Use The Ratio Test To Determine Whether N(-7)n! N=16 Converges Or Diverges. (a) Find The Ratio Of Successive

Use The Ratio Test To Determine Whether N(-7)n! N=16 Converges Or Diverges. (a) Find The Ratio Of Successive

---

Introduction to the Ratio Test and Its Application in Series Convergence

When analyzing infinite series, mathematicians often rely on various convergence tests to determine whether the series converges or diverges. One of the most powerful and commonly used methods is the Ratio Test, also known as the D'Alembert's ratio test. This test is particularly effective when dealing with series involving factorials, exponential functions, or sequences with ratios that simplify neatly.

In this discussion, we focus on the series:

$$
\sum_{n=1}^{\infty} N(-7)n! \quad \text{with} \quad N=16
$$

and aim to establish whether it converges or diverges using the Ratio Test. The problem begins by asking us to find the ratio of successive terms, which is fundamental to applying the test.

Understanding the Series and Its Terms

Before applying the Ratio Test, it's important to understand the structure of the series and its individual terms.

Series Description

- The series in question is:

$$
\sum_{n=1}^{\infty} N(-7)n!
$$

- Given that $N=16$, the terms are:

$$
a_n = 16 \times (-7) \times n! = -112 \times n!
$$

- The sequence of terms is therefore:

$$
a_1 = -112 \times 1! = -112, \quad a_2 = -112 \times 2! = -112 \times 2 = -224, \quad a_3 = -112 \times 6 = -672, \ \text{etc.}
$$

This sequence grows factorially in magnitude, and the negative sign indicates the terms alternate in sign, though the convergence primarily depends on the absolute value of the terms.

Applying the Ratio Test to the Series

The Ratio Test states that for a series $\sum a_n$, if we define:

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
$$

then:

- The series converges absolutely if $L < 1$,
- Diverges if $L > 1$,
- The test is inconclusive if $L = 1$.

Given our terms $a_n = -112 \times n!$, the absolute value simplifies to:

$$
|a_n| = 112 \times n!
$$

Now, we proceed to compute the ratio:

$$
\left| \frac{a_{n+1}}{a_n} \right| = \frac{112 \times (n+1)!}{112 \times n!} = \frac{(n+1)!}{n!}
$$

Since $\frac{(n+1)!}{n!} = n + 1$, the ratio reduces to:

$$
\left| \frac{a_{n+1}}{a_n} \right| = n + 1
$$

Calculating the Limit of the Ratio

To apply the Ratio Test, we need to evaluate:

$$
L = \lim_{n \to \infty} (n + 1)
$$

As $n \to \infty$, it's clear that:

$$
L = \infty
$$

Since $L > 1$, the Ratio Test indicates that the series diverges.

Conclusion on Series Convergence

Based on the above calculations:

- The limit of the ratio of successive terms tends to infinity.
- The series' terms grow factorially in magnitude, which outpaces any decay factors.
- The Ratio Test confirms divergence because $L > 1$.

Therefore, the series:

$$
\sum_{n=1}^{\infty} 16 \times (-7) \times n! \quad \text{diverges}.
$$

Additional Insights and Considerations

While the factorial growth directly leads to divergence, it is instructive to understand why this is consistent with other convergence tests and the general behavior of such series.

Absolute vs. Conditional Convergence

- Since the absolute value of the terms grows without bound, the series cannot converge absolutely.
- The presence of alternating signs does not aid convergence here because the magnitude of the terms grows rapidly.

Comparison with Other Series

- For example, the series $\sum_{n=1}^\infty \frac{1}{n!}$ converges, owing to the factorial in the denominator.
- In contrast, the series involving $n!$ in the numerator, as in this case, diverges quickly.

Implications for Infinite Series Analysis

- The factorial term's growth rate is critical in determining the convergence.
- When factorials appear in the numerator, the series typically diverges unless mitigated by other factors such as exponential decay.

Summary and Final Remarks

To summarize, applying the Ratio Test to the series:

$$
\sum_{n=1}^\infty 16 \times (-7) \times n!
$$

reveals that:

- The ratio of successive terms simplifies to $n + 1$.
- The limit of this ratio as $n \to \infty$ is infinity.
- Consequently, the series diverges.

This conclusion aligns with the intuition that factorial growth in the numerator leads to divergence, confirming the importance of the Ratio Test as an effective tool in analyzing such series.

Practical Tips for Applying the Ratio Test in Similar Problems

- Always start by expressing the a_{n+1} and a_n terms explicitly.
- Simplify the ratio thoroughly before taking the limit.
- Recognize the growth rates of factorials, exponentials, and polynomials to predict convergence behavior.
- Remember that if the ratio tends to infinity, the series diverges; if it tends to zero, it converges absolutely; and if it tends to a finite non-zero value, the test may be inconclusive, requiring further analysis.

Conclusion

Using the Ratio Test on the series involving factorials and constants, specifically:

$$
\sum_{n=1}^\infty 16 \times (-7) \times n!,
$$

we find that the ratio of successive terms grows without bound, resulting in divergence. This analysis underscores the importance of the Ratio Test in handling series with factorial components and highlights the growth behavior of factorial functions in determining the nature of infinite series.

Whether you're a student learning about convergence tests or a mathematician analyzing complex series, understanding how to properly apply the Ratio Test is invaluable for evaluating the behavior of series and ensuring accurate mathematical conclusions.

Frequently Asked Questions

How do you apply the ratio test to determine whether the series Σ N(-7)n! converges or diverges for N=16?

To apply the ratio test, compute the limit of the absolute value of the ratio of successive terms as n approaches infinity. If this limit is less than 1, the series converges; if greater than 1, it diverges; and if equal to 1, the test is inconclusive.

What is the general form of the terms in the series N(-7)n! for N=16?

The terms are given by $a_n = 16 (-7)^n n!$, where N=16 is a constant multiplier, and the series sums over n from 0 to infinity.

How do you find the ratio of successive terms in this series?

The ratio of successive terms is $|a_{n+1} / a_n| = |16 (-7)^{n+1} (n+1)!| / |16 (-7)^n n!|$, which simplifies to $|(-7) (n+1)|$.

What is the limit of the ratio of successive terms as n approaches infinity in this case?

The limit is $\lim_{n\to\infty} |(-7) (n+1)| = \infty$, since (n+1) grows without bound and multiplies by a constant factor -7.

Based on the ratio test, does the series N(-7)n! with N=16 converge or diverge?

Since the limit of the ratio of successive terms is infinity (greater than 1), the series diverges by the ratio test.

Additional Resources

Use The Ratio Test To Determine Whether $\sum_{n=16}^{\infty} -7n!$ Converges Or Diverges

---

Introduction

When studying infinite series in calculus and mathematical analysis, one of the primary objectives is to determine whether a given series converges or diverges. Convergence indicates that the sum of the entire infinite sequence approaches a finite value, while divergence implies that the sum grows without bound or oscillates indefinitely. Among the various tools available for this purpose, the ratio test is particularly powerful, especially for series involving factorials, exponentials, and other rapidly growing functions.

In this article, we explore the application of the ratio test to a specific series:

$$
\sum_{n=16}^\infty -7n!
$$

Although the series appears straightforward, its divergence or convergence hinges on the behavior of factorials and the constants involved. We will use the ratio test to analyze this series, focusing on the ratio of successive terms, and interpret the results in the context of convergence criteria.

Understanding the Series in Question

Before delving into the ratio test, it's crucial to understand the nature of the series:

\[
\sum_{n=16}^\infty -7n!
\]

This series starts at \(n=16 \) and continues indefinitely. Each term of the series is:

\[
a_n = -7n!
\]

Note that the factorial function \(n! \) grows extremely rapidly as \(n \) increases. Specifically,

\[
n! = 1 \times 2 \times 3 \times \ldots \times n,
\]

which leads to exponential growth in the magnitude of \(a_n \), but with an alternating sign pattern if the series involves negative terms. Here, the entire term is negative, consistent for all \(n \geq 16 \).

Key observation: Since each \(a_n \) is negative and grows unbounded in magnitude (due to \(n! \)), intuitively, the series diverges. But to confirm this rigorously, we turn to the ratio test.

The Ratio Test: An Overview

What Is the Ratio Test?

The ratio test, also known as the d'Alembert ratio test, provides a criterion for convergence based on the ratio of successive terms in a series. For a series \(\sum a_n\), the test involves calculating:

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|

\]

The series's behavior depends on the value of \(L \):

- If \(L < 1 \), the series converges absolutely.
- If \(L > 1 \), the series diverges.
- If \(L = 1 \), the test is inconclusive.

This test is particularly effective for series involving factorials, exponentials, and other functions with known growth rates.

Why Use the Ratio Test?

The ratio test is convenient because it simplifies the analysis of terms with factorials. Factorials satisfy the property:

\[
\frac{(n+1)!}{n!} = n + 1,
\]

which allows straightforward computation of the ratio of successive terms.

Applying the Ratio Test to \(\sum_{n=16}^\infty -7n!\)

Step 1: Expressing the Terms

Given:

\[
a_n = -7n!
\]

The absolute value of the terms:

\[
|a_n| = 7n!
\]

Since the ratio test involves absolute values, we focus on:

\[

$$
\left| \frac{a_{n+1}}{a_n} \right| = \frac{7(n+1)!}{7n!} = \frac{(n+1)!}{n!}
$$

Simplify:

$$
\left| \frac{a_{n+1}}{a_n} \right| = n + 1
$$

This is a critical step, as the factorial ratio simplifies to a linear function in n.

Step 2: Computing the Limit L

Calculate:

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} (n + 1)
$$

As $n \to \infty$, this limit clearly tends to infinity:

$$
L = \infty
$$

Step 3: Interpreting the Result

Since $L = \infty > 1$, the ratio test indicates that the series diverges. The intuition aligns with the growth rate of $n!$, which becomes unbounded as n increases.

Implications of the Ratio Test Result

The divergence result is consistent with the behavior of factorials. Because factorials grow faster than exponential functions, the terms $-7n!$ tend to become large in magnitude, and the partial sums of the series will not approach a finite limit.

Specifically:

- The terms $a_n = -7n!$ grow in absolute value without bound.
- The ratio of successive terms $(n+1)$ increases without bound.
- The series cannot converge to a finite sum.

Therefore, the series:

$$
\sum_{n=16}^\infty -7n!
$$

diverges.

Additional Considerations and Contextual Analysis

While the ratio test conclusively shows divergence, it’s informative to consider the broader context:

- Comparison to known divergent series: The series resembles a factorial series scaled by a constant. Since factorial growth dominates other types of series, it’s expected to diverge.
- Alternating series? Here, the terms are negative; if the signs alternated, the convergence behavior could differ, but in this case, all terms are negative and increasing in magnitude.
- Absolute convergence: The series does not converge absolutely, since the sum of $|a_n|$ diverges.

Summary and Final Conclusions

The key takeaways from applying the ratio test to the series $\sum_{n=16}^\infty -7n!$ are:

- The ratio of successive terms simplifies to $n + 1$.
- The limit of this ratio as $n \to \infty$ is infinity.
- Since this limit exceeds 1, the series diverges by the ratio test.

In essence, the factorial growth ensures the series cannot sum to a finite value.

Broader Significance and Practical Applications

Understanding the convergence or divergence of series involving factorials is fundamental in various fields:

- Mathematical analysis: Recognizing divergent factorial series prevents misapplication of summation techniques.
- Probability theory: Factorial terms appear in permutations and combinations; their

divergence informs combinatorial limits.
- Physics and engineering: Series involving factorials emerge in power series solutions to differential equations; knowing their convergence properties guides the formulation of solutions.

The ratio test remains a vital analytical tool in these disciplines, providing straightforward criteria for series behavior in cases involving factorials, exponentials, and other rapidly growing functions.

Conclusion

Applying the ratio test to the series $\sum_{n=16}^\infty -7n!$ reveals an unequivocal divergence. The key calculation of the ratio of successive terms simplifies to $n + 1$, which tends to infinity as $n \to \infty$. Hence, the series does not converge to a finite value, reaffirming the importance of the ratio test in assessing series with factorial components.

This comprehensive analysis underscores the significance of understanding growth rates and convergence criteria in advanced calculus. Recognizing divergence early prevents futile attempts to assign sums where none exist and guides mathematicians and scientists toward correct conclusions about series behavior.

References:

- Stewart, J. (2016). Calculus: Early Transcendentals. Cengage Learning.
- Apostol, T. M. (1967). Mathematical Analysis. Addison-Wesley.
- Stewart, J. (2012). Calculus: Concepts and Contexts. Brooks Cole.

Note: While the series diverges, similar techniques can be employed to analyze other series involving factorials, exponential functions, and power series, facilitating deeper insights into their convergence properties.

Use The Ratio Test To Determine Whether N 7 N N 16 Converges Or Diverges A Find The Ratio Of Successive

Use The Ratio Test To Determine Whether N 7 N N 16 Converges Or Diverges A Find The Ratio Of Successive

Related Articles

- Tarik Winds A Small Paper Tube Uniformly With 161 Turns Of Thin Wire To Form A Solenoid. The Tube's Diameter
- Which Of The Following Descriptions Fit Both Monopolies And Monopolistic Competitors, And Which Descriptions
- Suggest How AMAZON Should Improve Its Operations By Adopting Newtechnological Advancements. (Recommendations

Back to Home