

A Plane Progressive Wave Is Represented By The Equation $Y=0.5 \sin(1000T - 1000x/17)$ Where Y Is In Millimeters,

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Understanding wave phenomena is fundamental in physics and engineering, especially when analyzing how energy propagates through various mediums. The equation $Y=0.5 \sin(1000T - 1000x/17)$ provides a mathematical representation of a plane progressive wave, describing its behavior in space and time. This article aims to explore the details of this wave equation, interpret its components, and discuss its significance in real-world applications.

Deciphering the Wave Equation

The given wave equation is:

$$Y = 0.5 \sin(1000T - 1000x/17)$$

where:

- Y is the displacement in millimeters
- T is the time variable (seconds)
- x is the spatial coordinate (meters)

This sinusoidal function models a plane progressive wave propagating through a medium, characterized by its amplitude, frequency, wavelength, and speed.

Breaking Down the Components

Let's analyze each part of the equation:

1. Amplitude (A): 0.5 mm

- Represents the maximum displacement of the wave from its equilibrium position.
- Indicates the energy carried by the wave; larger amplitude corresponds to

higher energy.

2. Angular Frequency (ω): 1000 rad/sec

- Derived from the coefficient of T (time variable).
- Determines how many oscillations occur per second.

3. Wave Number (k): 1000 x 17 rad/m

- Derived from the coefficient of x (space variable).
- Indicates how many wave cycles exist per unit length.

4. Wave Function: $\sin(\omega T - k x)$

- Describes the wave's oscillatory nature and its propagation direction.

Interpretation of the Wave Equation

Understanding the physical meaning of the wave equation involves analyzing its parameters and their implications.

Wave Propagation Direction

The form $\sin(\omega T - k x)$ signifies a wave moving in the positive x-direction. If the sign before $k x$ was positive, it would indicate a wave traveling in the negative x-direction.

Wave Speed Calculation

The wave speed (v) can be derived using the relation between angular frequency and wave number:

$$v = \omega / k$$

Given:

- $\omega = 1000$ rad/sec
- $k = 1000 \times 17 = 17,000$ rad/m

Calculating:

$$v = 1000 / 17,000 \approx 0.0588 \text{ m/sec}$$

This implies that the wave propagates at approximately 0.0588 meters per second.

Wavelength Determination

Wavelength (λ) relates to wave number (k):

$$\lambda = 2\pi / k$$

Calculating:

$$\lambda = 2\pi / 17,000 \approx 0.000369 \text{ m or } 0.369 \text{ mm}$$

This indicates very short wavelengths, typical in high-frequency acoustic or mechanical waves.

Significance of the Wave Parameters

Understanding the parameters helps in applying the wave model to real-world scenarios.

Amplitude (0.5 mm)

- Acts as a measure of the maximum displacement.
- Relevant in applications like ultrasonic testing, where amplitude correlates with signal strength.

Frequency and Wavelength

- High frequency (1000 rad/sec) correlates with short wavelengths.
- Critical in applications requiring high resolution, like medical imaging.

Wave Velocity

- Low wave speed indicates slow propagation, common in dense or viscous media.

Applications of Plane Progressive Waves

Plane progressive waves are fundamental in various scientific and engineering fields.

Ultrasound Imaging

- High-frequency waves with short wavelengths are used for detailed internal imaging.
- The wave parameters influence image resolution and penetration depth.

Acoustic Engineering

- Analyzing sound wave propagation in different environments.
- Designing concert halls or soundproof rooms.

Material Testing

- Ultrasonic waves detect flaws or cracks in materials.
- Wave amplitude and speed help assess material integrity.

Communication Technologies

- Understanding wave behavior aids in designing efficient transmission systems.

Mathematical Properties and Wave Behavior

Delving deeper into the mathematical characteristics of the wave provides insights into its physical behavior.

Dispersion Relations

- The wave's dispersion relation links frequency and wavelength, impacting how different frequencies travel through a medium.

Superposition Principle

- Multiple waves can combine, leading to interference patterns—constructive or destructive.

Reflection and Transmission

- When encountering boundaries, waves can reflect or transmit depending on impedance differences.

Visualization of the Wave

Understanding the wave's behavior visually enhances comprehension.

Graphical Representation

- Plotting Y against x at a fixed T shows the wave's spatial profile.
- Plotting Y against T at a fixed x shows the wave's temporal oscillation.

Wavefronts and Propagation

- The wavefronts are surfaces of constant phase, moving in the positive x -direction at speed v .
- The wave's phase velocity (speed of the wavefronts) is key in understanding energy transmission.

Real-World Implications and Practical Considerations

Applying theoretical knowledge to practical scenarios involves considering limitations and real-world complexities.

Medium Properties

- Attenuation: Waves lose energy over distance due to medium absorption.
- Dispersion: Wave speed varies with frequency in some media, affecting wave shape.

Environmental Factors

- Temperature, pressure, and density influence wave speed and amplitude.

Design and Engineering Considerations

- Ensuring wave parameters are optimized for specific applications.
- Balancing amplitude and frequency for safety and efficiency.

Summary and Key Takeaways

- The wave equation $Y=0.5 \sin(1000T - 1000x/17)$ describes a plane progressive wave with specific amplitude, frequency, wavelength, and speed.
- The wave propagates in the positive x-direction at approximately 0.0588 m/sec.
- Short wavelengths and high frequencies make this wave suitable for high-resolution applications like ultrasonic imaging.
- Understanding these parameters helps in designing systems across acoustics, medical imaging, material testing, and communications.

Conclusion

The mathematical representation of a plane progressive wave provides valuable insights into its physical characteristics and behaviors. By analyzing the components of the equation, understanding wave propagation, and considering practical applications, engineers and scientists can harness wave phenomena for innovative solutions across diverse fields. The wave described by $Y=0.5 \sin(1000T - 1000x/17)$ exemplifies the intricate relationship between mathematical models and physical reality, emphasizing the importance of precise wave analysis in advancing technology and scientific understanding.

Keywords: plane progressive wave, wave equation, wave speed, wavelength, amplitude, ultrasonic waves, wave propagation, sinusoidal wave, wave analysis, physics, engineering

Frequently Asked Questions

What does the equation $Y = 0.5 \sin(1000 T - 1000 x 17)$ represent in the context of wave motion?

It represents a plane progressive wave with amplitude 0.5 mm, where the wave propagates with a specific phase relationship described by the sine function involving time T and position x .

How can we determine the wavelength of the wave from the given equation?

The wave number k is given as $1000 \times 17 = 17,000$ rad/m; thus, the wavelength $\lambda = 2\pi / k = 2\pi / 17,000$ meters.

What is the wave's frequency based on the given wave equation?

The angular frequency ω is 1000 rad/sec, so the frequency $f = \omega / (2\pi) \approx 159.15$ Hz.

What is the phase velocity of the wave described by the equation?

The phase velocity $v = \omega / k = 1000 / 17,000 \approx 0.0588$ m/sec.

In what direction does the wave propagate based on the equation?

Since the phase term is $(1000 T - 1000 x 17)$, the wave propagates in the positive x -direction.

What is the significance of the amplitude value 0.5 mm in the wave equation?

The amplitude indicates the maximum displacement of the medium from its equilibrium position, which is 0.5 millimeters in this wave.

Additional Resources

In-Depth Analysis of a Plane Progressive Wave: Understanding the Equation $Y = 0.5 \sin(1000 T - 1000 x / 17)$

Introduction to Plane Progressive Waves

A plane progressive wave is a fundamental concept in wave physics, representing a wave that propagates through a medium with uniform amplitude and phase surfaces that are planes perpendicular to the direction of wave travel. These waves are essential in understanding various physical phenomena such as sound propagation, electromagnetic waves, and mechanical vibrations.

The given wave equation:

$$Y = 0.5 \sin(1000 T - 1000 x / 17)$$

encapsulates the behavior of a plane progressive wave, where:

- Y : Displacement (in millimeters)
- T : Time (in seconds)
- x : Spatial coordinate (in meters or centimeters depending on the context)

This detailed review will explore every facet of this wave equation, including its physical significance, mathematical structure, wave parameters, and implications.

Deciphering the Wave Equation

Standard Form of a Plane Progressive Wave

The general mathematical form of a plane progressive wave traveling along the x-axis is:

$$y(x, t) = A \sin(kx - \omega t + \phi)$$

where:

- A : Amplitude (maximum displacement)
- k : Wave number (related to wavelength)

- ω : Angular frequency
- ϕ : Phase constant (initial phase)

In the given equation:

$$Y = 0.5 \sin(1000 T - 1000 \times 17)$$

the parameters correspond as:

- Amplitude $(A = 0.5)$ mm
- The argument of sine $(1000 T - 1000 \times 17)$ combines phase components

To interpret these, we need to identify k (wave number) and ω (angular frequency).

Analyzing the Wave Parameters

Amplitude (A)

- The maximum displacement from the mean position is 0.5 millimeters.
- This indicates a relatively small wave, typical in mechanical vibrations or small amplitude sound waves.

Angular Frequency (ω)

- The coefficient of T in the sine argument is 1000 .
- Since the argument of sine is in the form ωT , it follows:

$$\omega = 1000 \text{ radians/sec}$$

- Implication: The wave oscillates with an angular frequency of 1000 radians/sec, which relates to its temporal frequency.

Wave Number (k)

- The coefficient of x in the sine argument is (1000×17) , i.e., 17,000.
- The phase term involving position is:

$$- 1000 \times 17 \times x = - (17,000) x$$

- Comparing with the standard form $\cos(kx)$:

$$k = 17,000 \text{ radians/meter}$$

- Note: The units of x are crucial; assuming x is in meters, then k is in radians/meter.

Wave Speed (v)

- The wave propagates with a phase velocity given by:

$$v = \frac{\omega}{k}$$

Substituting the values:

$$v = \frac{1000}{17,000} \approx 0.0588 \text{ meters/sec}$$

- Interpretation: The wave travels at approximately 0.0588 m/sec, indicating a slow-moving mechanical wave.

Physical Interpretation of the Wave Equation

Direction of Propagation

- The wave phase is expressed as $\cos(1000T - 1000x/17)$.
- The negative sign before the x -term indicates the wave propagates in the positive x -direction.
- If the sign were reversed, the wave would propagate in the negative x -direction.

Wave Nature and Behavior

- The sinusoidal form indicates a harmonic wave, oscillating smoothly with constant amplitude.
- The wave is progressive, meaning it transports energy through the medium without attenuation (assuming ideal conditions).

Velocity and Wavelength

- The wavelength (λ) can be calculated from the wave number:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{17,000}$$

- Numerical calculation:

$$\lambda \approx \frac{6.2832}{17,000} \approx 3.696 \times 10^{-4} \text{ meters}$$

or approximately 0.3696 millimeters.

- Implication: The wave's spatial periodicity is about 0.37 mm, a very short wavelength, typical of high-frequency mechanical waves.

Key Wave Parameters and Their Significance

- Amplitude (0.5 mm): Indicates maximum displacement; relates to energy carried by the wave.
- Angular Frequency (1000 rad/sec): Determines how rapidly the wave oscillates in time.
- Wave Number (17,000 rad/m): Defines spatial periodicity; shorter wavelengths correspond to higher frequencies.
- Wave Speed (~0.0588 m/sec): The velocity at which the wave propagates; important in understanding energy transfer rate.

Physical Medium and Context

Given the parameters, this wave could represent:

- A mechanical wave traveling through a solid or viscous medium.
- A high-frequency acoustic wave in a thin medium.
- Vibrations on a string or membrane with small amplitude.

The small wavelength and slow velocity suggest a medium with high stiffness and low density or a specialized environment such as:

- Micro-scale mechanical vibrations.
- Acoustic waves in certain materials.

Energy and Power Considerations

The energy transported by a wave is proportional to the square of its amplitude:

- Since y oscillates between $+0.5$ mm and -0.5 mm, the maximum displacement is 0.5 mm.
- The energy flux depends on amplitude, frequency, and medium properties such as density and elasticity.

While the wave equation doesn't specify medium properties, understanding the amplitude and frequency allows estimation of energy transmission.

Applications and Practical Significance

- Engineering: Design of micro-mechanical systems, sensors, and actuators.
- Physics Research: Studying wave propagation at small scales.
- Acoustics: Understanding high-frequency sound waves in specialized materials.
- Material Science: Evaluating wave speed and wavelength for non-destructive testing.

Mathematical Insights and Further Analysis

Phase Velocity and Group Velocity

- For a simple harmonic wave like this, phase velocity (v) is as calculated.
- If multiple waves or wave packets are involved, group velocity considerations become relevant, but here, the wave is monochromatic.

Frequency and Wavelength Relationship

- The frequency (f) relates to the angular frequency:

$$\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi}$$

Calculating:

$$\left[f = \frac{1000}{6.2832} \approx 159.15 \text{ Hz} \right]$$

- The wave oscillates approximately 159 times per second.

Conclusion and Final Remarks

The wave equation:

$$\left[Y = 0.5 \sin (1000 T - 17,000 x) \right]$$

embodies a harmonic, plane progressive wave traveling in the positive x-direction with a small amplitude, high frequency, and very short wavelength. Its characteristics are typical of high-frequency mechanical waves, possibly relevant in micro-scale physical systems.

Understanding each parameter provides insights into the wave's behavior, energy transmission, and the medium involved. This detailed analysis underscores the importance of mathematical interpretation in physical wave phenomena and demonstrates how a seemingly simple equation encapsulates complex physical behavior.

Summary of Key Points

- Amplitude: 0.5 mm
- Angular frequency: 1000 rad/sec
- Frequency: ~159 Hz
- Wave number: 17,000 rad/m
- Wavelength: ~0.37 mm
- Wave velocity: ~0.0588 m/sec
- Direction: Propagates in the positive x-direction
- Physical context: Likely a high-frequency mechanical wave in a solid or micro-scale medium

This comprehensive understanding prepares physicists and engineers to analyze, design, and interpret wave phenomena in various scientific and technological applications.

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