

A Sample Of Size 45 Will Be Drawn From A Population With Mean 10 And Standard Deviation 5. Find The Probability

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Understanding the probability associated with sample means is a fundamental aspect of inferential statistics. When drawing a sample of size 45 from a population with a known mean of 10 and a standard deviation of 5, statisticians often seek to determine the likelihood that the sample mean falls within a specified range or deviates from the population mean by a certain amount. This article provides a comprehensive guide to calculating such probabilities, emphasizing key concepts, formulas, and practical applications.

Introduction to Sampling Distributions and Probability

Before diving into calculations, it's essential to understand the core concepts underpinning this problem:

What Is a Sampling Distribution?

A sampling distribution describes the probability distribution of a statistic—such as the sample mean—computed from a large number of samples drawn from a population.

Why Is the Sampling Distribution of the Mean Important?

It allows us to make inferences about the population mean based on sample data. The properties of this distribution are critical for calculating probabilities related to the sample mean.

Central Limit Theorem (CLT)

The CLT states that, for sufficiently large sample sizes (usually $n \geq 30$), the sampling distribution of the sample mean is approximately normal, regardless of the population's original distribution. This is key to performing probability calculations when the sample size is 45.

Given Data and Assumptions

Let's first clarify the data and assumptions involved:

- Population Mean (μ) = 10
- Population Standard Deviation (σ) = 5
- Sample Size (n) = 45

Assuming the population distribution is normal or the sample size is large enough (which it is), we can approximate the distribution of the sample mean as normal.

Understanding the Distribution of the Sample Mean

Given the data, the sample mean ($\bar{X}$) will follow a normal distribution with:

Mean of the sampling distribution

$$\mu_{\bar{X}} = \mu = 10$$

Standard deviation (Standard Error)

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{45}}$$

Calculating the Standard Error:

$$\sigma_{\bar{X}} = \frac{5}{\sqrt{45}} \approx \frac{5}{6.708} \approx 0.745$$

Thus, the sampling distribution of $\bar{X}$ is approximately:

$$\bar{X} \sim N(10, 0.745^2)$$

Calculating Probabilities for the Sample Mean

The core task involves computing the probability that the sample mean falls within a certain range or exceeds a specific value. The general approach involves the following steps:

Step 1: Standardize the sample mean

Convert the sample mean to a standard normal variable (Z-score):

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

Step 2: Use Standard Normal Distribution Tables or Technology

Calculate the probability using Z-tables or statistical software.

Step 3: Interpret the result

Translate the probability back into the context of the problem.

Example: Find the Probability That the Sample Mean Is Less Than 10.5

Suppose we want to find:

$$P(\bar{X} < 10.5)$$

Step-by-step calculation:

1. Calculate the Z-score:

$$Z = \frac{10.5 - 10}{0.745} \approx \frac{0.5}{0.745} \approx 0.671$$

2. Find the probability corresponding to $Z = 0.671$:

Using standard normal tables or software:

$$P(Z < 0.671) \approx 0.75$$

Interpretation:

There is approximately a 75% chance that the sample mean will be less than 10.5.

General Formula for Probability Calculations

To generalize, if you need to find the probability that the sample mean $\bar{X}$ falls between two values (a) and (b) , the formula is:

$$\begin{aligned} & \left[\right. \\ & P(a < \bar{X} < b) = P\left(\frac{a - \mu}{\sigma_{\bar{X}}} < Z < \frac{b - \mu}{\sigma_{\bar{X}}} \right) \\ & \left. \right] \end{aligned}$$

This involves:

- Standardizing both (a) and (b) into Z-scores.
- Using the standard normal distribution to find the probabilities.

Applications and Implications in Real-World Scenarios

Understanding how to compute these probabilities is crucial for various fields, including:

- Quality control in manufacturing
- Market research and opinion polling
- Medical studies and clinical trials
- Educational testing and assessment
- Financial risk analysis

In each case, determining the likelihood that a sample mean falls within a certain range helps in decision-making and policy formulation.

Additional Considerations

While the above calculations assume a known population standard deviation, in many real-world cases, (σ) is unknown, and the sample standard deviation (s) is used instead. This leads to the use of the Student's t-distribution, which accounts for additional uncertainty.

Key points to remember:

- When (σ) is known and $(n \geq 30)$, the normal distribution applies.
- For smaller samples or unknown (σ) , the t-distribution is more appropriate.
- Always verify assumptions before applying the formulas.

Conclusion

Calculating the probability associated with a sample mean drawn from a population involves understanding the sampling distribution, standardizing the sample mean, and using the properties of the normal distribution. In the

case of drawing a sample of size 45 from a population with mean 10 and standard deviation 5, the sample mean approximately follows a normal distribution with mean 10 and standard deviation about 0.745. By applying the Z-score formula and standard normal tables or software, you can determine probabilities such as the likelihood that the sample mean falls below a certain value, exceeds a threshold, or lies within a specific interval.

This process is vital for making informed decisions based on sample data, assessing risks, and conducting scientific research. Mastery of these probability calculations enhances your ability to interpret statistical results accurately and confidently.

Further Resources for Learning Probability and Statistics

To deepen your understanding of probabilities related to sample means and other statistical concepts, consider exploring:

- Online courses on statistics and probability
- Textbooks on inferential statistics
- Statistical software tutorials (e.g., R, SPSS, Python)
- Academic articles on the Central Limit Theorem and sampling distributions

By mastering these concepts, you'll be well-equipped to handle a wide range of statistical challenges across various disciplines.

Keywords for SEO Optimization:

- Sample size probability calculation
- Normal distribution sample mean
- Central Limit Theorem applications
- Probability of sample mean
- Standard Error calculation
- Inferential statistics tutorial
- Sampling distribution of the mean
- Statistical probability examples
- How to compute probability in statistics
- Population mean and standard deviation analysis

Frequently Asked Questions

What is the sampling distribution of the sample mean for a sample size of 45 drawn from a population with mean 10 and standard deviation 5?

The sampling distribution of the sample mean will be approximately normal with mean 10 and standard error (SE) = $5 / \sqrt{45} \approx 0.745$.

How do you calculate the standard error for the sample mean in this scenario?

Standard error (SE) = population standard deviation / $\sqrt{\text{sample size}}$ = 5 / $\sqrt{45}$ $\approx$ 0.745.

If asked to find the probability that the sample mean exceeds 11, how would you approach it?

Calculate the z-score: $z = (11 - 10) / 0.745 \approx 1.34$, then find the probability $P(Z > 1.34)$ using standard normal tables or software.

What is the probability that the sample mean is less than 9 in this scenario?

Calculate $z = (9 - 10) / 0.745 \approx -1.34$, then find $P(Z < -1.34)$ from the standard normal distribution, approximately 0.0901.

How does increasing the sample size affect the standard error and the probability calculations?

Increasing the sample size decreases the standard error, making the sampling distribution narrower, which affects the probabilities of observing sample means far from the population mean.

Can you find the probability that the sample mean lies between 9.5 and 10.5?

Calculate z-scores for 9.5 and 10.5: $z_1 = (9.5 - 10) / 0.745 \approx -0.67$, $z_2 = (10.5 - 10) / 0.745 \approx 0.67$. The probability is $P(-0.67 < Z < 0.67) \approx 0.496$.

What assumptions are made when using the normal approximation for this problem?

The key assumptions are that the population is either normally distributed or the sample size is large enough ($n=45$) to invoke the Central Limit Theorem, ensuring the sample mean distribution is approximately normal.

Additional Resources

A Sample Of Size 45 Will Be Drawn From A Population With Mean 10 And Standard Deviation 5. Find The Probability is a classic problem in the realm of probability and statistics, often encountered by students and professionals alike. This problem encapsulates fundamental concepts such as sampling distributions, the Central Limit Theorem, and the calculation of probabilities associated with sample means. Understanding this problem not only aids in grasping theoretical statistics but also equips practitioners with practical tools used in fields like quality control, research, and data analysis. In this article, we will delve into the various aspects of this problem, breaking it down step-by-step and exploring the underlying principles, methods, and implications.

Introduction to the Problem

The problem states that a sample of size 45 is drawn from a population with a known mean ($\mu = 10$) and standard deviation ($\sigma = 5$). The task is to find the probability associated with a certain event related to the sample mean, though the specific event—such as the probability that the sample mean exceeds or falls below a certain value—is not explicitly provided here. Typically, such problems involve calculating the probability that the sample mean is within or outside a specified range, or exceeding a certain threshold.

Why is this important?

Understanding how to compute such probabilities is crucial because:

- It helps determine the likelihood of observing certain sample results given the population parameters.
- It aids in hypothesis testing and confidence interval estimation.
- It provides insights into the variability inherent in sampling processes.

Fundamental Concepts

Before tackling the problem directly, it's essential to review some core concepts:

Population Parameters

- Mean (μ): The average value of the population, given as 10.
- Standard Deviation (σ): The measure of dispersion in the population, given as 5.

Sample Size (n)

- The number of observations drawn from the population, here $n=45$.

Sampling Distribution of the Sample Mean

- The sampling distribution describes how the sample mean ($\bar{X}$) varies from sample to sample.
- When sampling from a population with a known mean and standard deviation, the distribution of the sample mean is approximately normal if the sample size is large enough, thanks to the Central Limit Theorem.

Central Limit Theorem (CLT)

The CLT states that, regardless of the population's distribution, the distribution of the sample mean tends toward a normal distribution as the sample size increases, typically $n \geq 30$ is considered sufficient.

Implications for Our Problem

- Since $n=45$, which exceeds 30, the sampling distribution of $\bar{X}$ can be approximated as normal.

- The mean of this distribution is $(\mu_{\bar{X}} = \mu = 10)$.
- The standard deviation of this distribution, called the standard error (SE), is:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{45}}$$

Calculating this:

$$SE \approx \frac{5}{6.708} \approx 0.745$$

This standard error quantifies the expected variability of the sample mean.

Calculating Probabilities for the Sample Mean

Suppose the task is to compute the probability that the sample mean falls within a certain range or exceeds a specific value. The general approach involves:

1. Standardizing the sample mean to a z-score.
2. Using the standard normal distribution table to find the probability corresponding to that z-score.

Standardizing the Sample Mean

Given a value (x) , the z-score is:

$$z = \frac{x - \mu_{\bar{X}}}{SE}$$

For example, if we want to find the probability that $(\bar{X})$ exceeds a certain number (x) , we compute the z-score and then find $(P(Z > z))$.

Example: Computing a Specific Probability

Let's assume the problem asks: "What is the probability that the sample mean exceeds 11?"

Step 1: Compute the z-score

$$z = \frac{11 - 10}{0.745} \approx \frac{1}{0.745} \approx 1.34$$

Step 2: Use the standard normal distribution

- From the standard normal table, $(P(Z > 1.34) \approx 0.0901)$.

Result:

$[$


```
\boxed{
P(\bar{X} > 11) \approx 0.0901
}
\]
```

This indicates there's roughly a 9.01% chance that the sample mean exceeds 11.

Exploring Different Probabilities

Depending on the specific question, you might need to find:

- The probability that $(\bar{X})$ is less than a certain value.
- The probability that $(\bar{X})$ falls within a range.
- The probability that $(\bar{X})$ deviates from (μ) by a certain amount.

Each scenario involves similar calculations, primarily centered around standardizing the relevant boundary values and referencing the standard normal distribution.

Confidence Intervals and Hypothesis Testing

The problem's framework naturally extends to:

Confidence Intervals

- Estimating the range within which the true population mean lies with a certain confidence level (e.g., 95%).

Example:

A 95% confidence interval for (μ) is:

```
\[
\bar{X} \pm Z_{\alpha/2} \times SE
\]
```

Where $(Z_{0.025} \approx 1.96)$. If the sample mean $(\bar{X})$ was observed as 10, then:

```
\[
CI = 10 \pm 1.96 \times 0.745 \approx 10 \pm 1.46
\]
```

So, the interval is approximately (8.54, 11.46).

Hypothesis Testing

- Testing hypotheses such as whether the population mean equals a specific value.

For example:

Null hypothesis: $(\mu = 10)$

Alternative hypothesis: $(\mu \neq 10)$

Using the sample mean, standard error, and z-test, you can determine whether to reject (H_0) .

Pros and Cons of Using Normal Approximation

Pros:

- Simplifies calculations for large samples.
- Well-supported by the CLT.
- Facilitates the use of standard normal tables.

Cons:

- Less accurate if the population distribution is highly skewed or non-normal and the sample size is small.
- Assumes known population standard deviation; in practice, this may not be known, requiring the use of t-distribution.

Practical Applications

This type of probability calculation is widely applicable:

- Quality control: Determining whether the average of a batch exceeds acceptable limits.
- Medical research: Estimating the likelihood that a sample mean differs from a hypothesized population mean.
- Market analysis: Understanding consumer behavior averages and their variability.
- Environmental studies: Assessing pollutant levels in samples against regulatory thresholds.

Summary and Key Takeaways

- Drawing a sample of size 45 from a population with mean 10 and standard deviation 5 allows the use of the CLT to approximate the sampling distribution of the sample mean as normal.
- The standard error of the mean is approximately 0.745, which measures the variability of the sample mean.
- To find probabilities related to the sample mean, standardize the value to a z-score and reference the standard normal distribution.
- This approach enables the calculation of probabilities, confidence intervals, and hypothesis testing, forming the backbone of inferential statistics.
- While the normal approximation is powerful, it has limitations, especially with small samples or non-normal populations.

Final Remarks

The problem of finding the probability associated with a sample mean drawn

from a known population distribution exemplifies the core principles of inferential statistics. Mastery of this topic involves understanding the Central Limit Theorem, standardization techniques, and the interpretation of normal distribution probabilities. These tools are essential for making informed decisions based on sample data, whether in academic research, industry quality control, or policy formulation. As you continue exploring statistical problems, keep in mind the assumptions and conditions under which these methods are valid, and always consider the context and specifics of the data at hand.

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