Assuming The Utility Function Of An Individual Is As Follows. U= 18q+7q2-1/3q3determine The Utility Maximizing

Assuming The Utility Function Of An Individual Is As Follows. U= 18q + 7q2 - (1/3)q3 determine The Utility Maximizing Level of Quantity

Understanding how individuals make choices to maximize their utility is a fundamental concept in microeconomics. When faced with a specific utility function, such as U= 18q + 7q2 - (1/3)q3, the primary goal is to identify the quantity (q) that maximizes the individual's utility. This involves analyzing the function's behavior, finding critical points, and determining the maximum point through calculus techniques. In this article, we will explore the step-by-step process to determine the utility-maximizing level of q for the given function.

Understanding the Utility Function

What is a Utility Function?

A utility function represents an individual's preferences over a set of goods or services, assigning a numerical value (utility) to different levels of consumption. The higher the utility, the more preferred the combination of goods or services.

In our case, the utility function is:

U(q) = 18q + 7q2 - (1/3)q3

where:
- q represents the quantity of a good or service,
- U(q) indicates the utility derived from consuming q units.

Characteristics of the Given Utility Function

- It is a cubic function in q.
- The function likely exhibits increasing and decreasing marginal utility at different points.
- The goal is to find the value of q that yields the maximum utility.

Mathematical Approach to Utility Maximization

Step 1: Find the First Derivative (Marginal Utility)

The first derivative of the utility function with respect to q indicates the marginal utility (MU), which measures the rate of change of utility as q changes.

Differentiate U(q):

U'(q) = d/dq [18q + 7q2 - (1/3)q3]

Apply power rule:

U'(q) = 18 + 14q - q2

Step 2: Set the Marginal Utility to Zero to Find Critical Points

To find potential maxima or minima, set U'(q) = 0:

18 + 14q - q2 = 0

Rewrite:

-q2 + 14q + 18 = 0

Multiply both sides by -1 for clarity:

q2 - 14q - 18 = 0

This quadratic equation can be solved for q.

Step 3: Solve the Quadratic Equation

Use the quadratic formula:

q = [-b ± √(b2 - 4ac)] / 2a

where:
- a = 1
- b = -14
- c = -18

Calculate discriminant:

D = b2 - 4ac = (-14)2 - 4 1 (-18) = 196 + 72 = 268

Compute square root:

√D ≈ √268 ≈ 16.3707

Now find the roots:

q = [14 ± 16.3707] / 2

1. q = (14 + 16.3707) / 2 ≈ 30.3707 / 2 ≈ 15.1854
2. q = (14 - 16.3707) / 2 ≈ (-2.3707) / 2 ≈ -1.1854

Since quantity q cannot be negative in this context, discard q ≈ -1.1854.

Potential maximum at q ≈ 15.1854

Determining the Nature of Critical Points

Step 4: Find the Second Derivative (Marginal Utility's Rate of Change)

Differentiate U'(q):

U''(q) = d/dq [14q - q2] = 14 - 2q

Evaluate at q ≈ 15.1854:

U''(15.1854) = 14 - 2 15.1854 ≈ 14 - 30.3708 ≈ -16.3708

Since the second derivative is negative, the critical point at q ≈ 15.1854 corresponds to a local maximum.

Conclusion: Utility-Maximizing Quantity

Based on the calculus analysis, the individual maximizes utility at approximately q ≈ 15.19 units.

Additional Considerations in Utility Maximization

Feasibility and Context

- The calculated quantity assumes no constraints or budget limitations.
- In real-world scenarios, factors like income, prices, or availability can influence the optimal q.

Boundary Conditions

- Check utility at q = 0 if applicable, to confirm the maximum is within feasible bounds.
- For very large q, utility may decrease due to the negative cubic term, confirming the presence of a maximum.

Graphical Representation of the Utility Function

Visualizing the utility function can provide insights into the nature of its maximum:

- The function starts increasing, reaches a peak near $q \approx 15.19$, and then declines.
- The slope (marginal utility) is positive before the critical point and negative afterward.

Implications for Consumers and Economists

- Consumers aim to choose quantities that maximize their utility, given their preferences.
- Economists use calculus-based optimization to understand consumer behavior.
- The utility function's shape influences decision-making and market outcomes.

Summary of the Utility Maximization Process

1. Differentiate the utility function to find marginal utility.
2. Set the marginal utility equal to zero to find critical points.
3. Solve the resulting quadratic equation.
4. Determine the nature of critical points using the second derivative.
5. Identify the quantity (q) that maximizes utility.

Final Thoughts

Maximizing utility involves a systematic approach rooted in calculus. For the given utility function U= 18q + 7q2 - (1/3)q3, the optimal quantity is approximately 15.19 units, which provides the highest satisfaction level for the individual. Understanding this process is essential for economists and students studying consumer choice theory, as it forms the basis for analyzing demand, market behavior, and resource allocation.

Additional Resources for Further Study

- Microeconomics textbooks on consumer theory
- Calculus tutorials on optimization techniques
- Articles on utility functions and their applications in economic modeling
- Software tools for graphing and analysis, such as Desmos or Wolfram Alpha

In conclusion, the process of determining the utility-maximizing level of q from the given function illustrates the power of calculus in economic analysis. By carefully differentiating, solving, and analyzing the function, individuals and economists can better understand decision-making processes and optimize outcomes effectively.

Frequently Asked Questions

How do you determine the utility-maximizing quantity given the utility function U = 18q + 7q^2 - (1/3)q^3?

To find the utility-maximizing quantity, differentiate U with respect to q, set the derivative equal to zero, and solve for q: $dU/dq = 18 + 14q - q^2 = 0$. Then, analyze the second derivative or the critical points to identify the maximum.

What is the significance of setting the first derivative of the utility function to zero in this context?

Setting the first derivative to zero helps identify critical points where the utility could be at a maximum or minimum. In utility maximization, these points indicate potential optimal quantities, which are then tested to determine if they maximize utility.

How can we verify that the critical point found by solving dU/dq = 0 is a maximum utility point?

By calculating the second derivative, $d^2U/dq^2 = 14 - 2q$. If at the critical point the second derivative is negative, it indicates a maximum. Alternatively, analyzing the utility function's behavior around the critical point can confirm if it's a maximum.

What is the practical interpretation of the utility function U = 18q + 7q^2 - (1/3)q^3 for individual consumption choices?

This utility function represents how an individual's satisfaction varies with the quantity q of a good consumed. The quadratic and cubic terms capture increasing and decreasing marginal returns, helping determine the optimal consumption level that maximizes utility.

Are there any constraints or considerations to keep in mind when maximizing this utility function?

Yes. The quantity q should be non-negative ($q \geq 0$), and in some cases, an upper bound may exist depending on context. Additionally, ensuring the critical point falls within feasible ranges and verifying its nature as a maximum are essential steps.

Additional Resources

Assuming The Utility Function Of An Individual Is As Follows. $U = 18q + 7q^2 - (1/3)q^3$: Determine The Utility Maximizing Quantity

In the realm of microeconomics and consumer theory, understanding how individuals maximize their utility is fundamental. When presented with a specific utility function, such as $U = 18q + 7q^2 - (1/3)q^3$, economists and students alike seek to determine the utility-maximizing quantity of a good that an individual would consume. This guide provides a comprehensive, step-by-step approach to analyzing this utility function, illustrating how to find the optimal quantity q that maximizes utility, and explaining the underlying concepts involved.

Understanding the Utility Function

Before diving into the mathematics, it’s essential to interpret what the utility function represents. The function:

$$ U(q) = 18q + 7q^2 - \frac{1}{3}q^3 $$

expresses the total utility an individual derives from consuming q units of a particular good.

- The term 18q suggests a linear increase in utility with consumption.
- The quadratic term $7q^2$ indicates increasing marginal utility at lower levels or perhaps accelerating benefits.
- The cubic term $-(1/3)q^3$ introduces a diminishing or negative effect at higher quantities, typically modeling satiation or diminishing returns.

The goal is to determine the value of q that maximizes U(q), considering the typical assumptions such as the utility function being continuous and differentiable, and that q is non-negative (since negative consumption isn't meaningful).

Step 1: Find the Marginal Utility (MU)

Marginal Utility is the additional utility gained from consuming one more unit of the good. It is obtained by differentiating the utility function with respect to q:

$$
MU(q) = \frac{dU}{dq}
$$

Calculating the derivative:

$$
MU(q) = \frac{d}{dq} \left(18q + 7q^2 - \frac{1}{3}q^3 \right)
$$

$$
MU(q) = 18 + 14q - q^2
$$

This function tells us how utility changes as we increase q, and it is vital for identifying the maximum utility point.

Step 2: Set Marginal Utility to Zero to Find Critical Points

To locate potential maxima (or minima), set MU(q) equal to zero:

$$
18 + 14q - q^2 = 0
$$

Rearranged into standard quadratic form:

$$
-q^2 + 14q + 18 = 0
$$

Multiply through by -1 to simplify:

$$
q^2 - 14q - 18 = 0
$$

Now, apply the quadratic formula:

$$
q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where a = 1, b = -14, c = -18:

$$
q = \frac{14 \pm \sqrt{(-14)^2 - 4(1)(-18)}}{2}
$$

$$
q = \frac{14 \pm \sqrt{196 + 72}}{2}
$$

$$
q = \frac{14 \pm \sqrt{268}}{2}
$$

Simplify the square root:

$$
\sqrt{268} \approx 16.37
$$

Thus, the critical points are:

$$
q = \frac{14 \pm 16.37}{2}
$$

Calculating both:

- First solution:

$$
q = \frac{14 + 16.37}{2} = \frac{30.37}{2} \approx 15.19
$$

- Second solution:

$$
q = \frac{14 - 16.37}{2} = \frac{-2.37}{2} \approx -1.185
$$

Since negative consumption isn't feasible, the relevant critical point is $q \approx 15.19$.

Step 3: Confirm Whether the Critical Point Is a Maximum

To verify if $q \approx 15.19$ yields a maximum utility, examine the second derivative of the utility function, which indicates concavity:

$$
\frac{d^2U}{dq^2} = \frac{dMU}{dq} = \frac{d}{dq} (18 + 14q - q^2) = 14 - 2q
$$

Evaluate at $q \approx 15.19$:

$$
\frac{d^2U}{dq^2} = 14 - 2(15.19) = 14 - 30.38 = -16.38
$$

Since this is negative, the utility function is concave at this point, confirming that $q \approx 15.19$ is indeed a local maximum.

Step 4: Consider the Boundary Conditions

In many cases, consumer choice is constrained:

- Non-negativity constraint: $q \geq 0$
- Practical maximum: Usually determined by budget constraints, but since none are specified here, we focus on the critical point.

To ensure this is the global maximum, check the utility at $q = 0$:

$$
U(0) = 0
$$

Compare with U(15.19):

$$
U(15.19) = 18(15.19) + 7(15.19)^2 - \frac{1}{3}(15.19)^3
$$

Calculate step-wise:

- $18 \times 15.19 \approx 273.42$
- $7 \times (15.19)^2 \approx 7 \times 231.00 \approx 1617.00$
- $(15.19)^3 \approx 3518.86$, so:

$$
\frac{1}{3} \times 3518.86 \approx 1172.95
$$

Now, sum up:

$$
U(15.19) \approx 273.42 + 1617.00 - 1172.95 = 717.47
$$

Since U(15.19) is significantly higher than U(0) = 0, and the second derivative indicates a maximum, the utility-maximizing quantity is approximately 15.19 units.

Additional Considerations

- Marginal Utility at q = 0:

$$
MU(0) = 18 + 14 \times 0 - 0^2 = 18
$$

which is positive, indicating utility increases as q increases from zero.

- Behavior at large q:

Since the cubic term dominates for large q, and its coefficient is negative, utility will eventually decrease beyond the critical point, reaffirming that q ≈ 15.19 is the maximum.

- Implications of the Utility Function:

This utility function captures a typical pattern where initial consumption increases utility significantly, but after a certain point, additional units provide diminishing or even negative marginal utility.

Summary & Practical Steps to Maximize Utility

1. Differentiate the utility function to find the marginal utility:

$$
MU(q) = 18 + 14q - q^2
$$

2. Set MU(q) to zero and solve for q to find critical points:

$$
q \approx 15.19
$$

3. Verify the nature of the critical point using the second derivative:

$$
\frac{d^2U}{dq^2} = 14 - 2q
$$

At $q \approx 15.19$, the second derivative is negative, indicating a maximum.

4. Calculate utility at critical points and boundaries:

- Utility at $q \approx 15.19$ is approximately 717.47.
- Utility at $q=0$ is zero, confirming the maximum occurs at $q \approx 15.19$.

5. Conclusion: The utility-maximizing quantity is approximately 15.19 units.

Final Thoughts

This analysis demonstrates how to approach a utility maximization problem involving a polynomial utility function. The key steps involve deriving the marginal utility, solving for critical points, confirming the nature of these points, and comparing utility at boundaries. Such a method is foundational in consumer theory, helping to understand consumption choices and demand behavior under various utility functions.

By mastering these steps, students and practitioners can analyze a wide range of utility functions, facilitating more accurate modeling of consumer preferences and decision-making processes in economics.

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