A Metal Rod Has A Length Of 123. Cm At 200C. At What Temperature Will The Length Be 92.6 Cm If The Coefficient

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Introduction to Thermal Expansion of Metals

Understanding how materials respond to changes in temperature is fundamental in physics and engineering. Metals, in particular, exhibit a phenomenon known as thermal expansion, where their dimensions change when subjected to temperature variations. This behavior is crucial in designing structures, machinery, and various engineering components to prevent failure or deformation due to thermal stresses.

Imagine a metal rod that measures 123 centimeters at 200°C. If the temperature of this rod is decreased, its length will reduce accordingly. An important question for engineers and scientists is: At what temperature will the rod's length be reduced to 92.6 centimeters? To answer this, we need to understand the basic principles governing thermal expansion, specifically linear expansion, and how to apply the relevant formulas with the given data.

In this article, we will explore the concept of linear thermal expansion, derive the necessary formulas, and walk through a detailed, step-by-step solution to determine the temperature at which the metal rod's length becomes 92.6 cm. Our approach will include explanations suitable for students, educators, and professionals interested in material science, physics, or engineering.

Understanding Linear Thermal Expansion

What Is Linear Thermal Expansion?

Linear thermal expansion refers to the change in length of a material as a function of temperature change. When a metal is heated, its atoms vibrate more vigorously, causing the material to expand. Conversely, cooling causes contraction. The degree to which a material expands or contracts per degree of temperature change is characterized by its coefficient of linear expansion.

Coefficient of Linear Expansion (α)

The coefficient of linear expansion, denoted by α (alpha), is a material-specific property typically expressed in units of per degree Celsius ($°C^{-1}$) or per Kelvin (K^{-1}). It indicates the fractional change in length per unit change in temperature.

Mathematically, it is expressed as:

$$
\alpha = \frac{\Delta L / L_0}{\Delta T}
$$

where:

- L_0 is the original length of the material at the initial temperature,
- ΔL is the change in length,
- ΔT is the change in temperature.

Fundamental Formula for Linear Expansion

The linear expansion of a metal rod can be described by the formula:

$$
L = L_0 (1 + \alpha \Delta T)
$$

where:

- L is the length at the new temperature,
- L_0 is the original length at initial temperature,
- α is the coefficient of linear expansion,
- ΔT is the change in temperature ($T - T_0$).

This formula assumes the temperature change is moderate, and the material's properties remain constant over the temperature range.

Applying the Formula to the Problem

Given Data

- Initial length of the rod, $L_0 = 123\, \text{cm}$,
- Initial temperature, $T_0 = 200^\circ C$,
- Final length, $L = 92.6\, \text{cm}$,
- Coefficient of linear expansion, α (unknown in problem statement but typically provided or assumed).

Note: The problem statement seems to be incomplete regarding the coefficient α. For meaningful calculations, we need the specific value of α. Let's assume a typical value for a common metal, such as steel, which has $\alpha \approx 11 \times 10^{-6} \, ^\circ C^{-1}$.

Step-by-Step Solution

Step 1: Write the linear expansion formula:

$$
L = L_0 (1 + \alpha \Delta T)
$$

Step 2: Rearrange to find ΔT:

$$
\Delta T = \frac{L / L_0 - 1}{\alpha}
$$

Step 3: Substitute the known values:

$$
\frac{92.6}{123} - 1 = \frac{L / L_0 - 1}{\alpha}
$$

Calculate L / L_0:

$$
\frac{92.6}{123} \approx 0.7537
$$

Subtract 1:

$$
0.7537 - 1 = -0.2463
$$

Step 4: Calculate ΔT:

$$
\Delta T = \frac{-0.2463}{\alpha}
$$

Using $\alpha = 11 \times 10^{-6} \, ^\circ C^{-1}$:

$$
\Delta T = \frac{-0.2463}{11 \times 10^{-6}} \approx -22,395\, ^\circ C
$$

This negative value indicates a temperature decrease from the initial temperature.

Step 5: Find the final temperature:

$$
T = T_0 + \Delta T
$$

$$
T = 200^\circ C - 22,395^\circ C = -22,195^\circ C
$$

This temperature is physically unrealistic for typical metals, which suggests that either:

- The assumed α value is not appropriate for this problem,
- The problem's data may be inconsistent or incomplete,
- Or the initial and final lengths are such that the change exceeds typical thermal ranges.

Interpreting the Results

The calculation indicates an extremely low temperature, well below room temperature, leading to a significant contraction of the metal rod from 123 cm to 92.6 cm. Such a temperature is not practically achievable with ordinary cooling, implying that either:

- The coefficient α is much larger for this specific metal,
- Or the problem involves a different type of material with a higher thermal contraction coefficient.

Alternative Approach:

If the problem provides the coefficient α, the calculation can be more precise. For instance, if α is known or specified, the same formula applies, and the calculation becomes straightforward.

Generalized Solution with Variable Coefficient

In scenarios where the coefficient α is not specified, you can follow these steps:

1. Obtain the value of α for the specific metal.
2. Use the linear expansion formula:

$$
L = L_0 (1 + \alpha (T - T_0))
$$

3. Rearrange to solve for T:

$$
T = T_0 + \frac{L / L_0 - 1}{\alpha}
$$

4. Plug in the known values and compute.

Practical Applications of Thermal Expansion Calculations

Understanding how to calculate the temperature at which a metal rod reaches a specific length is vital in various real-world applications:

- Construction and Civil Engineering:
- Preventing structural failure due to thermal expansion and contraction.
- Designing expansion joints in bridges and buildings.

- Manufacturing and Mechanical Engineering:
- Ensuring components fit correctly over temperature ranges.
- Preventing warping or misalignment.

- Aerospace and Automotive Industries:
- Accounting for thermal stresses in vehicle components.

- Material Science:
- Selecting appropriate materials based on thermal properties for specific applications.

Summary and Key Takeaways

- Linear thermal expansion describes how the length of a metal changes with temperature.
- The fundamental formula:

$$
L = L_0 (1 + \alpha \Delta T)
$$

- To find the temperature at a specific length, rearranged as:

$$
T = T_0 + \frac{L / L_0 - 1}{\alpha}
$$

- The value of the coefficient α is crucial for accurate calculations.
- In real-world scenarios, the coefficient varies among different metals and alloys.

Conclusion

Calculating the temperature at which a metal rod reaches a certain length involves understanding the principles of thermal expansion and applying the appropriate formula. While the example discussed uses typical values and assumptions, actual calculations depend on the precise coefficient of linear expansion for the material in question.

This knowledge is essential for engineers and designers to ensure safety, durability, and precision in their projects involving temperature variations. Proper consideration of thermal expansion helps prevent structural failures, ensures proper fitting of components, and guides material selection for different environmental conditions.

Always remember to verify the specific properties of the materials you work with, and when in doubt, consult material property databases or conduct empirical tests to obtain accurate data for your calculations.

Note: If you have the specific value of the coefficient of linear expansion α for your metal, replace the assumed value in the calculation to obtain an accurate temperature estimate.

Frequently Asked Questions

What is the initial length of the metal rod at 200°C?

The initial length of the rod at 200°C is 123 cm.

What is the target length of the metal rod for temperature calculation?

The target length of the rod is 92.6 cm.

Which physical property relates the change in length of a metal rod to temperature?

The property is the coefficient of linear expansion.

What is the formula used to calculate the change in length due to temperature change?

The formula is $\Delta L = L_0 \times \alpha \times \Delta T$, where ΔL is change in length, L_0 is original length, α is the coefficient of linear expansion, and ΔT is the change in temperature.

How do you determine the temperature at which the rod's length becomes 92.6 cm?

By setting up the linear expansion formula for the given lengths and solving for the unknown temperature.

What is the significance of the coefficient in the calculation?

The coefficient of linear expansion determines how much the material's length changes per degree Celsius.

If the coefficient of linear expansion is not given, how can it be found?

It can be obtained from experimental data or from material property tables related to the specific metal.

Is the temperature change negative or positive when the rod length decreases?

The temperature change is negative, indicating cooling below the initial temperature.

What is the importance of understanding thermal expansion

in engineering?

It helps in designing structures and components that can withstand temperature variations without failure.

Can this calculation be applied to other materials with different coefficients?

Yes, but the specific coefficient of linear expansion for each material must be used in the calculation.

Additional Resources

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Introduction

In the realm of materials science and thermal physics, understanding how materials expand or contract with temperature is fundamental. This principle, known as thermal expansion, influences myriad applications—from engineering structures to manufacturing processes. Among the simplest yet most illustrative examples is the behavior of a metal rod subjected to temperature variations.

This investigative article explores a specific problem: Given a metal rod that measures 123 centimeters at 200°C, at what temperature will its length be reduced to 92.6 centimeters? The challenge involves calculating the temperature at which the rod contracts from its initial length, considering the material's linear coefficient of thermal expansion. This problem not only highlights key concepts in thermal physics but also demonstrates real-world applications where precise temperature control and material behavior are critical.

Fundamentals of Thermal Expansion

What Is Thermal Expansion?

Thermal expansion describes the tendency of matter to change in shape, area, and volume with temperature. For solids, the most notable form is linear expansion, where the length of a material changes in proportion to temperature variations.

The Linear Expansion Formula

The linear expansion of a material can be expressed as:

$$ \Delta L = \alpha L_0 \Delta T $$

where:
- ΔL = change in length,

- α = coefficient of linear expansion (per °C),
- L_0 = original length,
- ΔT = change in temperature (final temperature minus initial temperature).

Rearranged to find the final length:

$$L_f = L_0 + \Delta L = L_0 (1 + \alpha \Delta T)$$

This relation assumes the coefficient α remains constant over the temperature range, which is generally valid for small temperature changes.

The Problem Setup

Given:
- Initial length $L_0 = 123 \text{ cm}$,
- Initial temperature $T_0 = 200^\circ C$,
- Final length $L_f = 92.6 \text{ cm}$,
- Coefficient α (unspecified in the problem, but typically for metals 10^{-5} to $10^{-6} \text{ per } ^\circ C$).

The goal:
- Find the temperature T_f at which the rod's length is 92.6 cm.

Assumptions and Clarifications

Before proceeding, certain assumptions are necessary:
- The coefficient of linear expansion α is constant over the temperature range.
- The change in length is purely due to thermal effects, with no external forces causing deformation.
- The material behaves elastically during heating and cooling.

Given that the coefficient is unspecified, for the sake of calculations, we will proceed with a typical value for metals, such as $\alpha = 1.2 \times 10^{-5} \, /^\circ C$, which is representative of steel.

Step-by-Step Solution Approach

Step 1: Express the change in length in terms of temperature

From the linear expansion formula:

$$L_f = L_0 (1 + \alpha (T_f - T_0))$$

Rearranged to solve for T_f:

$$T_f = T_0 + \frac{L_f / L_0 - 1}{\alpha}$$

Step 2: Substitute known values

Calculate L_f / L_0:

$$\frac{92.6}{123} \approx 0.75366$$

Now, compute $L_f / L_0 - 1$:

$$0.75366 - 1 = -0.24634$$

Finally, solve for T_f:

$$T_f = 200^\circ C + \frac{-0.24634}{1.2 \times 10^{-5}}$$

Calculate:

$$\frac{-0.24634}{1.2 \times 10^{-5}} \approx -20,528.33^\circ C$$

Adding this to initial temperature:

$$T_f \approx 200^\circ C - 20,528.33^\circ C = -20,328.33^\circ C$$

This result indicates an extremely low temperature, well below typical physical conditions, suggesting that contracting the rod from 123 cm at 200°C down to 92.6 cm would require cooling to an unphysically low temperature, assuming the given α.

Deep Dive into Physical Implications

Is the Result Physically Realistic?

The calculated temperature is negative and exceedingly low, implying the contraction is beyond the physical limits of real metals. This suggests one of two possibilities:

1. The coefficient α is much larger than typical values, which is unlikely for metals but possible for certain alloys or anomalous materials.

2. The problem contains an inconsistency or error, such as the initial and final lengths being incompatible with the typical thermal expansion behavior.

Re-evaluating the Coefficient

Suppose the coefficient of expansion is significantly larger, say $1 \times 10^{-4} \, /^\circ C$:

$$T_f = 200^\circ C + \frac{-0.24634}{1 \times 10^{-4}} = 200^\circ C - 2,463.4^\circ C$$

While still very low, it is less extreme but still physically unrealistic for practical metals.

Alternative Interpretations

Given the unphysical results, perhaps the problem involves thermal contraction, which occurs when cooling a metal below a certain temperature, or perhaps the question involves a different coefficient related to volumetric expansion, or it is a theoretical exercise.

If the question is about contraction upon cooling from 200°C, and the coefficient is negative (which can happen in some materials), the same formula applies, but with a negative α.

Critical Analysis of Assumptions

This problem underscores the importance of understanding material properties:

- Coefficient of thermal expansion varies among different metals and alloys.
- Temperature-dependent coefficients: For large temperature ranges, α may not be constant.
- Physical limits: Materials cannot be cooled to absolute zero or below without phase changes.

Practical Applications and Significance

Understanding the relationship between length and temperature is vital in:

- Structural engineering: Bridges, railways, and pipelines are designed considering expansion and contraction.
- Manufacturing: Metal parts are machined at specific temperatures to ensure correct dimensions.
- Cryogenics and high-temperature applications: Materials are selected based on their expansion coefficients.

In real-world scenarios, engineers often rely on tables of material properties that specify the coefficient of thermal expansion over specific temperature ranges, acknowledging that the simple linear model is an approximation.

Final Remarks and Conclusions

The problem of determining at what temperature a metal rod contracts from 123 cm at 200°C to 92.6 cm highlights both the utility and limitations of classical thermal expansion formulas.

Key takeaways:

- The linear expansion formula provides a first-order approximation for small temperature ranges.
- Without a specified coefficient α, precise calculations are impossible; assumptions must be made based on typical material properties.
- The calculated unphysical temperature indicates the need for careful consideration of material properties and the range of validity of the linear model.
- Real applications require detailed material data and often more complex models accounting for nonlinear expansion.

In summary, while the mathematical approach is straightforward, real-world applications demand a nuanced understanding of material behavior, precise data, and recognition of the model's limitations. This problem exemplifies the importance of integrating theoretical physics with practical material science to solve engineering challenges effectively.

References

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Note: For specific calculations, always refer to the actual thermal expansion coefficient of the material in question, as values can vary significantly among different metals and alloys.

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