Consider The System Of Linear Equations Consider The System Of Linear Equations 6x+2y Z=4 X +5y+z=3 2x+y+4z=27A,

Consider The System Of Linear Equations Consider The System Of Linear Equations 6x+2y Z=4 X +5y+z=3 2x+y+4z=27A, a fundamental concept in algebra that involves finding the values of variables that satisfy multiple equations simultaneously. Linear systems are ubiquitous across various fields such as engineering, computer science, economics, and physics, where modeling real-world problems often boils down to solving systems of linear equations. Understanding how to analyze and solve these systems is essential for students and professionals alike, as it provides insights into underlying relationships within complex data.

In this comprehensive guide, we will explore the structure of the given system, methods to solve it, interpret the solutions, and discuss the significance of linear systems in different contexts. By the end of this article, you will have a clear understanding of how to approach such systems efficiently and accurately.

Understanding the Given System of Equations

Presentation of the System

The system in question consists of three equations involving three variables, typically denoted as x, y, and z:

1. $6x + 2y + z = 4$
2. $x + 5y + z = 3$
3. $2x + y + 4z = 27A$

Note: The notation "27A" suggests a parameter or constant, which may vary depending on context. For simplicity, assume that A is a known constant, and thus 27A is a known numerical value. If A is unknown, the system becomes parameterized.

Significance of the System

This system models a real-world scenario where multiple conditions or constraints must be satisfied simultaneously. For example, in economics, such equations could represent budget constraints; in engineering, they might describe system states or component interactions.

The goal is to determine the values of x, y, and z that satisfy all three equations concurrently. These solutions can be unique, infinite, or nonexistent, depending on the system's nature.

Methods for Solving the System of Equations

Several techniques are available for solving systems of linear equations, including substitution, elimination, and matrix methods like Gaussian elimination. The choice depends on the system's complexity and the context.

Method 1: Substitution

This method involves solving one of the equations for one variable and substituting into the others to reduce the system step by step.

1. Express z from the first equation:
   - $6x + 2y + z = 4 \Rightarrow z = 4 - 6x - 2y$
2. Substitute z into the second and third equations:
   - Equation 2: $x + 5y + (4 - 6x - 2y) = 3$
   - Equation 3: $2x + y + 4(4 - 6x - 2y) = 27A$
3. Simplify and solve for x and y, then back-substitute to find z.

Method 2: Elimination

This approach involves adding or subtracting equations to eliminate variables systematically.

1. Multiply equations if necessary to align coefficients.
2. Subtract equations to eliminate one variable, reducing to two equations with two variables.
3. Solve for remaining variables and substitute back to find the third.

Method 3: Matrix Approach (Gaussian Elimination)

This method uses matrices to systematically solve the system.

1. Write the augmented matrix:

6	2	1		4
1	5	1		3
2	1	4		27A

2. Apply row operations to reach row-echelon form.

3. Back-substitute to find solutions for x, y, and z.

Each method has its advantages, and choosing the right one depends on the problem's context and complexity.

Step-by-Step Solution of the System

Let's apply the substitution method for clarity and demonstrate the detailed process.

Step 1: Solve for z from the first equation

$$
z = 4 - 6x - 2y
$$

Step 2: Substitute z into the second equation

$$
x + 5y + z = 3 \Rightarrow x + 5y + (4 - 6x - 2y) = 3
$$

Simplify:

$$
x + 5y + 4 - 6x - 2y = 3
$$

$$
(x - 6x) + (5y - 2y) + 4 = 3
$$

$$
-5x + 3y + 4 = 3
$$

Subtract 4 from both sides:

$$
-5x + 3y = -1
$$

This simplifies to:

$$
5x - 3y = 1 \quad \text{(Equation A)}
$$

Step 3: Substitute z into the third equation

$$
2x + y + 4z = 27A
$$

Substitute z:

$$
2x + y + 4(4 - 6x - 2y) = 27A
$$

Expand:

$$
2x + y + 16 - 24x - 8y = 27A
$$

Combine like terms:

$$
(2x - 24x) + (y - 8y) + 16 = 27A
$$

$$
-22x - 7y + 16 = 27A
$$

Rearranged:

$$
-22x - 7y = 27A - 16 \quad \text{(Equation B)}
$$

Step 4: Solve the two equations for x and y

We now have the system:

$$
\begin{cases}
5x - 3y = 1 \quad \text{(Equation A)} \\
-22x - 7y = 27A - 16 \quad \text{(Equation B)}
\end{cases}
$$

Multiply Equation A by 7:

$$
35x - 21y = 7
$$

Multiply Equation B by 3:

$$
-66x - 21y = 3(27A - 16) = 81A - 48
$$

Subtract the second from the first:

$$
(35x - 21y) - (-66x - 21y) = 7 - (81A - 48)
$$

Simplify:

$$
35x + 66x = 7 - 81A + 48
$$

$$
101x = 55 - 81A
$$

Solve for x:

$$
x = \frac{55 - 81A}{101}
$$

Now, substitute x back into Equation A to find y:

$$
5x - 3y = 1
$$

$$
5 \times \frac{55 - 81A}{101} - 3y = 1
$$

Multiply through by 101 to clear denominator:

$$
5(55 - 81A) - 3y \times 101 = 101
$$

$$
275 - 405A - 303 y = 101
$$

Rearranged:

$$
-303 y = 101 - 275 + 405A
$$

$$
-303 y = -174 + 405A
$$

$$
y = \frac{174 - 405A}{303}
$$

Finally, find z:

$$
z = 4 - 6x - 2y
$$

Substitute x and y:

\[
z = 4 - 6 \times \frac{55 - 81A}{101} - 2 \times \frac{174 - 405A}{303}
\]

Note that:
\[
6 \times \frac{55 - 81A}{101} = \frac{6(55 - 81A)}{101} = \frac{330 - 486A}{101}
\]
and
\[
2 \times \frac{174 - 405A}{303} = \frac{2(174 - 405A)}{303} = \frac{348 - 810A}{303}
\]

Express all terms with a common denominator (303):
\[
z = \frac{4 \times 303}{303} - \frac{330 - 486A}{101} - \frac{348 - 810A}{303}
\]

Convert 4 to a fraction with denominator 303:
\[
4 = \frac{4 \times

Frequently Asked Questions

What is the first step to solve the system of equations 6x + 2y + z = 4, x + 5y + z = 3, 2x + y + 4z = 27?

The first step is to write the system in matrix form or use substitution or elimination methods to simplify the equations for solving.

How can substitution be applied to solve the system 6x + 2y + z = 4, x + 5y + z = 3, 2x + y + 4z = 27?

You can solve one equation for a variable, for example, express x from the second equation and substitute into the others to reduce the system to two variables.

What method is most efficient for solving this system: substitution, elimination, or matrix methods?

Using matrix methods such as Gaussian elimination or using augmented matrices is efficient, especially for larger systems, but elimination can also be effective here.

What are the solutions for x, y, and z in the system 6x + 2y + z = 4, x + 5y + z = 3, 2x + y + 4z = 27?

By solving the system, the solution is x = 5, y = -1, and z = -2.

How can the system be checked for consistency after finding the solutions?

Substitute the found values of x, y, and z back into all three original equations to verify that they satisfy each equation.

What real-world problems can be modeled using this system of linear equations?

Systems like this can model problems such as resource allocation, balancing chemical equations, or financial planning where multiple variables interact linearly.

Additional Resources

Consider The System Of Linear Equations Consider The System Of Linear Equations $6x + 2y - z = 4$, $x + 5y + z = 3$, $2x + y + 4z = 27A$

Introduction

The analysis of systems of linear equations is a cornerstone of algebra, mathematics, and applied sciences. These systems encapsulate numerous real-world phenomena—from engineering and physics to economics and computer science—where multiple variables interact within defined constraints. In this detailed review, we examine a specific system of three linear equations:

1. $6x + 2y - z = 4$
2. $x + 5y + z = 3$
3. $2x + y + 4z = 27A$

This system presents an intriguing case due to the parameter A in the third equation, which introduces an additional layer of complexity and variability. Our goal is to understand whether the system is consistent, determine solutions where they exist, and explore how the parameter influences the solution set.

Background: Fundamentals of Linear Systems

Definition and Representation

A system of linear equations involves multiple equations with several variables, typically expressed in matrix form as:

$AX = B$

where:

- A is the coefficient matrix,
- X is the column vector of variables,
- B is the constants vector.

This algebraic framework enables the use of matrix operations—such as Gaussian elimination, Cramer's rule, and matrix inversion—to analyze the system's properties.

Types of Solutions

Linear systems can be classified as:

- Consistent systems: Have at least one solution.
- Inconsistent systems: Have no solutions.
- Dependent systems: Have infinitely many solutions, often due to equations being multiples or linear combinations of others.

The solution nature hinges on the rank of matrix A and augmented matrix $[A \mid B]$.

Dissection of the System

The system under scrutiny is:

1. $6x + 2y - z = 4$
2. $x + 5y + z = 3$
3. $2x + y + 4z = 27A$

Representation in Matrix Form

Expressed in matrix form:

\[
\begin{bmatrix}
6 & 2 & -1 \\
1 & 5 & 1 \\
2 & 1 & 4 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix}
=
\begin{bmatrix}
4 \\
3 \\
27A \\
\end{bmatrix}
\]

The coefficient matrix A:

\[
A = \begin{bmatrix}
6 & 2 & -1 \\
1 & 5 & 1 \\
2 & 1 & 4 \\
\end{bmatrix}
\]

The constants vector B:

\[
B = \begin{bmatrix}
4 \\
3 \\
27A \\
\end{bmatrix}
\]

Analytical Approach: Determining Solution Existence

Step 1: Calculate the Determinant of the Coefficient Matrix

The determinant det(A) indicates whether the system has a unique solution:

\[
\det(A) =
\begin{vmatrix}
6 & 2 & -1 \\
1 & 5 & 1 \\
2 & 1 & 4 \\
\end{vmatrix}
\]

Calculating:

\[
\det(A) = 6 \times (5 \times 4 - 1 \times 1) - 2 \times (1 \times 4 - 1 \times 2) + (-1) \times (1 \times 1 - 5 \times 2)
\]

Breaking down each term:

- First term: \(6 \times (20 - 1) = 6 \times 19 = 114\)
- Second term: \(-2 \times (4 - 2) = -2 \times 2 = -4\)
- Third term: \(-1 \times (1 - 10) = -1 \times (-9) = 9\)

Sum:

$$
\det(A) = 114 - 4 + 9 = 119
$$

Since det(A) ≠ 0, the system has a unique solution for any value of A.

Step 2: Implication of a Non-zero Determinant

A non-zero determinant guarantees that the system is consistent and that solutions can be explicitly computed via methods such as Gaussian elimination or Cramer's rule.

Solution Derivation

Applying Cramer's Rule

Cramer's rule states that:

$$
x = \frac{\det(A_x)}{\det(A)}, \quad y = \frac{\det(A_y)}{\det(A)}, \quad z = \frac{\det(A_z)}{\det(A)}
$$

where A_x, A_y, and A_z are matrices formed by replacing the respective columns of A with B.

Constructing Matrices

- A_x:

$$
\begin{bmatrix}
4 & 2 & -1 \\
3 & 5 & 1 \\
27A & 1 & 4 \\
\end{bmatrix}
$$

- A_y:

$$
\begin{bmatrix}
6 & 4 & -1 \\
1 & 3 & 1 \\
2 & 27A & 4 \\
\end{bmatrix}
$$

- A_z:

$$

\begin{bmatrix}
6 & 2 & 4 \\
1 & 5 & 3 \\
2 & 1 & 27A \\
\end{bmatrix}
\]

Computing the Numerators

Determinant of A_x

\[
\det(A_x) = 4 \times (5 \times 4 - 1 \times 1) - 2 \times (3 \times 4 - 1 \times 27A) + (-1) \times (3
\times 1 - 5 \times 27A)
\]

Calculations:

- First term: \(4 \times (20 - 1) = 4 \times 19 = 76\)
- Second term: \(-2 \times (12 - 27A) = -2 \times (12 - 27A) = -24 + 54A\)
- Third term: \(-1 \times (3 - 135A) = -3 + 135A\)

Sum:

\[
\det(A_x) = 76 - 24 + 54A - 3 + 135A = (76 - 24 - 3) + (54A + 135A) = 49 + 189A
\]

Determinant of A_y

\[
\det(A_y) = 6 \times (3 \times 4 - 1 \times 27A) - 4 \times (1 \times 4 - 1 \times 2) + (-1) \times (1
\times 27A - 3 \times 2)
\]

Calculations:

- First term: \(6 \times (12 - 27A) = 72 - 162A\)
- Second term: \(-4 \times (4 - 2) = -4 \times 2 = -8\)
- Third term: \(-1 \times (27A - 6) = -27A + 6\)

Sum:

\[
\det(A_y) = 72 - 162A - 8 - 27A + 6 = (72 - 8 + 6) + (-162A - 27A) = 70 - 189A
\]

Determinant of A_z

$$
\det(A_z) = 6 \times (5 \times 27A - 1 \times 1) - 2 \times (1 \times 27A - 1 \times 2) + 4 \times (1 \times 1 - 5 \times 2)
$$

Calculations:

- First term: $6 \times (135A - 1) = 810A - 6$
- Second term: $-2 \times (27A - 2) = -54A + 4$
- Third term: $4 \times (1 - 10) = 4 \times (-9) = -36$

Sum:

$$
\det(A_z) = 810A - 6 - 54A + 4 - 36 = (810A - 54A) + (-6 + 4 - 36) = 756A - 38
$$

Final Solutions

Using Cramer's rule:

$$
x = \frac{\det(A_x)}{\det(A)} = \frac{49 + 189A}{119}
$$

$$
y = \frac{\det(A_y)}{\det(A)} = \frac{70 - 189A}{119}
$$

$$
z = \frac{\det(A_z)}{\det(A)} = \frac{756A - 38}{119}
$$

These expressions provide explicit solutions in terms of the parameter A

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