A Copper Wire Is Stretched With A Stress Of 70 MPa At 20C. If The Length Is Held Constant, To What Temperature

Understanding the Problem: Copper Wire Under Stress and Temperature Change

A Copper Wire Is Stretched With A Stress Of 70 MPa At 20°C. If The Length Is Held Constant, To What Temperature is a common question in the fields of materials science and mechanical engineering. It involves understanding how materials behave under stress and how temperature variations influence their physical properties. This scenario is typical when analyzing thermal expansion and stress-strain relationships in metals, especially copper, which is widely used in electrical wiring and electronic components.

In this article, we will explore the fundamental concepts necessary to determine the temperature at which a copper wire, subjected to a specific stress of 70 MPa at 20°C, must be heated or cooled to maintain its length constant. We will delve into the properties of copper, the principles of thermal expansion, and how stress interacts with temperature changes to influence the wire's dimensions.

Fundamental Concepts in Material Behavior Under Stress and Temperature

Stress and Strain in Materials

- Stress (σ): The internal force per unit area within a material. In this case, the copper wire is under a stress of 70 MPa.
- Strain (ε): The deformation or displacement per unit length resulting from stress.
- Elastic behavior: When materials return to their original shape after removing stress, following Hooke's Law.

Hooke's Law and Modulus of Elasticity

- The relationship between stress and strain in the elastic range is given by:

$$
\sigma = E \times \varepsilon
$$

where:

- E is the Young's modulus (modulus of elasticity) for copper, approximately 110 GPa.
- ε is the resulting strain.

Thermal Expansion of Metals

- Metals expand or contract with temperature changes.
- Coefficient of linear expansion (α) describes this behavior:

$$
\Delta L = L_0 \times \alpha \times \Delta T
$$

where:

- L_0 is the original length.
- ΔT is the change in temperature.
- α for copper is approximately $16.5 \times 10^{-6} \, /^\circ C$.

Relating Stress, Strain, and Temperature in Copper Wire

When a copper wire is subjected to stress and temperature changes, the resulting strain is a combination of mechanical deformation (due to stress) and thermal deformation (due to temperature change). To keep the length constant despite these factors, they must offset each other.

Elastic Strain Due to Stress

- Given the stress ($\sigma = 70\, \text{MPa}$) and Young's modulus ($E = 110\, \text{GPa}$):

$$
\varepsilon_{stress} = \frac{\sigma}{E} = \frac{70 \times 10^{6}}{110 \times 10^{9}} \approx 6.36 \times 10^{-4}
$$

This is the strain caused by the applied stress in the elastic region.

Thermal Strain

- The strain caused by a temperature change ΔT:

$$
\varepsilon_{thermal} = \alpha \times \Delta T
$$

With $\alpha = 16.5 \times 10^{-6} /^\circ C$.

Conditions for No Net Change in Length

- To keep the length constant, the total strain resulting from stress and temperature change must sum to zero:

$$
\varepsilon_{total} = \varepsilon_{stress} + \varepsilon_{thermal} = 0
$$

- Therefore:

$$
\alpha \times \Delta T = - \varepsilon_{stress}
$$

- Solving for ΔT:

$$
\Delta T = - \frac{\varepsilon_{stress}}{\alpha}
$$

Calculating the Temperature Change Needed

Using the values calculated:

$$
\Delta T = - \frac{6.36 \times 10^{-4}}{16.5 \times 10^{-6}} \approx -38.55^\circ C
$$

This indicates that, to maintain the same length under the applied stress, the copper wire must be cooled by approximately 38.55°C.

Final Temperature:

$$
T_{final} = T_{initial} + \Delta T = 20^\circ C - 38.55^\circ C \approx -18.55^\circ C
$$

Thus, the wire must be cooled to approximately -18.55°C to keep its length constant under the applied stress of 70 MPa.

Implications and Practical Applications

Engineering Considerations

- Stress relaxation: In real-world scenarios, materials may undergo plastic deformation if stresses exceed elastic limits. However, in our case, the stress is within elastic limits.
- Temperature control: Precise temperature management is essential in applications where maintaining dimensional stability is critical, such as in electrical wiring, precision instruments, and structural components.
- Material selection: Copper's high thermal conductivity and favorable expansion characteristics make it suitable for many applications requiring predictable behavior under thermal stress.

Applications in Industry

- Electrical wiring: Understanding thermal expansion helps prevent damage due to temperature fluctuations.
- Manufacturing: Controlling temperature during manufacturing processes ensures dimensional accuracy.
- Structural engineering: Designing components that can withstand thermal stresses without deformation.

Summary and Key Takeaways

- The stress of 70 MPa causes a measurable elastic strain in the copper wire.
- To keep the length constant, the thermal expansion or contraction must offset this strain.
- The calculated temperature change required is approximately -38.55°C, meaning the wire must be cooled.
- Understanding the interplay between stress and thermal expansion is vital for designing reliable, durable components in various engineering fields.

Conclusion

Determining the temperature at which a copper wire must be held to maintain its length under a given stress involves understanding the principles of elasticity and thermal expansion. By calculating the elastic strain caused by the applied stress and relating it to the thermal strain, engineers can predict the necessary temperature change. This knowledge informs the design and operation of systems where dimensional stability under mechanical and thermal loads is essential, ensuring safety, functionality, and longevity of components in diverse applications.

Keywords: Copper wire, stress, thermal expansion, temperature change, elastic strain, Young's modulus, coefficient of linear expansion, thermal stress, materials science, mechanical engineering

Frequently Asked Questions

What is the initial stress applied to the copper wire in the problem?

The initial stress applied to the copper wire is 70 MPa.

At what temperature is the copper wire initially stretched under the given stress?

The initial temperature is 20°C.

Why is the temperature of the copper wire adjusted if the length is to be held constant after stretching?

Adjusting the temperature compensates for thermal expansion or contraction, ensuring the length remains unchanged despite the applied stress.

What material property of copper is relevant when calculating the change in temperature to keep the length constant?

The coefficient of linear expansion of copper is relevant for calculating the temperature change needed.

What is the formula used to relate stress, strain, and temperature change in this scenario?

The relation involves the thermal strain and mechanical strain: $\Delta L = L(\alpha \Delta T) + (\sigma/E)L$, where α is the coefficient of linear expansion, ΔT is the temperature change, σ is the stress, and E is Young's modulus.

How does the coefficient of linear expansion affect the temperature calculation?

A higher coefficient of linear expansion means more thermal strain for a given temperature change, influencing the amount of temperature adjustment needed to keep the length constant.

If the applied stress is 70 MPa, what is the approximate temperature change required to keep the length constant?

Using the thermal expansion and elastic properties of copper, the approximate temperature change can be calculated, typically resulting in a small increase or decrease (around a few degrees Celsius).

What is the significance of holding the length constant in this problem?

Holding the length constant allows us to determine the temperature change needed to compensate for the mechanical deformation caused by the applied stress.

Additional Resources

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Introduction

In the realm of materials engineering and thermodynamics, understanding how materials respond to mechanical stresses and temperature variations is crucial. Imagine a copper wire subjected to a tensile stress of 70 MPa at room temperature (20°C). Now, consider a scenario where this wire is stretched under such stress but is constrained so that its length cannot change—meaning it is held fixed in length. The question then arises: if the temperature of the wire is allowed to change, to what temperature must it be heated (or cooled) so that its length remains unchanged despite the applied stress?

This intriguing problem combines principles of thermal expansion, mechanical stress, and material properties. It is not only academically stimulating but also practically relevant in designing electrical wiring, electronic components, and structural elements where thermal and mechanical stability are essential. In this article, we delve into the detailed analysis of this problem, exploring the fundamental concepts, mathematical formulations, and practical implications.

Understanding the Problem: Basic Concepts and Assumptions

Before jumping into calculations, it's essential to clarify the core concepts involved:

- Stress (σ): The force applied per unit area within a material. Here, the copper wire experiences a tensile stress of 70 MPa.

- Strain (ε): The relative deformation or change in length experienced by the material. For elastic deformation, strain is proportional to stress, governed by Hooke's law.

- Thermal Expansion: Materials expand or contract with temperature changes. The linear coefficient of thermal expansion (α) describes how much a material expands per degree change in temperature.

- Constraints: The problem specifies that the wire's length remains constant. This means the total length change due to thermal expansion and mechanical deformation must cancel out.

- Elastic Behavior: We assume the copper wire behaves elastically under the applied stress, which is valid for stresses well below the yield point (~70 MPa is within elastic limits for copper).

Mechanical Response of Copper Under Stress

Elastic Deformation and Hooke's Law

In elastic deformation, the strain produced by an applied stress is given by Hooke's law:

$$
\epsilon_{mech} = \frac{\sigma}{E}
$$

Where:

- ϵ_{mech} = mechanical strain (dimensionless)

- σ = applied stress (Pa)

- E = Young's modulus of copper (~110 GPa)

Given:

$$
\sigma = 70\, \text{MPa} = 70 \times 10^{6}\, \text{Pa}
$$
$$
E = 110\, \text{GPa} = 110 \times 10^{9}\, \text{Pa}
$$

Calculating mechanical strain:

$$
\epsilon_{mech} = \frac{70 \times 10^{6}}{110 \times 10^{9}} \approx 6.36 \times 10^{-4}
$$

This means the wire would tend to elongate by approximately 0.0636% under this stress if free to do so.

Thermal Expansion in Copper

Coefficient of Thermal Expansion

Copper's linear coefficient of thermal expansion (α) is approximately:

$$
\alpha \approx 16.8 \times 10^{-6}\, \text{°C}^{-1}
$$

This indicates that for each degree Celsius increase in temperature, the length of the copper wire increases by about 16.8 parts per million.

Thermal Strain

If the temperature changes by ΔT, the thermal strain is:

$$
\epsilon_{thermal} = \alpha \times \Delta T
$$

Combining Mechanical and Thermal Effects

The Key Constraint: Constant Length

The problem states that the total length change must be zero:

$$
\text{Total strain} = 0
$$

Since the total strain is the sum of mechanical strain and thermal strain:

$$
\epsilon_{total} = \epsilon_{mech} + \epsilon_{thermal}
$$

and

$$
\epsilon_{total} = 0
\Rightarrow \epsilon_{mech} + \alpha \times \Delta T = 0
$$

Rearranged:

$$
\Delta T = - \frac{\epsilon_{mech}}{\alpha}
$$

This indicates that to keep the length constant, the temperature must change in such a way that the thermal expansion cancels out the mechanical strain caused by the applied stress.

Calculating the Required Temperature Change

Plugging in the values:

$$
\Delta T = - \frac{6.36 \times 10^{-4}}{16.8 \times 10^{-6}} \approx -37.86\, \text{°C}
$$

So, the temperature must decrease by approximately 38°C to compensate for the mechanical elongation caused by the tensile stress.

Interpretation:

- Starting at 20°C, the temperature should be lowered to:

$$
T_{final} = 20\, \text{°C} - 37.86\, \text{°C} \approx -17.86\, \text{°C}
$$

- Result: To keep the copper wire's length constant under a tensile stress of 70 MPa, the temperature must be decreased to approximately $-18\, \text{°C}$.

Practical Implications and Limitations

Material Behavior at Low Temperatures

While the calculation suggests cooling the wire to around -18°C, real-world factors must be considered:

- Cryogenic Effects: Copper remains ductile at low temperatures, but mechanical properties can vary.

- Residual Stresses: Manufacturing processes might introduce residual stresses, complicating the analysis.

- Nonlinearities: At high strains or low temperatures, material behavior may deviate from ideal elastic assumptions.

Engineering Applications

This analysis is critical in environments where thermal and mechanical stability is vital, such as:

- Electrical wiring in cold environments: Ensuring that thermal contractions do not induce additional stress.

- Cryogenic systems: Where precise control of dimensions is required despite temperature fluctuations.

- Structural components: Preventing thermal stresses from causing deformation or failure.

Broader Perspectives and Related Concepts

Thermal Stress in Constrained Materials

When a material is constrained from expanding or contracting, temperature changes induce internal stresses:

$$
\sigma_{thermal} = E \times \alpha \times \Delta T
$$

This formula shows how thermal expansion mismatch or constraints generate stress, which can be critical in designing composite materials or assemblies.

Residual Stress Management

In manufacturing and assembly, understanding how thermal changes affect stress states helps in mitigating residual stresses, which can lead to fatigue or failure over time.

Material Selection and Design

Knowing the interplay between mechanical stress and temperature-dependent properties guides material selection, especially in applications involving temperature variations.

Summary

In summary, a copper wire subjected to a tensile stress of 70 MPa at 20°C will experience an elastic elongation of approximately 0.0636%. To maintain the same length despite this stress, the wire must be cooled by about 38°C, bringing its temperature down to roughly -18°C. This calculation hinges on fundamental principles of elasticity and thermal expansion, highlighting the importance of understanding how materials respond to combined thermal and mechanical stimuli.

Such analyses are vital for engineers and designers working in fields where dimensional stability under varying environmental conditions is crucial. Whether in electrical systems, structural components, or advanced manufacturing, appreciating these interactions ensures safety, reliability, and performance.

Final Note: Always consider real-world factors like material imperfections, nonlinear behavior at extreme conditions, and safety margins when applying these calculations to practical scenarios.

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