactive isotope in a sample, scientists can estimate the age of archaeological artifacts or geological formations. Knowledge of the half-life allows for precise calculations, especially when multiple decay stages are involved.

Medical and Industrial Uses

Radioactive materials with known half-lives are utilized in medical diagnostics and treatments. For example, isotopes like Technetium-99m, with a half-life of about 6 hours, are employed in imaging, while longer-lived isotopes are used in radiotherapy. Understanding decay patterns ensures safety and efficacy.

Nuclear Waste Management

The longevity and decay characteristics of radioactive waste influence storage strategies. Knowing how much of a radioactive isotope remains over decades or centuries guides policies to protect the environment and public health.

Factors Affecting Decay and Measurement Accuracy

Environmental Influences

While the decay of most isotopes is unaffected by external conditions, some factors like temperature, pressure, and chemical state can influence measurements or the behavior of radioactive materials in specific contexts.

Measurement Challenges

Accurate determination of remaining mass depends on precise detection methods, such as Geiger counters, scintillation counters, or mass spectrometry. Calibration and statistical analysis are vital for minimizing errors, especially when the remaining quantities are very small.

Summary and Key Takeaways

- The remaining mass of a radioactive element decreases exponentially over time, following the principles of nuclear decay.
- The half-life provides a practical measure for understanding how quickly a substance decays; for this element, it is 20 years.
- Mathematical models, either exponential decay formulas or half-life-based calculations, enable accurate predictions of remaining mass after any period.
- After one half-life (20 years), 50% of the initial mass remains; after two, 25%; after three, 12.5%, and so on.
- These calculations underpin numerous scientific, medical, and industrial applications, emphasizing the importance of understanding radioactive decay dynamics.

Final Thoughts

Understanding how much of a radioactive element remains after a certain period is essential for harnessing its properties safely and effectively. Whether it's determining the age of ancient artifacts, designing diagnostic procedures, or managing nuclear waste, the principles of decay and half-life provide the foundation for accurate, reliable calculations. The example of 28 grams with a 20-year half-life illustrates the exponential nature of decay and highlights the power of mathematical models in predicting the future state of radioactive materials. As science advances, these principles continue to underpin innovations and safety protocols across diverse fields, ensuring that we harness radioactive elements responsibly and knowledgeably.

The Remaining Mass Of 28 Grams Of A Radioactive Element Whose Half Life Is 20 Years Is Given By The Following

The Remaining Mass Of 28 Grams Of A Radioactive Element Whose Half Life Is 20 Years Is Given By The Following

Related Articles

- In Mesoamerica, Where Writing Appeared In The Last Centuries B.c.e., The Zapotec And Early Mayan Inscriptions
- If You Read A Text And Determined It Was Meant To Inform, What Did You Understand About The Text?(1 Point)the
- (100 POINTS!) What Is The Measure Of Each Interior Angle Of The Regular Polygon Pictured Below? If Necessary,

Back to Home