A) Graph The Function F(x) = X2 + 5x. - B) Draw The Tangent Lines To The Graph At The Points Whose X-coordinates

A) Graph The Function F(x) = X2 + 5x. - B) Draw The Tangent Lines To The Graph At The Points Whose X-coordinates

Understanding the graph of a function and its tangent lines is fundamental in calculus and analytic geometry. In this comprehensive guide, we will explore how to graph the quadratic function $F(x) = x^2 + 5x$, analyze its key features, and learn how to draw tangent lines at specific points based on given x-coordinates. This process not only deepens comprehension of the function's behavior but also illustrates essential concepts such as derivatives, slopes, and tangent line equations.

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1. Analyzing the Function $F(x) = x^2 + 5x$

Before plotting the graph or drawing tangent lines, it’s crucial to understand the function's properties, including its shape, key points, and features.

1.1. Recognizing the Type of Function

- The function $F(x) = x^2 + 5x$ is a quadratic function.
- Quadratic functions graph as parabolas, which are symmetrical and have a vertex as their highest or lowest point.

1.2. Standard Form and Vertex

- The quadratic can be written in the standard form $F(x) = ax^2 + bx + c$, where:
- $a = 1$
- $b = 5$
- $c = 0$
- To find the vertex, use the vertex formula $x_v = -\frac{b}{2a}$:
- $x_v = -\frac{5}{2 \times 1} = -\frac{5}{2} = -2.5$

- The y-coordinate of the vertex:
- $F(-2.5) = (-2.5)^2 + 5 \times (-2.5) = 6.25 - 12.5 = -6.25$

- Therefore, the vertex is at $(-2.5, -6.25)$.

1.3. Axis of Symmetry

- The axis of symmetry passes through the vertex:
- $x = -2.5$

1.4. Intercepts

- Y-intercept:
- Set $x = 0$:
- $F(0) = 0^2 + 5 \times 0 = 0$
- So, the y-intercept is at $(0, 0)$.

- X-intercepts:
- Set $F(x) = 0$:
- $x^2 + 5x = 0$
- $x(x + 5) = 0$
- $x = 0$ or $x = -5$
- X-intercepts at $(0, 0)$ and $(-5, 0)$.

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2. Plotting the Graph of $F(x) = x^2 + 5x$

Now that we understand key features, plotting the graph involves selecting points, sketching the parabola, and understanding its shape.

2.1. Selecting Points

- Besides the intercepts and vertex, choose additional x-values to get more points:
- For example, $x = -4, -3, -2, -1, 1, 2$.

- Calculate $F(x)$ for each:
- $x = -4$: $F(-4) = 16 - 20 = -4$
- $x = -3$: $F(-3) = 9 - 15 = -6$
- $x = -2$: $F(-2) = 4 - 10 = -6$
- $x = -1$: $F(-1) = 1 - 5 = -4$
- $x = 1$: $F(1) = 1 + 5 = 6$

- $x = 2$: $F(2) = 4 + 10 = 14$

- Plot these points along with the intercepts and vertex to sketch the parabola.

2.2. Sketching the Graph

- Connect the points smoothly, forming a parabola opening upward (since $a = 1 > 0$).
- Mark the vertex at $(-2.5, -6.25)$ and note the intercepts.

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3. Drawing Tangent Lines at Specific Points

Tangent lines touch a curve at a single point and have the same slope as the curve at that point. To draw tangent lines at points with given x-coordinates, follow these steps:

3.1. Understanding the Concept of Tangent Lines

- The tangent line at a point $x = x_0$ is the best linear approximation of the function at that point.
- Its slope is given by the derivative $F'(x)$ evaluated at x_0.

3.2. Calculating the Derivative $F'(x)$

- For $F(x) = x^2 + 5x$,
- $F'(x) = 2x + 5$.

3.3. Finding the Coordinates of Points for Tangent Lines

- For each given x-coordinate x_0:
- Compute $F(x_0)$ to find the point $(x_0, F(x_0))$.
- Compute $F'(x_0)$ to find the slope m.

3.4. Equation of the Tangent Line

- Use point-slope form:
- $y - y_0 = m(x - x_0)$,
where $(x_0, y_0) = (x_0, F(x_0))$,
and $m = F'(x_0)$.

3.5. Step-by-Step Example

Suppose we want to draw tangent lines at $x = -3$ and $x = 1$:

- At $x = -3$:
- $F(-3) = 9 - 15 = -6$,
- $F'(-3) = 2(-3) + 5 = -6 + 5 = -1$,
- Equation of tangent:
- $y - (-6) = -1 (x - (-3))$,
- $y + 6 = -1 (x + 3)$,
- $y + 6 = -x - 3$,
- $y = -x - 9$.

- At $x = 1$:
- $F(1) = 1 + 5 = 6$,
- $F'(1) = 2(1) + 5 = 2 + 5 = 7$,
- Equation of tangent:
- $y - 6 = 7 (x - 1)$,
- $y - 6 = 7x - 7$,
- $y = 7x - 1$.

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4. Practical Tips for Drawing Tangent Lines

Drawing tangent lines accurately involves some practical steps:

- Calculate the point of contact $(x_0, F(x_0))$.
- Determine the slope $m = F'(x_0)$.
- Use the point-slope form to write the tangent line equation.
- Plot the tangent line on the same graph as the parabola, ensuring it touches at exactly one point.
- For visual clarity, extend the tangent line sufficiently on both sides of the point.

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5. Applications and Importance of Tangent Lines

Understanding and drawing tangent lines have several practical applications:

- Approximating functions: Tangent lines provide linear approximations near a point.
- Finding slopes of curves: The derivative gives the slope at any point.
- Analyzing increasing/decreasing behavior: The sign of the derivative indicates whether the function is increasing or decreasing.
- Determining points of inflection and maxima/minima: Examining tangent lines and derivatives helps identify these critical points.

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6. Summary and Conclusion

Graphing the quadratic function $F(x) = x^2 + 5x$ involves understanding its key features such as the vertex, intercepts, and symmetry. Calculating the derivative enables us to find slopes at specific points, which are essential for drawing tangent lines. With the point-slope form and the derivative, drawing accurate tangent lines at chosen x-coordinates becomes straightforward, enhancing comprehension of the function's behavior.

Mastering these techniques is fundamental in calculus and aids in visualizing the dynamic relationship between a function and its tangent lines. Whether for academic purposes or practical applications, these skills form the backbone of analyzing and interpreting mathematical functions.

Additional

Frequently Asked Questions

What is the general shape of the graph for the function $F(x) = x^2 + 5x$?

The graph of $F(x) = x^2 + 5x$ is a parabola opening upwards, since the coefficient of x^2 is positive.

How do I find the slope of the tangent line to the graph at a specific

point?

You find the slope by differentiating the function: $F'(x) = 2x + 5$. Plug in the x-coordinate of the point into $F'(x)$ to get the slope.

What are the steps to draw the tangent line at a point with a given x-coordinate?

First, calculate the y-coordinate by evaluating $F(x)$ at that x. Next, find the slope using the derivative. Finally, use the point-slope form to draw the tangent line.

How do I determine the points where the tangent lines are horizontal?

Set the derivative $F'(x) = 0$, so $2x + 5 = 0$, solve for x: $x = -5/2$. The tangent line is horizontal at $x = -2.5$.

Can you explain how to draw tangent lines at multiple points on the graph?

Yes. For each x-coordinate, compute $F(x)$ and $F'(x)$, then draw a line passing through $(x, F(x))$ with slope $F'(x)$. Repeat for all points of interest.

What does the tangent line tell us about the behavior of the function at a point?

The tangent line indicates the instantaneous rate of change (slope) at that point, showing whether the function is increasing, decreasing, or at a maximum or minimum.

How can I use the tangent lines to analyze the function's increasing or decreasing intervals?

By examining the sign of the derivative $F'(x)$: if $F'(x) > 0$, the function is increasing; if $F'(x) < 0$, it is decreasing. Tangent lines help visualize this behavior at specific points.

Additional Resources

Graph The Function $F(x) = x^2 + 5x$: An In-Depth Exploration of Its Graph and Tangent Lines

Understanding the behavior of quadratic functions is fundamental in mathematics, especially in calculus and algebra. The function $F(x) = x^2 + 5x$ serves as a classic example of a parabola that opens upwards, with its unique features and properties offering valuable insights into graphing and calculus concepts. This article delves into the detailed analysis of this function's graph and explores how to draw tangent lines at specific points, enhancing comprehension of the function's behavior and its graphical representation.

Analyzing the Graph of $F(x) = x^2 + 5x$

The quadratic function $F(x) = x^2 + 5x$ can be examined thoroughly by exploring its key features: vertex, axis of symmetry, intercepts, domain, range, and overall shape. Visualizing these aspects helps to understand how the parabola behaves across the x- and y-axes.

1. Basic Characteristics

- Type of Function: Quadratic (parabola)
- Leading Coefficient: 1 (positive), indicating the parabola opens upward
- Domain: All real numbers $(-\infty, \infty)$
- Range: All real numbers greater than or equal to the vertex's y-coordinate

2. Finding the Vertex

The vertex of a parabola given by $ax^2 + bx + c$ can be found using the vertex formula:

$$x_{v} = -\frac{b}{2a}$$

For $F(x) = x^2 + 5x$, $a = 1$ and $b = 5$:

$$x_{v} = -\frac{5}{2 \times 1} = -\frac{5}{2} = -2.5$$

To find the y-coordinate of the vertex:

$$F(-2.5) = (-2.5)^2 + 5 \times (-2.5) = 6.25 - 12.5 = -6.25$$

Vertex Coordinates: $(-2.5, -6.25)$

This point represents the minimum of the parabola, where the graph reaches its lowest point.

3. Axis of Symmetry

The axis of symmetry passes through the vertex:

$$x = -2.5$$

This vertical line divides the parabola into mirror-image halves.

4. Intercepts

- Y-intercept: When $x = 0$:

$$F(0) = 0 + 0 = 0$$

- X-intercepts (Roots): Solving $x^2 + 5x = 0$:

$$x(x + 5) = 0$$

$$x = 0 \quad \text{or} \quad x = -5$$

Thus, the parabola intersects the x-axis at $(0, 0)$ and $(-5, 0)$.

5. Sketching the Graph

Using the vertex, intercepts, and symmetry, the graph can be sketched:

- Plot the vertex at $(-2.5, -6.25)$
- Plot the intercepts at $(0, 0)$ and $(-5, 0)$
- Draw a smooth parabola opening upward, symmetric about $x = -2.5$

Drawing Tangent Lines to the Graph at Specific Points

Tangent lines provide a linear approximation of the function at a specific point. To draw the tangent line at a given x-coordinate, we need to compute the derivative of F(x), evaluate it at the point, and then use the point-slope form for the line.

1. Derivative of $F(x) = x^2 + 5x$

Applying basic differentiation rules:

$$F'(x) = 2x + 5$$

This derivative represents the slope of the tangent line at any point x.

2. Calculating the Slope at Specific Points

Suppose we are asked to draw tangent lines at points with specific x-coordinates x_1 and x_2. For illustration, let's choose:

- $x = -2$ (near the vertex)
- $x = 1$

Calculate the slopes:

- At $x = -2$:

$$F'(-2) = 2 \times (-2) + 5 = -4 + 5 = 1$$

- At $x = 1$:

$$F'(1) = 2 \times 1 + 5 = 2 + 5 = 7$$

3. Finding Corresponding Points on the Graph

Evaluate the function at these points:

- $F(-2) = (-2)^2 + 5 \times (-2) = 4 - 10 = -6$
- $F(1) = 1 + 5 = 6$

The points are:

- $(-2, -6)$ with slope 1
- $(1, 6)$ with slope 7

4. Equation of the Tangent Lines

Using the point-slope form:

$$ y - y_1 = m (x - x_1) $$

- At $x = -2$:

$$ y + 6 = 1 (x + 2) \Rightarrow y = x + 2 - 6 = x - 4 $$

- At $x = 1$:

$$ y - 6 = 7 (x - 1) \Rightarrow y = 7x - 7 + 6 = 7x - 1 $$

These are the equations of the tangent lines at the respective points.

Visual Representation and Practical Applications

Plotting the parabola along with the tangent lines at various points offers a visual understanding of the function's behavior:

- The tangent at $(-2, -6)$ has a gentle slope, illustrating the slope's change near the vertex.
- The tangent at $(1, 6)$ has a steeper slope, indicating faster growth of the function as x increases.

Applications include:

- Approximate values of the function near specific points.
- Understanding the rate of change and concavity.
- Analyzing maximum and minimum points for optimization problems.

Pros and Cons of Graphing and Drawing Tangent Lines

Pros:

- Provides visual intuition about the function's behavior.

- Facilitates understanding of derivatives and slopes.
- Useful in approximation and analyzing function behavior locally.
- Enhances problem-solving skills in calculus.

Cons:

- Requires accurate calculation of derivatives and points.
- Can be complex for functions with higher degrees or more complicated expressions.
- Manual sketching may introduce inaccuracies; digital tools are recommended for precision.
- Overemphasis on tangent lines might overshadow the global behavior of the function.

Features and Final Remarks

- The quadratic function $F(x) = x^2 + 5x$ features a vertex at \((-2.5, -6.25)\), illustrating a parabola with a minimum point.
- Its axis of symmetry is at \(x = -2.5 \), dividing the parabola into mirror images.
- The function intersects the x-axis at \((0, 0)\) and \((-5, 0)\), with the y-intercept at the origin.
- Derivatives enable the calculation of tangent line slopes at any point, crucial for understanding local linearity.
- Drawing tangent lines at specific points helps visualize the rate of change and slope behavior.

In conclusion, the study of $F(x) = x^2 + 5x$ exemplifies fundamental concepts in graphing quadratic functions and applying calculus principles. Understanding how to plot the graph, identify key features, and draw tangent lines empowers students and mathematicians to analyze function behavior more thoroughly. Whether for academic purposes or real-world applications, mastering these techniques is essential in the toolkit of mathematical analysis.

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