Use The Shell Method To Find The Volume Of The Solid Generated By Revolving The Region Bounded By Y=6x5,y=x,

Use The Shell Method To Find The Volume Of The Solid Generated By Revolving The Region Bounded By Y=6x^5 and y=x

Understanding how to find the volume of a solid of revolution is an essential skill in calculus, especially when dealing with complex regions. One of the most effective techniques for such problems is the shell method. In this article, we will explore how to apply the shell method to determine the volume of a solid generated by revolving the region bounded by the curves $y = 6x^5$ and $y = x$ about a specified axis. We will walk through each step methodically, providing clarity and insight into the process.

Introduction to the Shell Method

What Is the Shell Method?

The shell method is a technique used to compute the volume of a solid of revolution, especially when the solid is generated around a vertical or horizontal axis. Instead of slicing the region into disks (as in the disk method), the shell method involves slicing the region into cylindrical shells.

The key idea is to:
- Imagine slicing the region into thin vertical or horizontal strips.
- Revolving each strip around an axis to create a shell.
- Summing the volumes of all these shells to find the total volume.

Advantages of the Shell Method

- Suitable for regions revolved around vertical lines (like y-axis) or horizontal lines (like x-axis).
- Simplifies calculations when the region is bounded by functions that are easier to integrate with respect to the variable of interest.
- Often yields more straightforward integrals compared to the disk/washer methods.

Analyzing the Region Bounded by $y=6x^5$ and $y=x$

Identifying the Region

The region is bounded by:
- The curve $y = 6x^5$.
- The line $y = x$.

To visualize:
- For $x > 0$, the line $y = x$ and the curve $y=6x^5$ intersect at points where $x = 6x^5$.

Finding the Points of Intersection

Set $y = x$ equal to $y = 6x^5$:
$$
x = 6x^5
$$
Rearranged:
$$
6x^5 - x = 0
$$
Factor out x:
$$
x(6x^4 - 1) = 0
$$
So, the solutions are:
- $x = 0$
- $6x^4 - 1 = 0 \Rightarrow x^4 = \frac{1}{6} \Rightarrow x = \pm \left(\frac{1}{6} \right)^{1/4}$

Since x is typically non-negative in these regions (assuming the region lies in the first quadrant), focus on positive solutions:
$$
x = \left(\frac{1}{6} \right)^{1/4}
$$
Numerically:
$$
x \approx \left(0.1667 \right)^{0.25} \approx 0.64
$$

Thus, the region is bounded between:
- $x=0$
- $x \approx 0.64$

The intersection points are at:
- $(0, 0)$
- $\left(\left(\frac{1}{6} \right)^{1/4}, \left(\frac{1}{6} \right)^{1/4} \right)$

Setting Up the Shell Method Integral

Choosing the Axis of Revolution

Let's consider revolving the region about the y-axis (vertical axis). The shell method is particularly effective here because the shells are generated by revolving vertical slices.

Formulating the Shells

- Each vertical slice at a position x has:
- Height: The difference between the top and bottom functions, which is $y = x$ (the line) and $y = 6x^5$ (the curve).
- Radius: The distance from the y-axis, which is simply x.
- Thickness: dx.

Since the region is bounded between $x=0$ and $x \approx 0.64$, the limits of integration are from 0 to $\left(\frac{1}{6} \right)^{1/4}$.

Expression for the Shell Volume

The volume of a typical shell is:

$$
dV = 2\pi \times \text{radius} \times \text{height} \times \text{thickness}
$$

which becomes:

$$
dV = 2\pi x \times [\text{top y} - \text{bottom y}] \times dx
$$

In our case:
- Top function: $y = x$
- Bottom function: $y = 6x^5$

Since $y = x$ is above $y=6x^5$ in the interval, the height of the shell is:

$$
h(x) = x - 6x^5
$$

Therefore, the volume is:

$$
V = \int_{x=0}^{x=\left(\frac{1}{6} \right)^{1/4}} 2\pi x \left(x - 6x^5 \right) dx
$$

Calculating the Volume

Setting Up the Integral

$$
V = 2\pi \int_{0}^{\left(\frac{1}{6} \right)^{1/4}} x (x - 6x^5) dx
$$

Simplify the integrand:

$$
V = 2\pi \int_{0}^{\left(\frac{1}{6} \right)^{1/4}} (x^2 - 6x^6) dx
$$

Performing the Integration

Integrate term by term:

$$
\int x^2 dx = \frac{x^3}{3}
$$

$$
\int 6x^6 dx = 6 \times \frac{x^7}{7} = \frac{6x^7}{7}
$$

So, the volume becomes:

$$
V = 2\pi \left[\frac{x^3}{3} - \frac{6x^7}{7} \right]_0^{\left(\frac{1}{6} \right)^{1/4}}
$$

Evaluate at the upper limit:

$$
x = \left(\frac{1}{6} \right)^{1/4}
$$

Calculating the Upper Limit Values

Let:

$$
x_0 = \left(\frac{1}{6} \right)^{1/4}
$$

Then:

$$
x_0^3 = \left(\frac{1}{6} \right)^{3/4}
$$

$$
x_0^7 = \left(\frac{1}{6} \right)^{7/4}
$$

Plug these into the expression:

$$
V = 2\pi \left[\frac{\left(\frac{1}{6} \right)^{3/4}}{3} - \frac{6 \times \left(\frac{1}{6} \right)^{7/4}}{7} \right]
$$

Simplify:

$$
V = 2\pi \left[\frac{1}{3} \times 6^{-3/4} - \frac{6 \times 6^{-7/4}}{7} \right]
$$

Express $6^{-3/4}$ and $6^{-7/4}$ explicitly:

$$
6^{-3/4} = \frac{1}{6^{3/4}}
$$

$$
6^{-7/4} = \frac{1}{6^{7/4}}
$$

\]

Putting it all together:
\[
V = 2\pi \left[\frac{1}{3 \times 6^{3/4}} - \frac{6 \times 1}{7 \times 6^{7/4}} \right]
\]

Further simplify:
\[
V = 2\pi \left[\frac{1}{3 \times 6^{3/4}} - \frac{6}{7 \times 6^{7/4}} \right]
\]

Note that:
\[
6^{7/4} = 6^{1 + 3/4} = 6^1 \times 6^{3/4} = 6 \times 6^{3/4}
\]

So:
\[
6^{7/4} = 6 \times 6^{3/4}
\]

Substitute back:
\[
V = 2\pi \left[\frac{1}{3 \times 6^{3/4}} - \frac{6}{7 \times 6 \times 6^{3/4}} \

Frequently Asked Questions

What is the shell method and how is it used to find the volume of a solid of revolution?

The shell method involves integrating cylindrical shells' volumes formed by revolving a region around an axis. It typically uses the formula $V = 2\pi \int (\text{radius})(\text{height})\, dx$ or dy, depending on the axis of rotation, to compute the volume.

How do you set up the integral for finding the volume when revolving the region bounded by y=6x^5 and y=x around the y-axis?

First, express x in terms of y from the given functions, then determine the limits of integration based on the intersection points. The volume is then found using $V = 2\pi \int (\text{radius})(\text{height})\, dy$, where radius is y (distance from y-axis) and height is x (from the region).

What are the steps to find the intersection points of y=6x^5

and y=x?

Set $6x^5 = x$, then solve: $x(6x^4 - 1) = 0$. This gives solutions at $x=0$ and $x=(1/6)^{1/4}$. These points determine the bounds of integration.

How do you express x as a function of y for the region bounded by y=6x^5?

Since $y=6x^5$, solve for x: $x = (y/6)^{1/5}$. This allows expressing the shell radius and height in terms of y.

What is the final integral setup for the volume using the shell method for this problem?

The volume $V = 2\pi \int_{0}^{(1/6)} y (x) dy = 2\pi \int_{0}^{(1/6)} y (y/6)^{1/5} dy$, integrating from $y=0$ to $y=(1/6)$.

How do you evaluate the integral to find the volume after setting it up?

Simplify the integrand, then perform the integral using substitution or power rule. After integrating, substitute the limits to find the numerical volume of the solid.

Additional Resources

Use The Shell Method To Find The Volume Of The Solid Generated By Revolving The Region Bounded By Y=6x^5 and y=x

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Introduction

The calculus of volumes of revolution is a fundamental aspect of integral calculus, offering insightful ways to visualize and compute the three-dimensional objects generated by rotating a plane region around a specified axis. Among the various techniques available, the shell method stands out for its versatility and efficiency, particularly when dealing with regions that are more naturally expressed in terms of horizontal slices or when the axis of revolution is vertical.

In this comprehensive analysis, we explore the application of the shell method to determine the volume of the solid generated by revolving the region bounded by the curves $y = 6x^5$ and $y = x$. This problem not only exemplifies the practical utility of the shell method but also provides a detailed walkthrough of the steps involved, including the identification of the region, setting up the integral, and performing the calculations. This investigation aims to serve as an authoritative guide for students, educators, and enthusiasts in the field of integral calculus.

Defining the Region and the Axis of Revolution

The Curves and Their Intersection

The first step in solving this problem involves understanding the bounded region. The curves given are:

- $y = 6x^5$
- $y = x$

To identify the bounded region, we need to find their points of intersection.

Solving for intersections:

Set $y = 6x^5$ equal to $y = x$:

$$
6x^5 = x
$$

Solution:

$$
6x^5 - x = 0
$$

$$
x (6x^4 - 1) = 0
$$

This yields two solutions:

1. $x = 0$

2. $6x^4 - 1 = 0 \Rightarrow x^4 = \frac{1}{6} \Rightarrow x = \pm \left(\frac{1}{6} \right)^{1/4}$

Since the region is typically considered in the first quadrant (assuming positive x and y), we focus on positive solutions:

$$
x = \left(\frac{1}{6} \right)^{1/4}
$$

Corresponding y values:

$$
y = x = \left(\frac{1}{6} \right)^{1/4}
$$

Summary of intersection points:

- $(0, 0)$
- $\left(\left(\frac{1}{6} \right)^{1/4}, \left(\frac{1}{6} \right)^{1/4} \right)$

The region is bounded between these two x-values, from 0 to $\left(\frac{1}{6} \right)^{1/4}$.

Choosing the Axis of Revolution and Methodology

The problem specifies the shell method, which is particularly advantageous when revolving around a vertical or horizontal axis.

Key considerations:

- The region's boundaries are expressed as functions of x, with y related to x.

- The shell method involves decomposing the volume into cylindrical shells, generated by revolving vertical slices about an axis.

Assuming the problem involves revolving the region around the y-axis, the shell method involves integrating with respect to x.

Setting Up the Shell Method Integral

Conceptual Overview

In the shell method, each "shell" is generated by revolving a vertical strip at position x between the curves, with thickness dx.

- The radius of each shell: the distance from the axis of revolution (here, y-axis) to the shell, which is simply x.

- The height of each shell: the difference between the upper and lower functions in y, i.e., from $y=x$ (lower) to $y=6x^5$ (upper). But since $y=6x^5$ is generally larger than $y=x$ for $x > 0$, the height is:

$$
h(x) = 6x^5 - x
$$

- The circumference of the shell: $2\pi \times \text{radius} = 2\pi x$.

- The volume of each shell:

$$
dV = \text{circumference} \times \text{height} \times dx = 2\pi x (6x^5 - x) dx
$$

Integral Setup

The volume V is obtained by integrating from the leftmost to rightmost intersection points:

$$
V = \int_{x=0}^{x=\left(\frac{1}{6} \right)^{1/4}} 2\pi x (6x^5 - x) dx
$$

Simplify the integrand:

$$
V = 2\pi \int_{0}^{\left(\frac{1}{6} \right)^{1/4}} x (6x^5 - x) dx
$$
$$
V = 2\pi \int_{0}^{\left(\frac{1}{6} \right)^{1/4}} (6x^6 - x^2) dx
$$

Calculating the Integral

Step 1: Write out the integral

$$
V = 2\pi \left[\int_{0}^{\left(\frac{1}{6} \right)^{1/4}} 6x^6 dx - \int_{0}^{\left(\frac{1}{6} \right)^{1/4}} x^2 dx \right]
$$

Step 2: Integrate term-by-term

- For $\int 6x^6 dx$:

$$
\int 6x^6 dx = 6 \times \frac{x^7}{7} = \frac{6}{7} x^7
$$

- For $\int x^2 dx$:

$$
\int x^2 dx = \frac{x^3}{3}
$$

Step 3: Evaluate the definite integrals at the bounds

Let $a = 0$, $b = \left(\frac{1}{6} \right)^{1/4}$:

$$
V = 2\pi \left[\frac{6}{7} b^7 - \frac{b^3}{3} \right]
$$

Calculate b, b^3, and b^7:

$$
b = \left(\frac{1}{6} \right)^{1/4} = 6^{-1/4}
$$

Then:

$$
b^3 = (6^{-1/4})^3 = 6^{-3/4}
$$
$$
b^7 = (6^{-1/4})^7 = 6^{-7/4}
$$

Putting it all together:

$$
V = 2\pi \left[\frac{6}{7} \times 6^{-7/4} - \frac{1}{3} \times 6^{-3/4} \right]
$$

Expressed with positive exponents:

$$
V = 2\pi \left[\frac{6}{7} \times 6^{-7/4} - \frac{1}{3} \times 6^{-3/4} \right]
$$

Simplification and Final Expression

To better interpret the result, rewrite the powers:

$$
6^{-7/4} = \frac{1}{6^{7/4}} \quad \text{and} \quad 6^{-3/4} = \frac{1}{6^{3/4}}
$$

Thus,

$$
V = 2\pi \left[\frac{6}{7} \times \frac{1}{6^{7/4}} - \frac{1}{3} \times \frac{1}{6^{3/4}} \right]
$$

Now, observe that:

$$
6^{7/4} = 6^{1 + 3/4} = 6^{1} \times 6^{3/4} = 6 \times 6^{3/4}
$$

Similarly,

$$
\frac{1}{6^{7/4}} = \frac{1}{6 \times 6^{3/4}} = \frac{1}{6} \times \frac{1}{6^{3/4}}
$$

\]

Substitute back:

$$
V = 2\pi \left[\frac{6}{7} \times \frac{1}{6} \times \frac{1}{6^{3/4}} - \frac{1}{3} \times \frac{1}{6^{3/4}} \right]
$$

Simplify:

$$
V = 2\pi \left[\frac{1}{7} \times \frac{1}{6^{3/4}} - \frac{1}{3} \times \frac{1}{6^{3/4}} \right]
$$

Factor

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