5. Determine The Cartesian Equation Of The Plane Which Contains The Point A (2,0,2) And Which Is Perpendicular

5. Determine The Cartesian Equation Of The Plane Which Contains The Point A (2,0,2) And Which Is Perpendicular

Understanding how to find the Cartesian equation of a plane is fundamental in vector and analytic geometry. When given specific conditions such as a point lying on the plane and the plane's perpendicularity to a certain vector or line, the problem becomes an excellent exercise in applying vector operations and geometric reasoning. In this article, we will explore the step-by-step process to determine the Cartesian equation of a plane that contains a specific point—A(2, 0, 2)—and is perpendicular to a given direction or line.

By the end of this guide, you'll be equipped to handle similar problems involving plane equations, perpendicular vectors, and point-plane relationships with confidence.

Understanding the Problem

Before diving into calculations, it's important to carefully interpret the problem statement. The key points are:

- The plane contains a specific point: A(2, 0, 2).
- The plane is perpendicular to a given vector or line (the problem statement may specify the direction vector or the line to which the plane is perpendicular).

This problem hinges on identifying the plane's normal vector, which is perpendicular to the plane. Once the normal vector is known, the equation of the plane can be formulated using the point-normal form.

Identifying the Normal Vector

What is the Normal Vector?

The normal vector (often denoted as n) of a plane is a vector perpendicular to the entire plane. It defines the orientation of the plane in space. When the problem states that the plane is perpendicular to a certain vector or line, it usually implies that this vector is the normal vector of the plane.

Determining the Normal Vector from the Given Conditions

- If the problem provides a specific vector to which the plane is perpendicular, then that vector itself is the normal vector.
- If the problem mentions a line, and the plane is perpendicular to that line, then the direction vector of the line is the normal vector of the plane.

Example Scenario:

Suppose the problem states: "Determine the Cartesian equation of the plane which contains A(2, 0, 2) and is perpendicular to the vector v = (a, b, c)."

In this case, v directly serves as the normal vector n.

Note: Since the original problem statement is incomplete about the exact vector or line to which the plane is perpendicular, we will assume a typical scenario where the plane is perpendicular to a known vector v = (a, b, c). If the problem specifies a different line or vector, the approach remains similar.

Formulating the Plane Equation

Once the normal vector n = (A, B, C) is identified, the general form of the Cartesian equation of the plane is:

$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

where (x_0, y_0, z_0) is a point on the plane—in this case, A(2, 0, 2).

Step-by-step Process:

1. Identify the normal vector n = (A, B, C).
2. Substitute the known point A(2, 0, 2) into the point-normal form:

$A(x - 2) + B(y - 0) + C(z - 2) = 0$

3. Simplify the equation to get the Cartesian form:

$A x + B y + C z + D = 0$

where D can be calculated accordingly.

Example: Plane Perpendicular to Vector v = (3, -2, 4)

Let's assume the normal vector n is (3, -2, 4). The point A(2, 0, 2) lies on the plane.

Step 1: Write the point-normal form:

$3(x - 2) - 2(y - 0) + 4(z - 2) = 0$

Step 2: Expand:

$3x - 6 - 2y + 4z - 8 = 0$

Step 3: Combine like terms:

$3x - 2y + 4z - 14 = 0$

This is the Cartesian equation of the plane.

Generalization: Handling Different Scenarios

Depending on what information is given, the approach adapts accordingly:

1. Plane Perpendicular to a Line

- If the line's direction vector is $d = (d_1, d_2, d_3)$ and the plane contains a point A, then:

- The normal vector n is aligned with d.
- Use A and n to formulate the plane's equation.

2. Plane Perpendicular to a Vector

- If the plane is perpendicular to a vector v, then v is the normal vector n.
- Use the point A to find the plane's equation.

3. Plane with Unknown Perpendicular Vector

- Additional information, such as another point on the plane or a line lying within it, is needed to determine the normal vector uniquely.

Final Equation and Summary

The general steps to find the Cartesian equation of a plane, given a point and a perpendicular vector, are:

1. Identify the normal vector n based on the problem's data.
2. Use the point-normal form:

$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

3. Simplify to obtain the standard form:

$A x + B y + C z + D = 0$

where

$D = - (A x_0 + B y_0 + C z_0)$

Conclusion

Determining the Cartesian equation of a plane that contains a specific point and is perpendicular to a given vector or line involves understanding the relationship between the plane's normal vector and the perpendicularity condition. The key steps include identifying or deducing the normal vector, applying the point-normal form, and simplifying the equation.

This process not only helps in solving geometric problems but also enhances understanding of the spatial relationships between points, lines, and planes in three-dimensional space. Whether dealing with academic exercises or real-world applications like engineering and computer graphics, mastering this technique is essential for accurate spatial analysis.

Remember: Always clarify the given data—especially the vector or line specifying perpendicularity—as this determines your normal vector and influences the entire solution process.

Frequently Asked Questions

What is the general method to find the Cartesian equation of a plane given a point and a perpendicular direction?

To find the Cartesian equation of a plane given a point and a perpendicular direction, identify the normal vector (perpendicular direction) and the point on the plane. Then, use the formula $n\cdot(r - r_0) = 0$, where n is the normal vector, r is a general point (x, y, z), and r_0 is the given point. Expand this to get the Cartesian equation.

How do you determine the normal vector of a plane if it's perpendicular to a specific line or vector?

If the plane is perpendicular to a given line or vector, that vector serves as the normal vector of the plane. For example, if the plane is perpendicular to a line with direction vector d, then the normal vector n of the plane is parallel to d.

Given point A(2,0,2) and the plane is perpendicular to a vector n, how do you write the Cartesian equation?

The Cartesian equation is given by: $n_1(x - 2) + n_2(y - 0) + n_3(z - 2) = 0$, where $n = (n_1, n_2, n_3)$ is the normal vector. Expand and simplify to get the standard form.

What additional information is needed to determine the complete Cartesian equation of the plane?

You need the normal vector (or a direction vector perpendicular to the plane) and a point on the plane. In this case, the point A(2,0,2) is given, but the normal vector must be specified or derived from the perpendicular condition.

How does the perpendicular condition influence the form of the plane's equation?

The perpendicular condition ensures that the normal vector of the plane is aligned with a given vector or line direction. This directly determines the coefficients in the plane's equation, making it perpendicular to that vector.

If the plane contains point A(2,0,2) and is perpendicular to the vector n = (a, b, c), what is the explicit Cartesian equation?

The equation is: $a(x - 2) + b(y - 0) + c(z - 2) = 0$, which simplifies to $ax + by + cz = 2a + 0 + 2c$. Alternatively, in standard form: $ax + by + cz = D$, where $D = 2a + 2c$.

Can you provide an example of the Cartesian equation of the plane if the normal vector is (1, -2, 3)?

Yes. Using point A(2,0,2), the equation is: $1(x - 2) - 2(y - 0) + 3(z - 2) = 0$. Simplifying, $x - 2 - 2y + 3z - 6 = 0$, which gives $x - 2y + 3z = 8$.

Additional Resources

Determine The Cartesian Equation Of The Plane Which Contains The Point A (2,0,2) And Which Is Perpendicular is a fundamental problem in vector geometry and analytical geometry. Understanding how to derive the equation of a plane given specific conditions is crucial for students and professionals working in fields such as engineering, computer graphics, physics, and mathematics. This problem combines the concepts of points, normals, and the Cartesian equation of planes, offering a comprehensive insight into spatial reasoning and algebraic techniques.

Introduction to Plane Equations in 3D Space

In three-dimensional space, a plane can be described mathematically in multiple ways. The most common representation is the Cartesian equation of the plane, which takes the form:

$$Ax + By + Cz + D = 0$$

where A, B, and C are the coefficients representing the components of the normal vector to the plane, and D is a constant. The normal vector, often denoted as $\vec{n} = (A, B, C)$, is perpendicular (orthogonal) to the plane.

To determine the Cartesian equation of a plane, two key elements are generally needed:

1. A point $A(x_1, y_1, z_1)$ lying on the plane.
2. The normal vector $\vec{n}$ to the plane.

Given these, the equation of the plane is derived straightforwardly.

Understanding the Problem Statement

The problem specifies that:

- The plane contains a point $A(2, 0, 2)$.
- The plane is perpendicular to a certain direction, which is usually defined by a vector or a line.

However, the statement as it stands appears incomplete—"which is perpendicular" is ambiguous without context. Typically, the problem involves:

- The plane being perpendicular to a given line or vector, or
- The plane being perpendicular to a certain coordinate axis or plane.

Assuming the intended problem is to find the Cartesian equation of the plane passing through point $A(2, 0, 2)$ and perpendicular to a given vector $\vec{n}$, the process involves:

- Identifying the normal vector $\vec{n}$.
- Using the point-normal form of the plane equation.

If the problem specifies a particular normal vector (say, $\vec{n} = (a, b, c)$), then the equation is straightforward. For the sake of this discussion, we'll explore the general case where the plane is perpendicular to a known vector or line.

Step-by-Step Process to Determine the Plane Equation

1. Identifying the Normal Vector

The normal vector $\vec{n}$ is perpendicular to the plane. If the problem states that the plane is perpendicular to a given vector $\vec{v} = (a, b, c)$, then $\vec{n} = \vec{v}$.

Example: Suppose the plane is perpendicular to the vector $\vec{v} = (1, 2, -1)$. Then, $\vec{n} = (1, 2, -1)$.

Pros:
- Directly gives the normal vector.
- Simplifies the process.

Cons:
- Requires knowledge of the vector $\vec{v}$.
- If the normal vector isn't given, it must be inferred from additional information.

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2. Applying the Point-Normal Form of a Plane

Once the normal vector $\vec{n} = (A, B, C)$ and a point $A(x_1, y_1, z_1)$ on the plane are known, the equation follows:

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

This is called the point-normal form.

Example: Using the point $A(2, 0, 2)$ and $\vec{n} = (1, 2, -1)$:

$$1(x - 2) + 2(y - 0) + (-1)(z - 2) = 0$$

Simplify:

$$(x - 2) + 2y - (z - 2) = 0$$
$$x - 2 + 2y - z + 2 = 0$$
$$x + 2y - z = 0$$

This is the Cartesian equation of the plane.

Features:
- Straightforward once the normal vector and point are known.
- Easily adaptable for different normal vectors and points.

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3. Generalizing for Any Normal Vector

In cases where the normal vector is not explicitly given, it can be deduced from other geometric conditions, such as:

- The plane being perpendicular to a line with a known direction vector.
- The plane being perpendicular to another plane or surface.

If the plane is perpendicular to a line with direction vector $\vec{d} = (a, b, c)$, then:

$$ \vec{n} \parallel \vec{d} $$

and the normal vector is $\vec{n} = (a, b, c)$.

Special Cases and Additional Considerations

Perpendicular to Coordinate Axes

- Perpendicular to x-axis: Normal vector $\vec{n} = (1, 0, 0)$. Equation simplifies to:

$$ x = x_0 $$

- Perpendicular to y-axis: Normal vector $\vec{n} = (0, 1, 0)$. Equation:

$$ y = y_0 $$

- Perpendicular to z-axis: Normal vector $\vec{n} = (0, 0, 1)$. Equation:

$$ z = z_0 $$

Advantages:
- Easy to derive.
- Useful in coordinate geometry applications.

Perpendicular to a Given Plane

If the plane is perpendicular to another plane, the normal vectors are orthogonal. For example, if the plane is perpendicular to $x + y + z + 5 = 0$, then the normal vector of the given plane is $\vec{n}_1 = (1, 1, 1)$. The normal vector of the desired plane must be orthogonal to $\vec{n}_1$, i.e., their dot product is zero.

Final Formulation and Derivation

Assuming the problem's missing detail is that the plane is perpendicular to a given vector, say $\vec{v} = (a, b, c)$, then the steps are:

1. Identify the point: $A(2, 0, 2)$.
2. Identify the normal vector: $\vec{n} = (a, b, c)$.
3. Write the equation:

$$a(x - 2) + b(y - 0) + c(z - 2) = 0$$

4. Expand and simplify:

$$a x - 2a + b y + c z - 2c = 0$$

or

$$a x + b y + c z = 2a + 2c$$

This is the Cartesian equation of the plane.

Conclusion and Summary

Determining the Cartesian equation of a plane that contains a specific point and is perpendicular to a given vector involves understanding vector-normal relationships and the point-normal form of the plane equation. The core steps include identifying the normal vector, applying the point-normal form, and simplifying to the standard Cartesian form. This process underscores the importance of vector algebra in spatial geometry, providing a foundation for more advanced topics such as planes intersecting, angles between planes, and three-dimensional modeling.

Key features:
- The normal vector is central to defining the plane.
- The point-normal form simplifies derivation.
- Flexibility exists depending on the given conditions (e.g., known vectors, axes, or other planes).

Pros:
- Clear geometric interpretation.
- Versatile for various conditions.
- Applicable in real-world contexts involving spatial relationships.

Cons:
- Requires understanding of vectors and their properties.
- Ambiguity can arise if problem statements are incomplete or vague.

In practice, always ensure the problem statement clearly specifies the conditions, especially the normal vector or the line to which the plane is perpendicular. This clarity is essential for accurate and efficient derivation of the Cartesian equation of the plane.

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