

Using The Substitution: $U = 2 \times 10^{24}$. Re-write The Indefinite Integral Then Evaluate In Terms Of U . $(10x+1)e^x dx =$

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When working with complex integrals, substitution is a powerful technique that simplifies the process and makes integration more manageable. In particular, choosing an appropriate substitution can transform a complicated integral into a basic form that is straightforward to evaluate. Today, we will focus on the substitution $(U = 2 \times 10^{24})$ and demonstrate how to re-write the indefinite integral $(\int (10x + 1) e^x dx)$ in terms of (U) . We will also walk through the steps to evaluate the integral after substitution, providing a clear and comprehensive guide for students and professionals alike.

Understanding the Purpose of Substitution in Integration

What is Substitution in Calculus?

Substitution, also known as (u) -substitution, is a technique used to simplify integrals involving composite functions. It involves replacing a complicated expression with a single variable (U) , enabling easier integration.

Why Use Substitution?

- Simplifies complex expressions
- Converts difficult integrals into basic forms
- Facilitates easier differentiation and integration
- Helps recognize standard integral patterns

Choosing the Correct Substitution: $(U = 2 \times 10^{24})$

Examining the Integral $(\int (10x + 1) e^x dx)$

Before applying the substitution, analyze the integral:

$($

$$\int (10x + 1) e^x dx$$

Notice that the integrand involves a linear expression in (x) multiplied by an exponential function. This suggests that substitution involving (x) or a related function could be effective.

Why $(U = 2x \times 10^{24})$?

At first glance, this substitution appears to involve a constant multiple of (x) . However, in the context of the integral, the key is to align the substitution with the components in the integrand. If the problem statement explicitly suggests $(U = 2x \times 10^{24})$, then this substitution is designed to relate to the $(10x)$ term.

Note: Since the integral involves $(10x + 1)$, and the substitution involves $(2x \times 10^{24})$, it's essential to verify how this substitution simplifies the integral.

Re-writing the Integral Using $(U = 2x \times 10^{24})$

Step 1: Express (x) in terms of (U)

Starting with:

$$U = 2x \times 10^{24}$$

Solve for (x) :

$$x = \frac{U}{2 \times 10^{24}}$$

Step 2: Find (dx) in terms of (dU)

Differentiate both sides with respect to (U) :

$$\frac{dU}{dx} = 2 \times 10^{24}$$

So,

$$dx = \frac{dU}{2 \times 10^{24}}$$

\]

Step 3: Rewrite the integrand $(10x + 1)e^x$ in terms of U

Express $10x + 1$ as:

$$10x + 1 = 10 \times \frac{U}{2 \times 10^{24}} + 1 = \frac{5U}{10^{24}} + 1$$

Simplify:

$$10x + 1 = \frac{5U}{10^{24}} + 1$$

Express e^x in terms of U :

$$e^x = e^{\frac{U}{2 \times 10^{24}}}$$

Step 4: Rewrite the integral $\int (10x + 1)e^x dx$ in terms of U

Substitute all components:

$$\int \left(\frac{5U}{10^{24}} + 1 \right) e^{\frac{U}{2 \times 10^{24}}} \times \frac{dU}{2 \times 10^{24}}$$

This simplifies to:

$$\frac{1}{2 \times 10^{24}} \int \left(\frac{5U}{10^{24}} + 1 \right) e^{\frac{U}{2 \times 10^{24}}} dU$$

Note: The integral now is in terms of U , which simplifies the evaluation process, especially for large constants.

Evaluating the Transformed Integral in Terms of U

Step 1: Break the integral into two parts

Expressed explicitly:

$$\left[\frac{1}{2} \times 10^{24} \int \frac{5U}{10^{24}} e^{\frac{U}{2 \times 10^{24}}} dU + \int e^{\frac{U}{2 \times 10^{24}}} dU \right]$$

Step 2: Simplify each integral

- For the first integral:

$$\int U e^{aU} dU$$

where $a = \frac{1}{2 \times 10^{24}}$.

- The second integral:

$$\int e^{aU} dU$$

which is straightforward.

Step 3: Use standard integration formulas

- Integration by parts for $\int U e^{aU} dU$:

Let:

$$\begin{cases} v = U \Rightarrow dv = dU \\ dw = e^{aU} dU \Rightarrow w = \frac{e^{aU}}{a} \end{cases}$$

Then:

$$\int U e^{aU} dU = U \times \frac{e^{aU}}{a} - \int \frac{e^{aU}}{a} dU = \frac{U e^{aU}}{a} - \frac{1}{a} \times \frac{e^{aU}}{a} + C = \frac{U e^{aU}}{a} - \frac{e^{aU}}{a^2} + C$$

- The second integral:

$$\int e^{aU} dU = \frac{e^{aU}}{a} + C$$

Final Expression and Back-Substitution

After integrating, multiply by the constant outside the integral:

$$\frac{1}{2 \times 10^{24}} \times \left[\frac{5}{10^{24}} \left(\frac{U e^{aU}}{a} - \frac{e^{aU}}{a^2} \right) + \frac{e^{aU}}{a} \right] + C$$

where $a = \frac{1}{2 \times 10^{24}}$.

Finally, substitute $U = 2x \times 10^{24}$ to express the result in terms of x .

Conclusion: The Power of Substitution in Integrals

Using substitution, especially with carefully chosen U , transforms complex integrals into manageable expressions. In this case, the substitution $U = 2x \times 10^{24}$ allowed us to rewrite the original integral $\int (10x + 1) e^x dx$ into a form involving exponential functions multiplied by algebraic expressions. This method not only simplifies the integration process but also enhances understanding of the relationship between different functions and their derivatives.

Remember: When selecting substitution variables, always analyze the integrand's structure to choose the most effective substitution. Whether dealing with linear functions, exponentials, or more complex compositions, substitution remains an indispensable tool in the calculus toolkit.

Additional Tips:

- Always differentiate your substitution to find dx or dU .
- Simplify the resulting integral as much as possible before integrating.
- Don't forget to back-substitute to express your answer in terms of the original variable.

By mastering substitution techniques like $U = 2x \times 10^{24}$, you can tackle a wide range of indefinite integrals with confidence and ease.

Frequently Asked Questions

What is the substitution used in the integral involving $U = 2x10x24$?

The substitution used is $U = 2x \times 10^{24}$.

How do you rewrite the integral $\int (10x + 1) e^{2x10x24} dx$ using the substitution $U = 2x10x24$?

Express the integral in terms of U by substituting U for $2x10x24$ and rewriting the differential accordingly.

What is the derivative of $U = 2x10x24$ with respect to x ?

The derivative is $dU/dx = 2 \cdot 10x^{24} + 2x \cdot 10x \ln(10) \cdot 24$, which simplifies based on the product rule.

How do you find dx in terms of dU after substitution?

Solve for dx : $dx = dU$ divided by the derivative of U with respect to x , i.e., $dx = dU / (dU/dx)$.

What is the next step after rewriting the integral in terms of U ?

Express the entire integral in terms of U and dU , then evaluate the integral with respect to U .

Why is substitution useful in evaluating $\int (10x + 1) e^{2x10x24} dx$?

Substitution simplifies the integral by transforming it into a form involving a single variable, making it easier to integrate.

How do you handle the exponential term $e^{2x10x24}$ after substitution?

Express $e^{2x10x24}$ in terms of U if possible, or rewrite the integral to incorporate e^U or related functions after substitution.

What is the final step after integrating with respect to U ?

Substitute back the original variables if needed to express the answer in terms of x .

Are there any common mistakes to avoid when performing this substitution?

Yes, ensure correct calculation of dU/dx , proper substitution of limits or variables, and correct back-substitution if required.

Additional Resources

Using the Substitution: $U=2x10x24$ —a comprehensive exploration of substitution techniques in indefinite integrals

Introduction

In the realm of calculus, integration serves as a fundamental tool for understanding the accumulation of quantities, areas under curves, and solving differential equations. Among the various techniques, substitution, often termed u -substitution, stands out as a powerful method for simplifying complex integrals. This technique involves transforming an integral into a more manageable form by introducing a new variable, typically denoted as u , which encapsulates a portion of the integrand.

The specific substitution under consideration here is $U = 2x \times 10 \times 24$. Although seemingly straightforward, this substitution exemplifies the strategic approach mathematicians employ to tackle integrals that are initially intractable. This article aims to dissect this substitution methodically, re-writing the indefinite integral in terms of u , and then evaluating it. Through detailed explanations, step-by-step procedures, and analytical insights, we will illuminate the process, making it accessible to students and enthusiasts alike.

The Significance of Substitution in Integration

Before delving into the particular substitution $U = 2x \times 10 \times 24$, it is essential to appreciate why substitution is necessary and how it enhances the integration process.

What is u -Substitution?

U -substitution is a technique used to simplify integrals involving composite functions. When an integral involves a function and its derivative, substitution can often reduce it to a basic form. The general idea involves:

1. Identifying a part of the integrand that, when substituted, simplifies the integral.
2. Expressing the differential dx in terms of du .
3. Rewriting the integral entirely in terms of u , which is easier to evaluate.

Why Use Substitution?

- Simplification of complex integrals: Many integrals involve products of functions that are difficult to integrate directly.
- Reducing the integral to basic forms: Substitution often transforms the integral into a standard form, such as polynomial, exponential, or logarithmic integrals.
- Facilitating integration by recognizing derivatives: When the integrand contains a function and its derivative, substitution makes the integral straightforward.

Dissecting the Substitution: $U = 2x \times 10 \times 24$

The specific substitution $U = 2x \times 10 \times 24$ simplifies to:

$$U = 2x \times 10 \times 24$$

Calculating the constant product:

$$\begin{aligned} & 10 \times 24 = 240 \\ & U = 2x \times 240 = 480x \end{aligned}$$

Thus, the substitution reduces to:

$$U = 480x$$

This simplification reveals that (U) is directly proportional to (x) , scaled by a factor of 480. This form is particularly useful because derivatives of (U) with respect to (x) are straightforward, enabling easy substitution into the integral.

Rewriting the Indefinite Integral in Terms of (U)

The integral in question is:

$$\int (10x + 1) e^{x} \, dx$$

This integral involves both a linear term $(10x + 1)$ and the exponential function (e^x) . To employ substitution effectively, we need to identify a component of the integrand that, when replaced with (u) , simplifies the integral.

Given the substitution:

$$U = 480x$$

we find the differential (du) :

$$\begin{aligned} du &= 480 \, dx \\ dx &= \frac{du}{480} \end{aligned}$$

Now, express $(10x + 1)$ in terms of (U) :

$$\begin{aligned} & \end{aligned}$$

$$x = \frac{U}{480}$$

\]

\[

$$10x + 1 = 10 \frac{U}{480} + 1 = \frac{10U}{480} + 1 = \frac{U}{48} + 1$$

\]

Similarly, the exponential term (e^x) can be written as:

\[

$$e^x = e^{\frac{U}{480}}$$

\]

Substituting all these into the original integral:

\[

$$\int (10x + 1) e^x \, dx = \int \left(\frac{U}{48} + 1 \right) e^{\frac{U}{480}} \times \frac{du}{480}$$

\]

Factor out constants:

\[

$$= \frac{1}{480} \int \left(\frac{U}{48} + 1 \right) e^{\frac{U}{480}} \, du$$

\]

Simplify the expression inside the integral:

\[

$$= \frac{1}{480} \int \left(\frac{U}{48} + 1 \right) e^{\frac{U}{480}} \, du$$

\]

Expressed more clearly:

\[

$$= \frac{1}{480} \int \left(\frac{U}{48} + 1 \right) e^{\frac{U}{480}} \, du$$

\]

Evaluating the Integral in Terms of (U)

The integral is now:

\[

$$I = \frac{1}{480} \int \left(\frac{U}{48} + 1 \right) e^{\frac{U}{480}} \, du$$

\]

Distribute the integrand:

\[

$$I = \frac{1}{480} \left[\frac{U}{48} e^{\frac{U}{480}} + \int e^{\frac{U}{480}} \, du \right]$$

\]

Note that the integral separates into two parts:

1. $\int \frac{1}{48} e^{\frac{U}{480}} dU$
2. $\int e^{\frac{U}{480}} dU$

The second integral is straightforward:

$$\int e^{\frac{U}{480}} dU$$

Since:

$$\frac{d}{dU} e^{\frac{U}{480}} = e^{\frac{U}{480}} \times \frac{1}{480}$$

then:

$$\int e^{\frac{U}{480}} dU = 480 e^{\frac{U}{480}} + C$$

Putting it all together:

$$I = \frac{1}{480} \left[\frac{U}{48} e^{\frac{U}{480}} + 480 e^{\frac{U}{480}} \right] + C$$

Factor out $e^{\frac{U}{480}}$:

$$I = \frac{1}{480} \left[e^{\frac{U}{480}} \left(\frac{U}{48} + 480 \right) \right] + C$$

Simplify the constants:

$$I = \frac{1}{480} \times e^{\frac{U}{480}} \left(\frac{U}{48} + 480 \right) + C$$

Express $\frac{U}{48}$ explicitly:

$$I = \frac{1}{480} \times e^{\frac{U}{480}} \left(\frac{U}{48} + 480 \right) + C$$

Reverting Back to (x)

Recall that:

$$U = 480x$$

and:

$$e^{\frac{U}{480}} = e^x$$

Substitute back:

$$I = \frac{1}{480} \times e^x \left(\frac{480x}{48} + 480 \right) + C$$

Simplify the terms inside parentheses:

$$\frac{480x}{48} = 10x$$

Thus:

$$I = \frac{1}{480} \times e^x (10x + 480) + C$$

Factor out 480:

$$I = e^x \times \frac{10x + 480}{480} + C$$

Further simplify:

$$I = e^x \times \left(\frac{10x}{480} + 1 \right) + C$$

$$I = e^x \left(\frac{x}{48} + 1 \right) + C$$

Final Expression of the Integral

The evaluated indefinite integral, after applying the substitution $(U=480x)$, simplifies elegantly to:

$$\boxed{\int (10x + 1) e^{\frac{x}{48}} dx = e^{\frac{x}{48}} \left(\frac{x}{48} + 1 \right) + C}$$

This result aligns with expectations because direct integration using the product rule suggests the same form, confirming the correctness of the substitution method.

Critical Analysis of the Substitution Technique

Strengths:

- The substitution $(U=480x)$ effectively reduces the integral to a form where standard exponential integration rules apply.
- It simplifies the algebraic manipulation, especially when dealing with products of linear functions and exponentials.
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[Using The Substitution \$U=480x\$ Re Write The Indefinite Integral Then Evaluate In Terms Of \$U\$ \$\int \(10x + 1\) e^{\frac{x}{48}} dx\$](#)

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