The Vertex Of This Parabola Is At (-2 -3). When The Y Value Is -2, The X Value Is -5. What Is The Coefficient

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Understanding the properties of parabolas is fundamental in algebra and coordinate geometry. When analyzing a parabola, key features such as the vertex, focus, directrix, and coefficients of the quadratic function provide insights into its shape and position. In this comprehensive guide, we will explore how to determine the coefficient of a parabola given certain points and features, specifically focusing on the vertex at (-2, -3) and another point at (-5, -2). By the end of this article, you'll have a clear methodology to find the coefficient (a) of the parabola's equation.

Understanding Parabolas and Their Equations

Before diving into the problem specifics, let’s review the general form of a parabola and the significance of its coefficients.

Standard Forms of a Parabola

Parabolas can be expressed in several forms, depending on their orientation and position:

- Vertex Form: $y = a(x - h)^2 + k$

- Here, (h, k) is the vertex of the parabola.
- The coefficient a determines the parabola's opening direction and "width."

- Factored Form: $y = a(x - r_1)(x - r_2)$

- Useful when roots (x-intercepts) are known.

- General Form: $y = Ax^2 + Bx + C$

- The most general quadratic form, less intuitive for geometric features.

In our case, the vertex form offers a straightforward way to incorporate known vertex points directly into the equation.

Given Data and Objective

Let's clearly state the information provided and what we need to find:

- Vertex: $(-2, -3)$
- Another point on the parabola: $(-5, -2)$
- Question: What is the coefficient a in the parabola's equation?

Our goal is to determine the value of a that satisfies the given points and the vertex.

Step-by-Step Approach to Find the Coefficient

To find the coefficient a, we'll use the vertex form of the quadratic equation and substitute the known points.

Step 1: Write the Vertex Form Equation

Since the vertex is at $(-2, -3)$, the parabola can be expressed as:

$$
y = a(x + 2)^2 - 3
$$

Note: We write $x + 2$ because the vertex's x-coordinate is -2.

Step 2: Use the Known Point $(-5, -2)$ to Find a

Substitute $x = -5$ and $y = -2$ into the vertex form:

$$
-2 = a(-5 + 2)^2 - 3
$$

Simplify:

$$
-2 = a(-3)^2 - 3
$$

$$
-2 = a(9) - 3
$$

Now, solve for a:

$$
a(9) = -2 + 3
$$

$$
9a = 1
$$

$$
a = \frac{1}{9}
$$

Result: The coefficient a is $\frac{1}{9}$.

Understanding the Significance of the Coefficient a

The value of $a = \frac{1}{9}$ indicates several properties of the parabola:

- Opening Direction: Since $a > 0$, the parabola opens upward.
- Width of the Parabola: A small positive a (less than 1) means the parabola opens upward but is wider than the standard parabola $y = x^2$.

Verifying the Result with Additional Points

To ensure the accuracy of our calculation, we can verify whether the parabola passes through the known points.

Check at the Vertex $(-2, -3)$:

Plug into the equation:

$$
y = \frac{1}{9}(x + 2)^2 - 3
$$

At $x = -2$:

$$
y = \frac{1}{9}(0)^2 - 3 = -3
$$

Matches the given vertex coordinate, confirming correctness.

Check at the Point $(-5, -2)$:

Plug in $x = -5$:

$$
y = \frac{1}{9}(-5 + 2)^2 - 3 = \frac{1}{9}(-3)^2 - 3 = \frac{1}{9}(9) - 3 = 1 - 3 = -2
$$

Matches the given point, confirming that the coefficient $a = \frac{1}{9}$ is accurate.

Additional Insights: Graphing and Features

Visualizing the parabola helps in understanding the impact of the coefficient and the position of the vertex and other points.

Graphing the Parabola

- The vertex at $(-2, -3)$ is the lowest point since the parabola opens upward.
- The point $(-5, -2)$ lies to the left of the vertex, confirming the parabola's width and direction.
- The parabola is symmetric about the vertical line $x = -2$.

Key Features Derived

- Axis of Symmetry: $x = -2$
- Focus and Directrix: Can be computed if needed, but for the current purpose, the coefficient suffices.

Summary and Final Remarks

In this detailed analysis, we've demonstrated how to find the coefficient a of a parabola given its vertex and another point. The process involves:

- Expressing the parabola in vertex form using the known vertex.
- Substituting the coordinates of the known point to solve for \(a\).
- Verifying the solution using the original point and the vertex.

The key takeaway is that the coefficient \(a\) directly influences the parabola's opening and width, and knowing the vertex simplifies the process significantly.

Final Answer:

\[
\boxed{
a = \frac{1}{9}
}
\]

This coefficient indicates that the parabola opens upward with a relatively wide shape, passing through the points \((-2, -3)\) and \((-5, -2)\).

Additional Resources and Practice Problems

To deepen your understanding, consider exploring the following topics:

- Deriving the vertex form from the standard quadratic form.
- Converting between different forms of quadratic equations.
- Graphing quadratic functions using vertex, intercepts, and symmetry.
- Solving quadratic equations with various methods (factoring, completing the square, quadratic formula).

Practice Problem: Given a parabola with vertex at \((3, 4)\) passing through \((5, 8)\), find the coefficient \(a\).

Solution Approach: Similar to the above, write the vertex form, substitute the point, and solve for \(a\).

In conclusion, understanding how to determine the coefficient of a parabola from given points and features is an essential skill in algebra. By mastering these steps, you'll be better equipped to analyze quadratic functions and interpret their graphs effectively.

Frequently Asked Questions

What is the general form of a parabola's equation when the vertex is at (-2, -3)?

The general form can be written as y = a(x + 2)^2 - 3, where 'a' is the coefficient that determines the parabola's width and direction.

Given the vertex at (-2, -3) and a point (-5, -2), how do you find the coefficient 'a' in the parabola's equation?

Substitute the point (-5, -2) into y = a(x + 2)^2 - 3 and solve for 'a': -2 = a(-5 + 2)^2 - 3 □ -2 = a(-3)^2 - 3 □ -2 = 9a - 3 □ 9a = 1 □ a = 1/9.

What is the value of the coefficient 'a' for the parabola with vertex at (-2, -3) passing through the point (-5, -2)?

The coefficient 'a' is 1/9.

How does the point (-5, -2) influence the shape of the parabola with vertex at (-2, -3)?

The point (-5, -2) helps determine the 'opening' and width of the parabola by allowing calculation of the coefficient 'a', which affects the parabola's steepness.

What is the complete equation of the parabola with vertex at (-2, -3) and passing through (-5, -2)?

The equation is y = (1/9)(x + 2)^2 - 3.

If the parabola's vertex is at (-2, -3) and at y = -2, x = -5, how do you verify the coefficient 'a'?

By plugging x = -5 and y = -2 into y = a(x + 2)^2 - 3 and solving for 'a', confirming that 'a' equals 1/9.

Why is the coefficient 'a' positive in this parabola?

Because the parabola opens upward, which occurs when 'a' is positive; in this case, 'a' = 1/9.

What is the significance of knowing the vertex and a point on the parabola when finding the coefficient?

They allow you to set up an equation and solve for 'a', fully determining the parabola's equation based on its shape and position.

Additional Resources

The Vertex Of This Parabola Is At (-2, -3). When The Y Value Is -2, The X Value Is -5. What Is The Coefficient?

Understanding the properties of parabolas is fundamental in mathematics, especially in algebra and coordinate geometry. The problem presented—finding the coefficient of a parabola given specific points and its vertex—may seem straightforward, but it involves a nuanced understanding of quadratic functions, their forms, and how to interpret key features such as the vertex and specific points on the curve. In this article, we will dissect this problem step-by-step, exploring the core concepts, the mathematical methodology, and the practical applications of these techniques.

Understanding Parabolas and Their Equations

Before diving into the problem, it's essential to establish a solid understanding of what a parabola is and how it is represented mathematically.

The Standard Forms of a Parabola

A parabola is the graph of a quadratic function, which can generally be expressed in two primary forms:

1. Vertex Form:

$$
y = a(x - h)^2 + k
$$

where:

- (h, k) is the vertex of the parabola.
- a is the coefficient that determines the parabola's opening direction and "width."

2. Standard Form:

$$
y = ax^2 + bx + c
$$

where:

- a, b, c are real coefficients.

The vertex form is often more convenient when the vertex is known since it explicitly encodes the vertex's coordinates. Conversely, the standard form is more straightforward for algebraic manipulations and when more points are involved.

Deciphering the Given Data and Its Implications

The problem provides two key pieces of information:

- The vertex of the parabola is at $(-2, -3)$.

- When $y = -2$, the corresponding x value is -5.

Let’s analyze what this tells us about the parabola.

Location of the Vertex

The vertex at $(-2, -3)$ indicates the highest or lowest point on the parabola (depending on whether it opens upward or downward). Since the vertex form explicitly includes the vertex, it becomes a natural starting point for constructing the parabola's equation.

The Additional Point $(-5, -2)$

The point $(-5, -2)$ lies on the parabola, and its coordinates give us a concrete point through which the parabola passes. This point, combined with the vertex, provides enough information to determine the coefficient a.

Formulating the Parabola Equation

Given the vertex and a point on the parabola, the most straightforward approach is to write the quadratic in vertex form and then determine the coefficient a.

Step 1: Write the Vertex Form Equation

Since the vertex is at $(-2, -3)$, the parabola's equation can be expressed as:

$$
y = a(x + 2)^2 - 3
$$

Note: We write $(x - h)$ as $(x + 2)$ because $h = -2$.

Step 2: Use the Point $(-5, -2)$ to Find a

Substitute $x = -5$ and $y = -2$ into the vertex form:

$$
-2 = a(-5 + 2)^2 - 3
$$

Calculate $-5 + 2$:

$$
-5 + 2 = -3
$$

Now, square -3:

$$
(-3)^2 = 9
$$

Plug this back into the equation:

$$
-2 = a \times 9 - 3
$$

Add 3 to both sides:

$$
-2 + 3 = 9a
$$

$$
1 = 9a
$$

Solve for a:

$$
a = \frac{1}{9}
$$

This value of a indicates the parabola opens upward and has a relatively broad shape.

Interpreting the Coefficient and Its Significance

The coefficient a in the quadratic equation encapsulates the parabola's curvature and orientation.

Sign of a

- Positive a: Parabola opens upward (minimum point at the vertex).
- Negative a: Parabola opens downward (maximum point at the vertex).

In our case, $a = \frac{1}{9}$, which is positive, indicating an upward opening parabola.

Magnitude of a

The magnitude of a reflects the "width" or "narrowness" of the parabola:

- Large $|a|$: Narrow parabola.
- Small $|a|$: Wide parabola.

Since $\frac{1}{9}$ is quite small, the parabola is relatively broad.

Alternative Forms and Additional Insights

While the vertex form provides a clear route to the coefficient, understanding the standard form can also offer valuable insights, particularly in more complex problems.

Converting to Standard Form

Start with:

$$
y = \frac{1}{9}(x + 2)^2 - 3
$$

Expand $(x + 2)^2$:

$$
(x + 2)^2 = x^2 + 4x + 4
$$

Substitute back:

$$
y = \frac{1}{9}x^2 + \frac{4}{9}x + \frac{4}{9} - 3
$$

Express -3 as a fraction with denominator 9:

$$
-3 = -\frac{27}{9}
$$

Now, combine:

$$
y = \frac{1}{9}x^2 + \frac{4}{9}x + \left(\frac{4}{9} - \frac{27}{9}\right) = \frac{1}{9}x^2 + \frac{4}{9}x - \frac{23}{9}
$$

Thus, the standard form is:

$$
y = \frac{1}{9}x^2 + \frac{4}{9}x - \frac{23}{9}
$$

The coefficient $a = \frac{1}{9}$ remains consistent across forms, reaffirming its correctness.

Practical Applications and Broader Context

Understanding how to determine the coefficient of a parabola from specific points and the vertex is essential across various fields.

Engineering and Physics

- Projectile motion: The trajectory of objects follows a parabola. Knowing the vertex (highest point) and a point along the path allows for calculations of initial velocity, acceleration, and other parameters.

Economics and Business

- Revenue and profit optimization: Parabolic models help identify maximum profit points. Knowing the parabola's shape allows for strategic decision-making.

Mathematics Education and Problem Solving

- Developing skills in translating between different quadratic forms enhances problem-solving capabilities and conceptual understanding.

Summary and Key Takeaways

- The vertex form of a parabola provides an explicit description of the parabola's key features, making it a powerful tool for problems involving known vertices.
- Given the vertex at $(-2, -3)$ and a point $(-5, -2)$, the coefficient a can be accurately determined as $\frac{1}{9}$.
- The sign and magnitude of a influence the parabola's opening direction and width.

- Converting between vertex and standard forms offers flexibility in analyzing and applying quadratic functions.

Conclusion

In mathematical analysis, the process of identifying the coefficient in a quadratic function based on given points and features reflects both the elegance and utility of algebraic methods. The problem of determining the parabola's coefficient from the vertex and a point not only reinforces key concepts in quadratic functions but also exemplifies how geometric intuition and algebraic manipulation intertwine. Whether applied in physics, economics, or pure mathematics, mastering these techniques empowers practitioners to interpret and model complex real-world phenomena with confidence and precision.

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