A. What Is The Present Value Of A 3-year Annuity Of $100 If The Discount Rate Is 6%? (Do Not Round Intermediate

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Introduction

Understanding the concept of present value (PV) is fundamental in finance and investment decision-making. It enables investors, financial analysts, and business owners to determine the current worth of a series of future cash flows, considering the time value of money. When dealing with annuities—fixed payments made at regular intervals—calculating the present value becomes a vital skill, particularly for evaluating loans, investments, or retirement plans.

This article focuses on a specific scenario: calculating the present value of a 3-year annuity of $100 payments each year, with a discount rate of 6%. The question is straightforward but involves intricate calculations that require precision, especially when intermediate steps are not rounded. By the end of this guide, you will understand how to approach such problems methodically, appreciate the significance of discount rates, and apply the relevant formulas accurately.

Understanding Annuities and Present Value

What Is An Annuity?

An annuity is a series of equal payments made at regular intervals over a specified period. These payments can be received or paid out, making annuities common in various financial contexts such as:

- Retirement income plans
- Loan repayments
- Investment schemes
- Insurance products

In our scenario, the annuity consists of annual payments of $100 for three years.

Present Value Concept

The present value is the current worth of future cash flows discounted at a particular rate. The core idea is that receiving a sum today is more valuable than receiving the same sum in the future because of the potential earning capacity of money—this is known as the "time value of money."

The formula for calculating present value takes into account the discount rate, which reflects the risk or opportunity cost of capital.

Why Is Discount Rate Important?

The discount rate influences how future payments are valued today. A higher discount rate decreases

the present value of future cash flows, and vice versa. In our case, a 6% rate is used, which is considered a modest rate aligned with average investment returns or risk-free rates.

The Formula for Present Value of an Annuity

The Standard Present Value of an Ordinary Annuity Formula

For an ordinary annuity (where payments are made at the end of each period), the present value (PV) can be calculated using the formula:

$$
PV = P \times \left(\frac{1 - (1 + r)^{-n}}{r} \right)
$$

Where:

- P = payment amount per period ($100)
- r = discount rate per period (6% or 0.06)
- n = number of periods (3 years)

This formula sums the present value of each individual payment, considering the number of periods until each payment.

Applying the Formula with Precise Calculations

To ensure accuracy, intermediate calculations should not be rounded. This involves calculating powers and ratios precisely, then multiplying by the payment amount.

Step-by-Step Calculation

Step 1: Define the Variables

- $P = 100$
- $r = 0.06$
- $n = 3$

Step 2: Calculate $(1 + r)^{-n}$

$$
(1 + r)^{-n} = (1 + 0.06)^{-3} = 1.06^{-3}
$$

Using precise calculation:

$$
1.06^3 = 1.191016
$$

Therefore,

$$
1.06^{-3} = \frac{1}{1.191016} \approx 0.839619
$$

Note: To maintain precision, avoid rounding until the final step.

Step 3: Calculate the numerator $1 - (1 + r)^{-n}$

$$
1 - 0.839619 = 0.160381
$$

Step 4: Divide by the discount rate r

$$
\frac{0.160381}{0.06} \approx 2.673017
$$

Step 5: Calculate the present value PV

$$
PV = P \times \left(\frac{1 - (1 + r)^{-n}}{r} \right) = 100 \times 2.673017 = 267.3017
$$

Final Present Value

$$
\boxed{\text{PV} \approx \$267.3017}
$$

Note: Since the instruction emphasizes not to round intermediate calculations, the final answer is presented with four decimal places for accuracy.

Additional Insights and Implications

Impact of Discount Rate on Annuity Valuation

The discount rate significantly influences the present value. For example:

- If the rate were higher (say 8%), the present value would decrease.
- If the rate were lower (say 4%), the present value would increase.

This sensitivity underscores the importance of selecting an appropriate discount rate based on risk, opportunity cost, or prevailing market conditions.

Real-World Applications

- Retirement Planning: Calculating how much a series of future payments is worth today to determine if an investment is viable.
- Loan Amortization: Understanding how much a series of payments is worth at the outset.
- Valuation of Financial Instruments: Assessing bonds, annuities, or structured products.

Limitations and Assumptions

- The payments are assumed to be made at the end of each period (ordinary annuity).
- The discount rate remains constant over the period.
- Payments are fixed and occur regularly.

Conclusion

Calculating the present value of a 3-year annuity of $100 payments at a 6% discount rate involves understanding the fundamental principles of the time value of money, applying the correct formula, and executing precise calculations without intermediate rounding. The key steps include computing the present value factor using the formula, accurately calculating powers and ratios, and then multiplying by the payment amount.

The precise present value for this specific scenario is approximately \$267.3017. Recognizing how the discount rate affects this valuation is crucial for sound financial decision-making, whether in personal finance, corporate investments, or financial analysis.

By mastering these calculations, investors and financial professionals can better assess the worth of future cash flows, optimize investment strategies, and make informed economic decisions.

Frequently Asked Questions

What is the formula to calculate the present value of a 3-year annuity of $100 with a 6% discount rate?

The present value (PV) of an annuity can be calculated using the formula: PV = P × [(1 - (1 + r)^-n) / r], where P = payment amount ($100), r = discount rate (6% or 0.06), and n = number of periods (3).

What is the present value of a $100 annual annuity over 3 years at a 6% discount rate, without rounding intermediate steps?

Using the formula: PV = 100 × [(1 - (1 + 0.06)^-3) / 0.06] = 100 × [(1 - (1.06)^-3) / 0.06] = 100 × [(1 - 0.83962) / 0.06] = 100 × [0.16038 / 0.06] = 100 × 2.673. So, the present value is approximately $267.30.

How does the discount rate affect the present value of a 3-

year annuity?

A higher discount rate decreases the present value of the annuity because future payments are discounted more heavily, while a lower rate increases the present value, reflecting less discounting of future cash flows.

Why is it important not to round intermediate calculations when determining the present value of an annuity?

Not rounding intermediate calculations ensures maximum accuracy in the final result, preventing small rounding errors from accumulating and leading to an incorrect present value estimate.

Can the present value of an annuity be used for investment decision-making? If so, how?

Yes, the present value helps determine whether the future cash flows from an investment are worth their current cost, enabling investors to compare different investment options and assess profitability based on discounted cash flows.

Additional Resources

A. What Is The Present Value Of A 3-year Annuity Of $100 If The Discount Rate Is 6%? (Do Not Round Intermediate

Understanding the concept of present value is fundamental in finance and investment decision-making. When evaluating cash flows that occur over multiple periods, such as annuities, determining their present worth helps investors, businesses, and individuals make informed choices. This article delves into the calculation of the present value of a 3-year annuity paying $100 annually, with a discount rate of 6%. We will explore the underlying principles, mathematical formulas, step-by-step calculations, and practical implications, ensuring clarity for readers with varied backgrounds.

Foundations of Present Value and Annuities

Before diving into the calculations, it’s essential to understand the basic concepts underpinning present value and annuities.

What Is Present Value?

Present value (PV) is a financial metric that determines the current worth of a sum of money or stream of cash flows that will be received or paid in the future. It accounts for the time value of money—the idea that money available today is worth more than the same amount in the future due to its potential earning capacity.

Mathematically, the present value is calculated by discounting future cash flows back to the present using a discount rate, which reflects the opportunity cost, risk, inflation, and other factors.

Understanding Annuities

An annuity is a series of equal payments made at regular intervals over a specified period. Examples include loan repayments, insurance payouts, or investment payments.

Key features of annuities:

- Regular payments of equal amount ($100 in this case)
- Fixed period (3 years)
- Payments occur at the end of each period (ordinary annuity)

Different types of annuities exist, but in this context, we're considering an ordinary annuity, where payments are made at the end of each period.

Mathematical Framework for Present Value of an Annuity

The calculation of the present value of an annuity involves summing the discounted values of each individual payment, or, more efficiently, using the annuity present value formula.

The Standard Present Value of an Ordinary Annuity Formula

The formula for the present value (PV) of an ordinary annuity with periodic payment P, discount rate r, and number of periods n is:

$$
PV = P \times \left(\frac{1 - (1 + r)^{-n}}{r} \right)
$$

Where:

- P: payment amount per period ($100)
- r: discount rate per period (6% or 0.06)
- n: total number of periods (3 years)

This formula calculates the current worth of all future payments by discounting each at the rate r.

Step-by-Step Calculation

Applying the formula to our specific case:

- $P = 100$
- $r = 0.06$
- $n = 3$

Let's proceed with the calculation, maintaining the precision throughout without rounding intermediate steps.

Step 1: Calculate $(1 + r)^{-n}$

$$
(1 + 0.06)^{-3} = 1.06^{-3}
$$

Calculating 1.06^3:

$$
1.06^3 = 1.06 \times 1.06 \times 1.06
$$

- First, $1.06 \times 1.06 = 1.1236$
- Then, $1.1236 \times 1.06 = 1.191016$

Thus,

$$
1.06^3 = 1.191016
$$

Now, take the reciprocal to find 1.06^{-3}:

$$
1.06^{-3} = \frac{1}{1.191016} \approx 0.839619
$$

Step 2: Compute the numerator $1 - (1 + r)^{-n}$

$$
1 - 0.839619 = 0.160381
$$

Step 3: Divide by the discount rate r

$$
\frac{0.160381}{0.06} \approx 2.673017
$$

Step 4: Multiply by the payment amount P

$$
PV = 100 \times 2.673017 = 267.3017
$$

Therefore, the present value of the 3-year $100 annuity at a 6% discount rate is approximately $267.3017.

Interpreting the Result

The computed present value of approximately $267.30 reflects the current worth of receiving $100 at the end of each of the next three years, discounted at 6% annually. This means that, given a 6% opportunity cost, an investor should be willing to pay up to about $267.30 today to secure these future payments.

This valuation is crucial for various financial decisions. For instance:

- Investment Analysis: A company considering issuing a 3-year bond paying $100 annually can assess whether its cost of capital aligns with this valuation.
- Loan Structuring: Borrowers or lenders can determine fair terms for loans with similar payment schedules.
- Personal Finance Planning: Individuals planning future cash flows can evaluate the present worth of annuity income.

Practical Implications and Variations

While the calculation above assumes a fixed discount rate and payments at the end of each period, real-world scenarios may introduce complexities:

- Changing Discount Rates: Over time, interest rates fluctuate due to economic conditions, impacting present value calculations.
- Different Payment Frequencies: Payments could be made more frequently (monthly, quarterly), requiring adjustment of the rate and periods.

- Variable Payments: Some annuities have payments that change over time, necessitating more complex valuation models.
- Inflation Effects: Inflation diminishes the real value of future cash flows, influencing discount rates and valuation.

Understanding these factors allows financial professionals to adapt the basic formula to more complex situations, ensuring accurate valuation.

Conclusion

Calculating the present value of an annuity is a cornerstone skill in finance, enabling stakeholders to appraise the worth of future cash flows accurately. For the specific case of a 3-year $100 annuity at a 6% discount rate, the present value approximates $267.30, illustrating the principle that future payments are worth less today due to the time value of money.

By mastering these calculations and understanding their implications, investors and financial managers can make more informed, strategic decisions—whether evaluating investments, structuring loans, or planning personal finances. The key takeaway is that the present value provides a meaningful measure of the worth of future cash flows, rooted in fundamental financial mathematics and economic reasoning.

A What Is The Present Value Of A 3 Year Annuity Of 100 If The Discount Rate Is 6 Do Not Round Intermediate

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