What Is The Equation In Slope-intercept Form Of The Line That Passes Through The Point (3,4) And Is Parallel

What Is The Equation In Slope-intercept Form Of The Line That Passes Through The Point (3,4) And Is Parallel to a given line? Understanding this question involves exploring fundamental concepts in algebra and coordinate geometry, specifically the properties of lines, slopes, and how to formulate their equations in slope-intercept form. This article provides an in-depth explanation of how to determine the equation of a line passing through a specific point and parallel to another line, emphasizing clarity and practical application.

Understanding the Slope-Intercept Form of a Line

What Is the Slope-Intercept Form?

The slope-intercept form is one of the most common ways to represent a linear equation. It is written as:

$$y = mx + b$$

where:
- **y** is the dependent variable (the output),
- **x** is the independent variable (the input),
- **m** is the slope of the line,
- **b** is the y-intercept, the point where the line crosses the y-axis.

This form is especially useful because it directly shows the slope and the y-intercept, making graphing and understanding the line's behavior straightforward.

The Significance of Slope and Y-Intercept

- Slope (m): Indicates the steepness of the line. A positive slope means the line rises as x increases; a negative slope means it falls.
- Y-intercept (b): The point (0, b) where the line crosses the y-axis.

Knowing these allows us to quickly sketch the line or understand its properties without plotting numerous points.

Determining the Slope of a Parallel Line

What Does Parallel Mean in Geometry?

Parallel lines are lines in the same plane that never intersect. Their defining characteristic is that they have identical slopes. Therefore, to find a line parallel to a given line, we need to determine the slope of that line and then use it for our new line.

Finding the Slope of the Given Line

Suppose the question references a specific line, such as the line passing through points (x_1, y_1) and (x_2, y_2). Its slope is calculated as:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

If the equation of the original line is given explicitly, for example, $y = 2x + 3$, then its slope is 2.

In our case:
- The line passing through the point (3, 4) is to be parallel to some given line. If the original line's equation is not provided, let's assume it's $y = 2x + c$ (or any specific line). Then, the slope for the parallel line is $m = 2$.

Note: If the original line is not specified, you can only find the slope based on the given information about the line you're paralleling.

Formulating the Equation of the Parallel Line

Using the Point-Slope Form

To find the equation of a line passing through a point (x_0, y_0) with slope m, the point-slope form is:

$$y - y_0 = m(x - x_0)$$

For our specific point (3, 4), and a slope m (which equals the slope of the original line), the equation is:

$$y - 4 = m(x - 3)$$

Converting to Slope-Intercept Form

To express this in the slope-intercept form $y = mx + b$, we solve for y:

1. Expand the right side:

$y - 4 = m(x - 3)$
$y - 4 = mx - 3m$

2. Add 4 to both sides:

$y = mx - 3m + 4$

Thus, the slope-intercept form of the line passing through (3, 4) and parallel to the original line is:

$$y = mx + (4 - 3m)$$

where m is the slope of the original line.

Example: Finding the Equation When the Original Line Is Known

Suppose the original line is $y = 2x + 1$. Its slope is 2, so any line parallel to it will also have a slope of 2.

Step-by-step:

1. Identify the slope (m):
$m = 2$

2. Use the point (3, 4):
Plug into the point-slope form:

$y - 4 = 2(x - 3)$

3. Convert to slope-intercept form:

$y - 4 = 2x - 6$
$y = 2x - 6 + 4$
$y = 2x - 2$

Result:
The equation of the line passing through (3, 4) and parallel to $y = 2x + 1$ is:

$$y = 2x - 2$$

This line has the same slope as the original, ensuring parallelism, and passes through the given point.

General Process for Finding the Equation of a Parallel Line

To summarize, the steps are:

1. Identify or find the slope of the original line. If the original line's equation is given, extract the slope directly.

2. Set the slope of the new line equal to that of the original line, since parallel lines share the same slope.

3. Use the point-slope form with the point (3, 4) and the identified slope.

4. Convert the resulting equation into slope-intercept form for clarity and ease of understanding.

Special Cases and Additional Considerations

Vertical Lines

- If the original line is vertical (e.g., $x = 5$), its slope is undefined.
- A line parallel to a vertical line is also vertical.
- The equation of the parallel line will be a vertical line passing through (3, 4), which is $x = 3$.
- Note that vertical lines cannot be expressed in slope-intercept form.

Horizontal Lines

- If the original line is horizontal (e.g., $y = 4$), its slope is zero.
- The parallel line will also be horizontal, with the same y-value.
- Equation: $y = 4$.

Practical Applications and Importance

Understanding how to find the equation of a line passing through a point and parallel to another line has numerous applications:

- **Graphing:** Quickly sketch lines that are parallel to known lines through specific points.
- **Design and Engineering:** Ensuring components are aligned parallel to each other.
- **Navigation and Mapping:** Plotting parallel paths or routes.
- **Data Analysis:** Creating models that maintain consistent slopes for comparison.

Moreover, mastering these concepts reinforces a solid understanding of linear equations, slopes, and the coordinate plane's geometric properties.

Summary

- The equation of a line in slope-intercept form is $y = mx + b$.
- To find a line passing through (3, 4) and parallel to another line, identify the slope of that line.
- Use the point-slope form with the known point and slope, then convert to slope-intercept form.
- Parallel lines share the same slope; vertical and horizontal lines have special considerations.
- Applying these principles aids in graphing, problem-solving, and real-world applications.

In conclusion, the key to determining the equation in slope-intercept form of a line passing through a specific point and parallel to another line lies in understanding the relationship between slopes and points, and applying algebraic techniques systematically. Whether dealing with explicit equations or general cases, these methods form the foundation of analytical geometry and linear modeling.

Note: For specific cases where the original line's equation is provided, always substitute the actual slope into the formulas. If the original line is unknown, additional information is needed to determine the parallel line's equation.

Frequently Asked Questions

What is the general form of the slope-intercept equation of a line passing through the point (3, 4) and parallel to a given line?

The general form is $y = m x + b$, where m is the slope of the given line, and b is calculated using the point (3, 4).

How do I find the slope of the line parallel to another line given in slope-intercept form?

The slope of a line parallel to another is equal to the slope of the original line. So, you use the same m value for the parallel line.

What is the equation of a line passing through (3, 4) with a slope of 2?

Using $y = m x + b$, plug in the point: $4 = 2\ 3 + b$, so $b = 4 - 6 = -2$. The equation is $y = 2 x - 2$.

How do I write the slope-intercept form of a line passing through (3, 4) with an arbitrary slope m?

First, plug in the point: $4 = m\ 3 + b$, so $b = 4 - 3 m$. The equation is $y = m x + (4 - 3 m)$.

If a line passes through (3, 4) and is parallel to y = 5 x + 1, what is its equation?

Since the lines are parallel, they share the same slope m = 5. Using the point: 4 = 5 3 + b, so b = 4 - 15 = -11. The line's equation is y = 5 x - 11.

Can the equation of the line passing through (3, 4) and parallel to a line with negative slope be positive?

Yes, if the original line has a negative slope, the parallel line will also have that same negative slope. The equation depends on the specific slope value.

Additional Resources

What Is The Equation In Slope-intercept Form Of The Line That Passes Through The Point (3,4) And Is Parallel

Understanding the concepts of lines, slopes, and equations is fundamental in algebra and analytical geometry. When dealing with lines in a coordinate plane, one of the most common forms used for representation is the slope-intercept form, expressed as y = mx + b. This form succinctly captures the slope of the line (m) and the y-intercept (b), providing an immediate visual of how the line behaves and where it crosses the y-axis.

In particular, determining the equation of a line that passes through a specific point and is parallel to another line involves understanding the properties of parallel lines and the role of slopes in defining their orientation. The question, "What is the equation in slope-intercept form of the line that passes through the point (3, 4) and is parallel," requires a detailed exploration of these concepts, including how to find the slope, apply the point-slope and slope-intercept forms, and interpret the geometric implications.

This article will provide a comprehensive, analytical overview of the steps needed to derive such an equation, emphasizing both the theoretical foundations and practical applications.

Understanding the Fundamentals: Lines, Slopes, and the Slope-Intercept Form

What Is a Line in the Coordinate Plane?

A line in the coordinate plane is a set of points that satisfy a linear equation. These points extend infinitely in both directions, forming a straight path. The equation of a line encapsulates its geometric properties, such as steepness and position.

In the Cartesian coordinate system, lines are typically expressed using various forms such as:

- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By = C$

Among these, the slope-intercept form is especially intuitive because it directly reveals the line’s slope and y-intercept.

The Slope of a Line and Its Significance

The slope (m) measures the steepness and direction of a line:

- A positive slope indicates the line rises as it moves from left to right.
- A negative slope indicates it falls.
- A zero slope corresponds to a horizontal line.
- An undefined slope (vertical line) is not expressed in slope-intercept form but is important contextually.

Mathematically, the slope between two points, (x_1, y_1) and (x_2, y_2), is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

This ratio reflects the rate of change of y with respect to x.

Why Focus on Parallel Lines?

Parallel lines are lines in the same plane that never intersect, regardless of how far they extend. The key property of parallel lines is that they have equal slopes:

$$
\text{If two lines are parallel, their slopes are equal:} \quad m_1 = m_2
$$

This fact simplifies the process of finding equations of lines parallel to a given line when the slope is known.

Deriving the Equation of a Line Parallel to a Given Line

Step 1: Identifying the Known Point

In the problem, the line passes through the point (3, 4). This point is essential because it will serve as the anchor for the new line's equation.

Step 2: Determining the Slope of the Original Line

Since the question involves finding a line parallel to another, the critical missing piece is the slope of the original line. Typically, this is given or can be inferred from additional information.

- If the original line's equation is known, for example, $y = 2x + 1$, then the slope m is 2.
- If the slope is not given, but the line's equation is known in some form, extract the slope directly.
- If the original line's equation is not specified, and the task is to find a line parallel to an unspecified line passing through the point (3,4), then the question is incomplete. Usually, the problem provides the original line's equation or slope.

For the purpose of this analysis, we proceed under the common scenario where the original line's equation or slope is known.

Step 3: Applying the Parallel Line Property

Since parallel lines share the same slope, the new line passing through (3,4) will have the same slope m as the original line.

Constructing the Equation in Slope-Intercept Form

Step 4: Using the Point-Slope Form

The point-slope form of a line's equation is an initial step:

$$
y - y_1 = m(x - x_1)
$$

Where:

- (x_1, y_1) is a point on the line, in this case, (3, 4).
- m is the slope, identical to the original line's slope.

Substituting the known point:

\[
y - 4 = m(x - 3)
\]

This form is convenient for deriving the slope-intercept form.

Step 5: Converting to Slope-Intercept Form

To express the line in y = mx + b form:

1. Distribute m:

\[
y - 4 = m x - 3 m
\]

2. Isolate y:

\[
y = m x - 3 m + 4
\]

3. Rewrite as:

\[
y = m x + \left(4 - 3 m\right)
\]

Here, b = 4 - 3m is the y-intercept corresponding to the given slope.

Application with a Specific Slope: An Illustrative Example

Suppose the original line has a slope of m = 2. Then:

\[
b = 4 - 3 \times 2 = 4 - 6 = -2
\]

Thus, the equation of the line passing through (3, 4) and parallel to the original line is:

\[
\boxed{
y = 2x - 2

}
\]

This line maintains the slope of 2 and passes through the point (3, 4). To verify, plug in x = 3:

\[
y = 2(3) - 2 = 6 - 2 = 4
\]

which matches the y-coordinate of the point.

General Formula for the Equation of the Parallel Line

When the slope of the original line is known as m, the general form of the equation passing through (x_1, y_1) = (3, 4) is:

\[
\boxed{
y = m x + \left(4 - 3 m \right)
}
\]

This formula allows for quick computation once m is specified.

Special Cases and Additional Considerations

Vertical Lines: When the Slope Is Undefined

If the original line is vertical (e.g., x = a), the equation in slope-intercept form does not exist because the slope is undefined.

- For a vertical line passing through (3, 4), the equation is simply x = 3.
- Parallel vertical lines are also x = 3, so the equation in this case is:

\[
x = 3
\]

which is not in slope-intercept form but remains a valid equation of the line passing through (3, 4) and parallel to the vertical line x = a.

Implications for Different Types of Lines

The process described applies primarily to lines with finite slopes. For horizontal lines ($m=0$), the equation is $y = y_1$, which simplifies the process.

Conclusion and Practical Applications

The process of determining the equation of a line passing through a specific point and parallel to another involves understanding the core properties of slopes and line equations. The key steps include:

- Identifying the slope of the original line (or the given line's equation).
- Using the point-slope form with the known point and the identified slope.
- Converting to slope-intercept form to get a clear, usable equation.

In practice, this approach is invaluable in fields like engineering, computer graphics, navigation, and data analysis, where constructing lines with specific properties is routine.

To summarize:

- The slope of the new line is equal to the slope of the original line.
- The line passes through the point (3, 4), anchoring its position.
- The general form of the equation in slope-intercept is:

$$
\boxed{
y = m x + (4 - 3 m)
}
$$

where m is the slope of the original line.

This analytical method ensures clarity, precision, and adaptability to various scenarios involving parallel lines and their equations. Whether dealing with straight-line equations in algebra or more complex geometric constructions, mastering these concepts is foundational to further mathematical explorations and real-world problem solving.

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