

Construct A 90% Confidence Interval For The Population Mean, . Assume The Population Has A Normal Distribution.

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Understanding how to estimate a population mean accurately is essential in statistics, especially when dealing with small sample sizes or when the population standard deviation is unknown. When the population is normally distributed, constructing a confidence interval provides a range of plausible values for the true population mean. A 90% confidence interval offers a balance between precision and confidence, indicating that if we were to repeat the sampling process multiple times, approximately 90% of such intervals would contain the true population mean. This comprehensive guide will walk you through the process of constructing a 90% confidence interval for the population mean, assuming the population follows a normal distribution.

Introduction to Confidence Intervals

What Is a Confidence Interval?

A confidence interval (CI) is a statistical range, derived from sample data, that is believed to contain the true population parameter with a certain level of confidence. In this context, the parameter is the population mean (μ).

Key points:

- The CI provides an estimate of the population mean.
- The confidence level (e.g., 90%) indicates the probability that the interval contains the true mean in repeated sampling.

Why Use Confidence Intervals?

- To quantify uncertainty associated with sample estimates.
- To make informed decisions based on statistical data.
- To communicate the precision of the estimate.

Assumptions for Constructing a 90% Confidence Interval

Constructing a valid confidence interval relies on certain assumptions:

1. **Normality of the Population:** The population from which the sample is drawn should follow a normal distribution, especially important when the sample size is small ($n < 30$).
2. **Random Sampling:** The sample should be randomly selected to avoid bias.
3. **Known or Estimated Standard Deviation:** When the population standard deviation (σ) is unknown, the sample standard deviation (s) is used, and the t-distribution applies.

In this guide, we assume the population is normally distributed and that the population standard deviation is unknown, which is a common scenario.

Step-by-Step Process for Constructing a 90% Confidence Interval

1. Gather Sample Data

Obtain a random sample from the population, noting:

- The sample size (n)
- The sample mean ($\bar{x}$)
- The sample standard deviation (s)

Example: Suppose a sample of 25 data points yields:

- $\bar{x} = 50$
- $s = 8$

2. Determine the Appropriate Distribution

Since the population standard deviation is unknown and the population is normal, use the Student's t-distribution.

- Degrees of freedom (df) = $n - 1$
- For our example: $df = 25 - 1 = 24$

3. Find the Critical t-Value

Identify the t-value corresponding to the desired confidence level (90%) and degrees of freedom.

- Use a t-distribution table or statistical software.
- For 90% confidence and $df = 24$, the two-tailed critical t-value (t^*) is approximately 1.711.

Note: The critical value divides the remaining 10% equally into two tails of 5% each.

4. Calculate the Standard Error (SE)

Standard Error measures the variability of the sample mean:

$$SE = \frac{s}{\sqrt{n}}$$

- For our example:

$$SE = \frac{8}{\sqrt{25}} = \frac{8}{5} = 1.6$$

5. Construct the Confidence Interval

The formula for the confidence interval is:

$$\boxed{\bar{x} \pm t^* \times SE}$$

- Plugging in the values:

$$50 \pm 1.711 \times 1.6$$

- Calculate the margin of error (ME):

$$ME = 1.711 \times 1.6 \approx 2.738$$

- Determine the interval:

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\[
\text{Lower limit} = 50 - 2.738 \approx 47.262
\]
\[
\text{Upper limit} = 50 + 2.738 \approx 52.738
\]
```

Final 90% Confidence Interval: (47.262, 52.738)

This means we are 90% confident that the true population mean lies between approximately 47.262 and 52.738.

Interpreting the Confidence Interval

Meaning of the 90% Confidence Level

- If we repeated the sampling process multiple times and constructed a confidence interval each time, about 90% of those intervals would contain the true population mean.
- The interval calculated from our sample is just one of those possible intervals.

Limitations

- The confidence interval does not guarantee the population mean is within the interval for this specific sample.
- It reflects the method's reliability over numerous samples, not certainty about a particular interval.

Factors Affecting the Width of the Confidence Interval

The precision of the estimate depends on several factors:

1. **Sample Size (n):** Larger samples reduce the standard error, resulting in narrower intervals.
2. **Sample Variability (s):** More variability increases the standard error, widening the interval.

3. **Confidence Level:** Higher confidence levels (e.g., 95%, 99%) require larger t-values, leading to wider intervals.

Tip: To achieve a more precise estimate, increase the sample size where feasible.

Practical Applications of Constructing a 90% Confidence Interval

Constructing confidence intervals is vital across various fields:

- Medical Research: Estimating average patient response times to treatments.
- Manufacturing: Determining the average lifespan of a product.
- Economics: Estimating average household income within a region.
- Education: Gauging average test scores of students.

By understanding and applying the process, professionals can make informed decisions based on data.

Common Mistakes to Avoid

- Using the Normal Distribution When Not Appropriate: For small samples with unknown population variance and non-normal populations, the t-distribution should be used.
- Misinterpreting the Confidence Level: Remember that a 90% confidence level does not mean there's a 90% probability the specific interval contains the population mean; rather, it's about the long-term reliability of the method.
- Neglecting Assumptions: Ensure the data meets the necessary assumptions before constructing the interval.

Summary of the Construction Process

To succinctly recap:

1. Gather a random sample and compute $\bar{x}$ and s .

2. Determine degrees of freedom: $df = n - 1$.
3. Find the critical t-value for 90% confidence and df.
4. Calculate the standard error: $SE = s / \sqrt{n}$.
5. Compute the margin of error: $ME = t^* \times SE$.
6. Construct the confidence interval: $(\bar{x} \pm ME)$.

Conclusion

Constructing a 90% confidence interval for the population mean when the population has a normal distribution is a fundamental skill in statistical analysis. It allows researchers and analysts to quantify the uncertainty associated with their sample estimates effectively. Remember to verify assumptions, select the correct distribution, and interpret the results appropriately. Whether for academic research, business decisions, or policy formulation, confidence intervals provide a robust framework for making informed, data-driven conclusions.

Additional Resources

- Statistical Tables: t-distribution critical values for various degrees of freedom and confidence levels.
- Software Tools: Use statistical software such as R, SPSS, or Python (SciPy) to automate calculations.
- Further Reading: "Introduction to Statistical Inference" by Robert V. Hogg and Allen Craig provides an in-depth understanding.

By mastering the construction and interpretation of confidence intervals, you improve your statistical literacy and enhance your ability to analyze data confidently and accurately.

Frequently Asked Questions

What is a 90% confidence interval for the population mean when the population is normally distributed?

A 90% confidence interval for the population mean provides a range within which we are 90% confident the true population mean lies, calculated using the sample mean, the standard deviation, and the appropriate z-value for 90% confidence.

How do you construct a 90% confidence interval for a population mean with a known population standard deviation?

When the population standard deviation is known, the confidence interval is calculated as: $\text{sample mean} \pm z(\text{population standard deviation} / \sqrt{n})$, where z is the z-value corresponding to 90% confidence (approximately 1.645).

What is the formula for constructing a 90% confidence interval when the population standard deviation is unknown?

When the population standard deviation is unknown, use the t-distribution: $\text{sample mean} \pm t(\text{sample standard deviation} / \sqrt{n})$, where t is the t-value with $n-1$ degrees of freedom for 90% confidence.

How do sample size and variability affect the width of a 90% confidence interval?

Larger sample sizes and lower variability (smaller standard deviation) result in narrower confidence intervals, indicating more precise estimates of the population mean.

What role does the normal distribution assumption play in constructing a 90% confidence interval?

The assumption of normality ensures that the sampling distribution of the sample mean is approximately normal, which validates using z or t distributions to construct the confidence interval.

Can you use the same method to construct a 90% confidence interval if the population distribution is not normal?

If the sample size is large (typically $n > 30$), the Central Limit Theorem allows for approximate normality, and the same method can be used. For small samples from non-normal populations, alternative methods or transformations may be necessary.

How do you interpret a 90% confidence interval for the population mean?

It means that if we were to take many samples and construct a confidence interval from each, approximately 90% of those intervals would contain the true population mean.

What steps are involved in constructing a 90% confidence interval for the population mean?

First, gather a sample and calculate the sample mean and standard deviation. Then, determine the appropriate z or t value for 90% confidence. Finally, compute the margin of error and add/subtract it from the sample mean to get the interval.

How do you determine whether to use the z-distribution or t-distribution when constructing the interval?

Use the z-distribution if the population standard deviation is known and the sample size is large. Use the t-distribution if the population standard deviation is unknown and the sample size is small ($n \leq 30$).

What are common pitfalls to avoid when constructing a 90% confidence interval for a population mean?

Avoid using the wrong distribution (z vs. t), ignoring the normality assumption for small samples, using a small sample size without verifying normality, and misinterpreting the confidence level as the probability that the interval contains the population mean.

Additional Resources

Construct A 90% Confidence Interval For The Population Mean is a fundamental topic in statistical inference, essential for researchers, data analysts, and students aiming to draw meaningful conclusions from sample data. When working with normally distributed populations, constructing confidence intervals provides a range within which the true population mean is likely to fall, with a specified level of confidence—in this case, 90%. This process combines sample data, statistical theory, and probability principles to estimate an unknown parameter accurately. Understanding how to construct and interpret these intervals is critical for making informed decisions in fields such as medicine, economics, engineering, and social sciences.

Understanding the Concept of Confidence Intervals

What is a Confidence Interval?

A confidence interval (CI) is a range of values, derived from sample data, that is believed to contain the true population parameter with a specified probability, known as the confidence level. For example, a 90% confidence interval suggests that if the same population is sampled repeatedly and intervals are constructed each time, approximately 90% of those intervals will contain the true mean.

Importance in Statistical Analysis

Confidence intervals serve multiple purposes:

- Estimate Precision: They provide an estimate of the population parameter along with an indication of the uncertainty.
- Decision Making: They help determine whether observed effects are statistically significant or practically meaningful.
- Comparison: They allow for comparison between groups or conditions, assessing overlaps or differences.

Assumptions in Constructing a 90% Confidence Interval for the Mean

Before constructing a confidence interval, certain assumptions must be satisfied to ensure validity:

- Normal Distribution: The population from which the sample is drawn is assumed to be normally distributed. If the sample size is large (typically $n > 30$), the Central Limit Theorem allows for some relaxation of this assumption.
- Random Sampling: The data should come from a random sample to avoid bias.
- Known or Estimated Standard Deviation: If the population standard deviation (σ) is known, the z-distribution is used. If σ is unknown, the sample standard deviation (s) is used, and the t-distribution applies.

Constructing the 90% Confidence Interval

The core process involves calculating the interval using sample data and the appropriate statistical distribution.

Step 1: Gather Sample Data

Obtain a sample of size n from the population, recording:

- Sample mean ($\bar{x}$)
- Sample standard deviation (s)

Step 2: Determine the Appropriate Distribution

- If the population standard deviation σ is known, use the standard normal (z) distribution.
- If σ is unknown and the population is normally distributed (or the sample size is large), use the Student's t -distribution with $n-1$ degrees of freedom.

Step 3: Find the Critical Value

- For a 90% confidence level, the critical value depends on the distribution:
 - z -distribution (if σ known): Find $z_{0.95}$ (since 90% confidence leaves 5% in each tail).
 $z_{0.95} \approx 1.645$
 - t -distribution (if σ unknown): Find $t_{0.95, n-1}$ based on the sample size.

Step 4: Calculate the Margin of Error (ME)

- For known σ :
$$ME = z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$
- For unknown σ :
$$ME = t_{\alpha/2, n-1} \times \frac{s}{\sqrt{n}}$$

Step 5: Construct the Confidence Interval

- The interval is given by:

$$\left[\bar{x} - ME, \quad \bar{x} + ME \right]$$

Worked Example

Suppose a researcher measures the heights of 25 adult males and finds:

- Sample mean height ($\bar{x}$) = 175 cm
- Sample standard deviation (s) = 10 cm

Construct a 90% confidence interval for the true population mean height.

Step 1: Sample size $n = 25$

Step 2: Since σ is unknown, use the t-distribution.

Step 3: Find critical t-value for 90% confidence and 24 degrees of freedom:

$$t_{0.95, 24} \approx 1.711$$

Step 4: Calculate Margin of Error:

$$ME = 1.711 \times \frac{10}{\sqrt{25}} = 1.711 \times 2 = 3.422$$

Step 5: Construct the interval:

$$(175 - 3.422, \quad 175 + 3.422) = (171.578, \quad 178.422)$$

Interpretation: We are 90% confident that the true mean height of adult males in the population lies between approximately 171.58 cm and 178.42 cm.

Features, Pros, and Cons of Constructing

Confidence Intervals

Features:

- Based on probability theory
- Accounts for sample variability
- Applicable for various confidence levels (e.g., 90%, 95%, 99%)
- Requires assumptions about the distribution

Pros:

- Provides a range of plausible values for the population mean
- Incorporates sample size and variability
- Offers a quantifiable measure of uncertainty
- Useful for hypothesis testing and decision making

Cons:

- Relies on the assumption of normality (particularly for small samples)
- Sensitive to outliers or skewed data
- The choice of confidence level affects the width of the interval (higher confidence levels produce wider intervals)
- Cannot confirm the true population parameter; it only estimates probabilistic bounds

Challenges and Considerations

- Sample Size: Small samples can lead to wider intervals and less precise estimates. When n is small, using the t-distribution is essential.
- Distributional Assumption: If the population is not normal and the sample size is small, the interval may not be accurate. Nonparametric methods might be needed.
- Confidence Level Selection: Higher confidence levels (e.g., 99%) increase the interval width, providing more certainty but less precision.
- Interpretation: A common misconception is that there is a 90% probability that the specific interval contains the true mean; technically, the probability applies to the method over many samples, not a single interval.

Extensions and Variations

- Constructing Confidence Intervals for Other Parameters: Such as

proportions, variances, or differences between means.

- One-sided Confidence Intervals: Useful for testing whether the mean exceeds or is less than a certain value.

- Bootstrap Methods: When assumptions are violated, resampling techniques can estimate confidence intervals without strict distributional assumptions.

Conclusion

Constructing a 90% confidence interval for the population mean when assuming a normal distribution is a vital skill in statistical analysis. It combines sample data, distribution theory, and probability to provide a plausible range for an unknown parameter, enabling informed decision-making. By carefully selecting the appropriate distribution (z or t), calculating the margin of error, and interpreting the interval correctly, analysts can effectively communicate the uncertainty inherent in sample estimates. While the method has limitations—particularly regarding assumptions and sample size—its broad applicability and interpretability make it an indispensable tool in statistical inference. Mastery of constructing and interpreting these intervals enhances the robustness and credibility of data-driven conclusions across diverse fields.

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