5- Find Dy/dx In The Following Cases, Evaluate It At X=2: A. (2x+1)(3x-2) B. (x2-3x+2)/(2x+5x-1) C. Y=3u4-4u+5

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Introduction

Understanding how to find the derivative of a function, commonly denoted as Dy/dx, is a fundamental concept in calculus. Derivatives provide us with insights into the rate at which a function changes with respect to a variable—crucial information in fields like physics, engineering, economics, and beyond.

In this article, we will explore detailed methods to compute derivatives for three specific cases, evaluate these derivatives at a particular point (x=2), and interpret the results. The three cases include a product of two functions, a rational function, and a function involving an exponential form. Each case will be broken down step by step, ensuring clarity for learners at various levels.

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Understanding Derivatives

Before delving into specific cases, it's essential to understand what derivatives are and the basic rules used to compute them:

- Derivative Definition: The derivative of a function y=f(x) at a point x measures the instantaneous rate of change of y with respect to x.
- Common Rules:
- Power Rule: d/dx [x^n] = n x^(n-1)
- Constant Multiple Rule: d/dx [k·f(x)] = k · f'(x)
- Sum Rule: d/dx [f(x) + g(x)] = f'(x) + g'(x)
- Product Rule: d/dx [u·v] = u'·v + u·v'
- Quotient Rule: d/dx [u/v] = (u'·v - u·v') / v^2
- Chain Rule: d/dx [f(g(x))] = f'(g(x)) · g'(x)

Now, let's apply these rules to specific cases.

Case A: Differentiating $(2x+1)(3x-2)$

Step 1: Recognize the Structure

This expression is a product of two functions: $u(x) = (2x+1)$ and $v(x) = (3x-2)$.

Step 2: Apply the Product Rule

The product rule states:

$$
\frac{d}{dx}[u \cdot v] = u' \cdot v + u \cdot v'
$$

Step 3: Find u' and v'

- For $u(x) = 2x + 1$:
$$
u' = \frac{d}{dx}(2x + 1) = 2
$$
- For $v(x) = 3x - 2$:
$$
v' = \frac{d}{dx}(3x - 2) = 3
$$

Step 4: Compute Dy/dx

$$
Dy/dx = u' \cdot v + u \cdot v' = 2 \cdot (3x - 2) + (2x + 1) \cdot 3
$$

Simplify:

$$
Dy/dx = 2(3x - 2) + 3(2x + 1) = (6x - 4) + (6x + 3) = 12x - 1
$$

Step 5: Evaluate at x=2

Substitute $x=2$:

$$
Dy/dx|_{x=2} = 12(2) - 1 = 24 - 1 = 23
$$

Summary for Case A:

- Derivative expression: $\frac{dy}{dx} = 12x - 1$
- At x=2: 23

Case B: Differentiating $\frac{x^2 - 3x + 2}{2x + 5x - 1}$

Step 1: Simplify the denominator

Note that the denominator is:

$$
2x + 5x - 1 = 7x - 1
$$

So, the function becomes:

$$
y = \frac{x^2 - 3x + 2}{7x - 1}
$$

Step 2: Recognize the structure for the Quotient Rule

Let $u = x^2 - 3x + 2$ and $v = 7x - 1$.

Step 3: Find u' and v'

- $u' = \frac{d}{dx} (x^2 - 3x + 2) = 2x - 3$
- $v' = \frac{d}{dx} (7x - 1) = 7$

Step 4: Apply the Quotient Rule

$$
Dy/dx = \frac{u'v - uv'}{v^2}
$$

Plugging in:

$$
Dy/dx = \frac{(2x - 3)(7x - 1) - (x^2 - 3x + 2) \cdot 7}{(7x - 1)^2}
$$

Step 5: Expand numerator

First, expand $(2x - 3)(7x - 1)$:

$$
(2x)(7x) + (2x)(-1) + (-3)(7x) + (-3)(-1) = 14x^2 - 2x - 21x + 3 = 14x^2 - 23x + 3
$$

Next, expand $(x^2 - 3x + 2) \cdot 7$:

$$
7x^2 - 21x + 14
$$

Now, numerator:

$$
14x^2 - 23x + 3 - (7x^2 - 21x + 14) = 14x^2 - 23x + 3 - 7x^2 + 21x - 14
$$

Simplify:

$$
(14x^2 - 7x^2) + (-23x + 21x) + (3 - 14) = 7x^2 - 2x - 11
$$

Step 6: Write the derivative expression

$$
Dy/dx = \frac{7x^2 - 2x - 11}{(7x - 1)^2}
$$

Step 7: Evaluate at x=2

Substitute x=2:

- Numerator:

$$
7(2)^2 - 2(2) - 11 = 7 \cdot 4 - 4 - 11 = 28 - 4 - 11 = 13
$$

- Denominator:

$$
(7 \cdot 2 - 1)^2 = (14 - 1)^2 = 13^2 = 169
$$

Therefore:

$$
Dy/dx|_{x=2} = \frac{13}{169} = \frac{1}{13}
$$

Summary for Case B:

- Derivative expression: $\frac{dy}{dx} = \frac{7x^2 - 2x - 11}{(7x - 1)^2}$
- At x=2: 1/13

Case C: Differentiating \(Y = 3u^4 - 4u + 5 \)

Step 1: Clarify the function

This function involves a variable \(u \), which is likely a function of \(x \) (i.e., \(u = u(x) \)). To find \(dY/dx \), we need to apply the chain rule:

\[
\frac{dY}{dx} = \frac{dY}{du} \times \frac{du}{dx}
\]

Step 2: Find \(dY/du \)

\[
Y = 3u^4 - 4u + 5
\]
\[
dY/du = 12u^3 - 4
\]

Step 3: Express \(dY/dx \)

\[
dY/dx = (12u^3 - 4) \times \frac{du}{dx}
\]

To evaluate at \(x=2 \), we need the value of \(u \) at \(x=2 \):

- If \(u \) is explicitly given as a function of \(x \), substitute \(x=2 \) into \(u(x) \), then compute \(dY/dx \).

Assumption: Since \(u \) is not explicitly provided, we'll assume \(u \) is known at \(x=2 \). For illustration, suppose \(u(2) = u_0 \).

Let's assume \(u(2) = 1 \) for the sake of demonstration, and \(du/dx \

Frequently Asked Questions

What is the derivative dy/dx for the function (2x + 1)(3x - 2) and its value at x=2?

Using the product rule, dy/dx = (2)(3x - 2) + (2x + 1)(3). At x=2, dy/dx = 2(6 - 2) + (5)(3) = 2(4) + 15 = 8 + 15 = 23.

How do you differentiate the function (x^2 - 3x + 2)/(2x + 5x - 1) and evaluate at x=2?

Simplify denominator: $2x + 5x - 1 = 7x - 1$. Then, differentiate numerator and denominator using quotient rule. At $x=2$, substitute to find dy/dx.

What is the derivative dy/dx of Y=3u^4 - 4u + 5 with respect to u?

$dy/du = 12u^3 - 4$, as the derivative of $3u^4$ is $12u^3$, and the derivative of $-4u$ is -4.

How do you evaluate dy/dx for the function (2x + 1)(3x - 2) at x=2?

First, find dy/dx using the product rule: $2(3x - 2) + (2x + 1)(3)$. Substitute $x=2$: $2(6 - 2) + 5(3) = 8 + 15 = 23$.

What is the step-by-step process to differentiate and evaluate (x^2 - 3x + 2)/(7x - 1) at x=2?

Apply quotient rule: $dy/dx = [(2x - 3)(7x - 1) - (x^2 - 3x + 2)(7)] / (7x - 1)^2$. Substitute $x=2$ to compute the value.

Can you explain how to differentiate Y=3u^4 - 4u + 5 and interpret its derivative?

Yes, differentiate term-by-term: $dy/du = 12u^3 - 4$. This represents the rate of change of Y with respect to u at any point u.

Evaluate the derivative dy/dx of (2x + 1)(3x - 2) at x=2 in detail.

Using product rule: $dy/dx = 2(3x - 2) + (2x + 1)(3)$. At $x=2$: $2(6 - 2) + 5(3) = 8 + 15 = 23$.

What is the derivative of the function (x^2 - 3x + 2)/(7x - 1) at x=2?

Differentiate numerator and denominator separately using quotient rule, then substitute $x=2$ to find the derivative's value.

How does the chain rule apply to differentiate Y=3u^4 - 4u + 5 if u is a function of x?

If $u = u(x)$, then $dy/dx = dy/du \, du/dx = (12u^3 - 4) \, du/dx$. You need u's derivative to complete the calculation.

Additional Resources

Comprehensive Guide to Finding Dy/dx in Various Cases and Evaluating It at X=2

Understanding how to differentiate functions is fundamental in calculus, especially when dealing with complex expressions. The process of finding the derivative, denoted as Dy/dx, provides insight into the rate of change of a function relative to its independent variable. This guide delves into five different cases, focusing specifically on three prominent examples, illustrating the step-by-step techniques to compute derivatives, and evaluating them at a specific point, X=2. The discussion is comprehensive, covering the underlying principles, differentiation rules, and practical approaches to handling diverse types of functions.

Introduction to Differentiation and Its Significance

Differentiation, the process of calculating the derivative of a function, is central to calculus. It serves multiple purposes:

- Understanding the rate of change of a function at any point.
- Finding slopes of tangent lines to curves.
- Optimizing functions by locating maximum and minimum values.
- Analyzing the behavior of functions—whether increasing or decreasing.

The derivative Dy/dx, when computed correctly, reveals how a small change in x affects the value of y. This understanding is crucial in fields like physics, economics, engineering, and data science.

Fundamental Differentiation Rules and Techniques

Before diving into specific cases, it’s essential to review key rules and techniques:

- Constant Rule: The derivative of a constant is zero.
- Power Rule: $\frac{d}{dx} x^n = n x^{n-1}$
- Sum and Difference Rule: Derivative of a sum/difference is the sum/difference of derivatives.
- Product Rule: $\frac{d}{dx} [u \cdot v] = u' v + u v'$
- Quotient Rule: $\frac{d}{dx} \left[\frac{u}{v}\right] = \frac{u' v - u v'}{v^2}$
- Chain Rule: Used for composite functions, $y = f(g(x))$, then $dy/dx = f'(g(x)) \cdot g'(x)$

Applying these rules accurately is critical in different scenarios, especially when functions involve products, quotients, or compositions.

Case A: Differentiating $(2x+1)(3x-2)$

Understanding the Function

This function is a product of two linear expressions:

$$
y = (2x + 1)(3x - 2)
$$

Since it's a product, the product rule is the appropriate differentiation technique.

Step-by-Step Differentiation

1. Identify u and v:

$$
u = 2x + 1 \quad \text{and} \quad v = 3x - 2
$$

2. Compute derivatives u' and v':

$$
u' = \frac{d}{dx}(2x + 1) = 2
$$
$$
v' = \frac{d}{dx}(3x - 2) = 3
$$

3. Apply the product rule:

$$
\frac{dy}{dx} = u' v + u v' = (2)(3x - 2) + (2x + 1)(3)
$$

4. Simplify the expression:

$$
\frac{dy}{dx} = 2(3x - 2) + 3(2x + 1) = 6x - 4 + 6x + 3
$$

$$
\Rightarrow \frac{dy}{dx} = (6x + 6x) + (-4 + 3) = 12x - 1
$$

Evaluation at $x=2$

Substitute $x=2$:

$$
\frac{dy}{dx}\bigg|_{x=2} = 12(2) - 1 = 24 - 1 = 23
$$

Result: The derivative of $(2x+1)(3x-2)$ at $x=2$ is 23.

Case B: Differentiating $\frac{x^2 - 3x + 2}{2x + 5x - 1}$

Understanding the Function

The function involves a quotient:

$$
y = \frac{x^2 - 3x + 2}{2x + 5x - 1}
$$

Note that the denominator simplifies:

$$
2x + 5x - 1 = 7x - 1
$$

Thus, the function reduces to:

$$
y = \frac{x^2 - 3x + 2}{7x - 1}
$$

This is a quotient, so the quotient rule is essential here.

Step-by-Step Differentiation

1. Identify numerator and denominator:

$$
u = x^2 - 3x + 2
$$

\[
v = 7x - 1
\]

2. Calculate derivatives:

\[
u' = 2x - 3
\]
\[
v' = 7
\]

3. Apply the quotient rule:

\[
\frac{dy}{dx} = \frac{u' v - u v'}{v^2}
\]

4. Substitute the derivatives:

\[
\frac{dy}{dx} = \frac{(2x - 3)(7x - 1) - (x^2 - 3x + 2)(7)}{(7x - 1)^2}
\]

5. Expand numerator terms:

- First term:

\[
(2x - 3)(7x - 1) = 2x \cdot 7x + 2x \cdot (-1) - 3 \cdot 7x + (-3) \cdot (-1) = 14x^2 - 2x - 21x + 3 = 14x^2 - 23x + 3
\]

- Second term:

\[
(x^2 - 3x + 2) \cdot 7 = 7x^2 - 21x + 14
\]

6. Combine numerator:

\[
14x^2 - 23x + 3 - (7x^2 - 21x + 14) = 14x^2 - 23x + 3 - 7x^2 + 21x - 14
\]

\[
= (14x^2 - 7x^2) + (-23x + 21x) + (3 - 14) = 7x^2 - 2x - 11
\]

7. Final derivative expression:

$$
\frac{dy}{dx} = \frac{7x^2 - 2x - 11}{(7x - 1)^2}
$$

Evaluation at $x=2$

Calculate numerator:

$$
7(2)^2 - 2(2) - 11 = 7 \times 4 - 4 - 11 = 28 - 4 - 11 = 13
$$

Calculate denominator:

$$
(7 \times 2 - 1)^2 = (14 - 1)^2 = 13^2 = 169
$$

Therefore,

$$
\frac{dy}{dx}\bigg|_{x=2} = \frac{13}{169} = \frac{1}{13}
$$

Result: The derivative at $x=2$ is $\frac{1}{13}$.

Case C: Differentiating $y = 3u^4 - 4u + 5$

Understanding the Function

This function involves the variable u, which is often a function of x. Since the original problem states $y = 3u^4 - 4u + 5$, it hints at the application of the chain rule if u is a function of x. However, if u is an independent variable, then the differentiation treats u as a constant.

Assuming u is a function of x, i.e., $u = u(x)$, the derivative becomes:

$$
\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}
$$

Step-by-Step Differentiation

1. Differentiate \(y \) with respect to \(u \):

\[
\frac{dy}{du} = 12u^3 - 4
\]

2. Apply the chain rule:

\[
\frac{dy}{dx} = (12u

5 Find Dy Dx In The Following Cases Evaluate It At X 2 A 2x 1 3x 2 B X2 3x 2 2x 5x 1 C Y 3u4 4u 5

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