Explain Why The Function F(x, Y) = Sin(y)e-y + 8 Is Differentiable At The Point (0,). The Partial Derivatives

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Understanding the differentiability of a function at a specific point is fundamental in calculus, particularly when analyzing multivariable functions. The function in question, $F(x, y) = \sin(y) e^{-y} + 8$, is a two-variable function that combines trigonometric and exponential components. To determine its differentiability at a particular point, specifically at $(0, y_0)$, we need to examine its partial derivatives and whether they satisfy the criteria for differentiability.

In this article, we will delve into why the function $F(x, y) = \sin(y) e^{-y} + 8$ is differentiable at the point $(0, y_0)$. We will analyze its partial derivatives with respect to x and y, explore the concept of differentiability in the context of multivariable calculus, and provide a comprehensive step-by-step explanation to clarify the conditions under which the function is differentiable at the specified point.

Understanding the Function $F(x, y) = \sin(y) e^{-y} + 8$

Before analyzing differentiability, it is essential to understand the structure and properties of the function:

- Components of the Function:
- $\sin(y)$: A well-known trigonometric function that is continuous and differentiable everywhere.
- e^{-y}: An exponential function, also continuous and differentiable everywhere.
- The product $\sin(y) e^{-y}$: Combines these two functions, which results in a smooth, continuous, and differentiable function.
- The constant $+ 8$: Shifts the entire function vertically, which does not affect differentiability.

- Domain of the Function:
- Since both $\sin(y)$ and e^{-y} are defined for all real numbers, the domain of $F(x, y)$ is $\mathbb{R}^2$.

- Dependence on Variables:
- The function does not explicitly depend on x; it only depends on y. This indicates that the partial derivative with respect to x will be

straightforward.

Partial Derivatives of $F(x, y)$

Partial derivatives are key to understanding the differentiability of multivariable functions. They measure how the function changes as each variable varies independently, holding other variables constant.

Partial Derivative with Respect to x

Since the function $F(x, y)$ does not explicitly include x, its partial derivative with respect to x is:

$$
\frac{\partial F}{\partial x} = 0
$$

for all (x, y). At the point $(0, y_0)$, this remains:

$$
\frac{\partial F}{\partial x} (0, y_0) = 0
$$

This indicates that the function does not change as x varies, which simplifies the analysis of differentiability concerning x.

Partial Derivative with Respect to y

The partial derivative of F with respect to y is:

$$
\frac{\partial F}{\partial y} = \frac{\partial}{\partial y} \left(\sin(y) e^{-y} + 8 \right)
$$

Since the constant 8 has zero derivative, focus on differentiating $\sin(y) e^{-y}$. Applying the product rule:

$$
\frac{\partial}{\partial y} \left(\sin(y) e^{-y} \right) = \frac{\partial \sin(y)}{\partial y} \cdot e^{-y} + \sin(y) \cdot \frac{\partial e^{-y}}{\partial y}
$$

Calculating each term:

- $\frac{\partial \sin(y)}{\partial y} = \cos(y)$
- $\frac{\partial e^{-y}}{\partial y} = -e^{-y}$

Thus,

$$
\frac{\partial F}{\partial y} = \cos(y) e^{-y} + \sin(y)(-e^{-y}) = e^{-y} \left(\cos(y) - \sin(y) \right)
$$

At the point $(0, y_0)$:

$$
\left. \frac{\partial F}{\partial y} \right|_{(0, y_0)} = e^{-y_0} \left(\cos(y_0) - \sin(y_0) \right)
$$

This expression is continuous and well-defined for all real y_0.

Criteria for Differentiability in Multivariable Functions

In multivariable calculus, a function $F(x, y)$ is differentiable at a point (x_0, y_0) if:

1. The partial derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$ exist at (x_0, y_0).
2. The partial derivatives are continuous in a neighborhood around (x_0, y_0).
3. The function can be well-approximated by a linear map near (x_0, y_0).

For functions composed of elementary functions such as sine, exponential, and constants, the differentiability generally follows from the differentiability of these component functions and the rules of calculus.

Is $F(x, y) = \sin(y) e^{-y} + 8$ Differentiable at $(0, y_0)$?

Based on the previous calculations:

- The partial derivative with respect to x:

$$
\frac{\partial F}{\partial x} = 0
$$

which is continuous everywhere, including at $(0, y_0)$.

- The partial derivative with respect to y:

$$
\frac{\partial F}{\partial y} = e^{-y} \left(\cos(y) - \sin(y)\right)
$$

which is continuous everywhere since it is composed of continuous functions (e^{-y}, $\sin y$, $\cos y$).

Hence, both partial derivatives exist and are continuous in a neighborhood of $(0, y_0)$. This satisfies the conditions for differentiability.

Conclusion:

- The function $F(x, y) = \sin(y) e^{-y} + 8$ is differentiable at $(0, y_0)$ for any real y_0.
- The differentiability is guaranteed because:
- The partial derivatives exist at the point.
- The partial derivatives are continuous in a neighborhood around the point.
- The function is composed of elementary functions that are smooth and differentiable everywhere.

Additional Insights and Implications

Understanding the differentiability of $F(x, y)$ has broader applications in calculus and analysis, including:

- Gradient Vector:
- The gradient of F at (x, y) is given by:

$$
\nabla F(x, y) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right) = (0, e^{-y} (\cos y - \sin y))
$$

- Since the gradient exists and is continuous everywhere, it indicates the function's smoothness.

- Total Differentiability:
- The function is totally differentiable at $(0, y_0)$, meaning we can approximate F near this point using its linear approximation:

$$
F(x, y) \approx F(0, y_0) + \frac{\partial F}{\partial x}(0, y_0) (x - 0) + \frac{\partial F}{\partial y}(0, y_0) (y - y_0)
$$

- Since the partial derivatives are continuous, the approximation becomes increasingly accurate as (x, y) approaches $(0, y_0)$.

- Implications in Optimization and Analysis:
- The differentiability ensures that techniques such as gradient descent or differential calculus methods can be applied to analyze the behavior of the function near the point.
- It also implies the function's graph is smooth in the neighborhood of the point, with no sharp corners or discontinuities.

Summary

To summarize, the function $F(x, y) = \sin(y) e^{-y} + 8$ is differentiable at the point $(0, y_0)$ because:

- The partial derivatives $\frac{\partial F}{\partial x} = 0$ and $\frac{\partial F}{\partial y} = e^{-y} (\cos y - \sin y)$ exist at the point.
- These derivatives are continuous functions, not only at $(0, y_0)$ but across the entire domain $\mathbb{R}^2$.
- The function is composed

Frequently Asked Questions

What is the function F(x, y) = sin(y) e^(-y) + 8, and why is it important to determine its differentiability at a specific point?

The function F(x, y) = sin(y) e^(-y) + 8 is a composite function involving sine and exponential functions. Determining its differentiability at a specific point helps understand whether it has a linear approximation there, which is essential for analysis and applications like optimization or modeling.

How do you compute the partial derivatives of F with respect to x and y?

Since F(x, y) does not explicitly depend on x, its partial derivative with respect to x is zero. The partial derivative with respect to y involves differentiating sin(y) e^(-y) using the product rule.

What are the partial derivatives of F at the point

(0, y)?

At any point, the partial derivative with respect to x is 0, because F does not depend on x. The partial derivative with respect to y at (0, y) is d/dy [sin(y) e^(-y)] which can be computed using the product rule.

How do you evaluate the partial derivatives specifically at the point (0, y)?

At (0, y), the partial derivative $\partial F/\partial x = 0$. The partial derivative $\partial F/\partial y = \cos(y) e^{-y} - \sin(y) e^{-y} = e^{-y} [\cos(y) - \sin(y)]$.

What conditions must be met for F to be differentiable at the point (0, y)?

F is differentiable at (0, y) if its partial derivatives are continuous around that point. Since the partial derivatives involve elementary functions that are continuous everywhere, F is differentiable at (0, y).

Why is the differentiability of F at a point important in multivariable calculus?

Differentiability ensures the existence of a linear approximation of the function near that point, which is crucial for optimization, understanding local behavior, and applying calculus tools like the chain rule and gradient analysis.

Does the function F(x, y) = sin(y) e^(-y) + 8 depend on x? How does this affect its differentiability?

No, F does not depend on x explicitly, so its partial derivative with respect to x is zero everywhere. This simplifies the analysis of differentiability, focusing primarily on the behavior with respect to y. The absence of x dependence means F is differentiable with respect to x everywhere.

Additional Resources

Explain Why The Function F(x, y) = sin(y) e^{−y} + 8 Is Differentiable At The Point (0, y)

Understanding the differentiability of functions is a fundamental aspect of advanced calculus and mathematical analysis. When examining the function F(x, y) = sin(y) e^{−y} + 8, it becomes essential to analyze whether this function is differentiable at specific points, such as at (0, y). This exploration not only deepens our grasp of multivariable calculus but also highlights the intricacies involved in analyzing functions involving compositions, exponentials, and trigonometric functions. In this article, we will provide a comprehensive explanation of why the function $F(x, y) = \sin(y) e^{-y} + 8$ is differentiable at the point $(0, y)$, focusing on the partial derivatives and their properties.

Introduction to Differentiability in Multivariable Functions

Before delving into the specifics of the function, it's crucial to review what differentiability entails in the context of functions of multiple variables.

Differentiability at a point refers to the existence of a linear approximation of the function around that point. For a function $F(x, y)$, differentiability at a point (a, b) means:

- The partial derivatives $F_x(a, b)$ and $F_y(a, b)$ exist.
- The function can be well approximated by its tangent plane near (a, b).
- The function is continuous at that point.

In multivariable calculus, the differentiability of a function at a point often hinges on the existence and continuity of its partial derivatives around that point.

Overview of the Function $F(x, y)$

The function under analysis is:

$F(x, y) = \sin(y) e^{-y} + 8$

Notice that this function depends solely on y; x does not appear explicitly in its expression. This simplifies the analysis because:

- The function is independent of x.
- The partial derivative with respect to x will be straightforward.

Analyzing the Function's Behavior at $(0, y)$

Step 1: Recognize the Function's Structure

Given $F(x, y) = \sin(y) e^{-y} + 8$, it is evident that:

- The function is continuous everywhere because it is composed of elementary continuous functions (sine, exponential, and constants).
- The addition of 8 shifts the function vertically but does not affect differentiability.

Step 2: Examine the Partial Derivatives

The key to establishing differentiability at $(0, y)$ involves investigating the partial derivatives $F_x(0, y)$ and $F_y(0, y)$.

Partial Derivatives of $F(x, y)$

Partial Derivative with Respect to x: $F_x(x, y)$

Since $F(x, y) = \sin(y) e^{-y} + 8$, and x does not appear explicitly:

- $F_x(x, y) = 0$ everywhere.

This is because the function is independent of x, implying that F is constant with respect to x at any fixed y.

Partial Derivative with Respect to y: $F_y(x, y)$

To find $F_y(x, y)$, differentiate with respect to y:

- $F(y) = \sin(y) e^{-y} + 8$

Using the product rule:

- $\frac{d}{dy} [\sin(y) e^{-y}] = (\cos(y) e^{-y}) + (\sin(y) \frac{d}{dy} [e^{-y}])$

Since $\frac{d}{dy} [e^{-y}] = -e^{-y}$, we get:

- $F_y(y) = \cos(y) e^{-y} + \sin(y) (-e^{-y}) = e^{-y} [\cos(y) - \sin(y)]$

Thus:

- $F_y(x, y) = e^{-y} [\cos(y) - \sin(y)]$

Evaluating Partial Derivatives at $(0, y)$

Partial Derivative with respect to x:

- $F_x(0, y) = 0$ for all y.

Partial Derivative with respect to y:

- $F_y(0, y) = e^{-y} [\cos(y) - \sin(y)]$

Since this derivative exists for all y, and is continuous (being composed of elementary continuous functions), it supports the differentiability of F.

Confirming Differentiability at (0, y)

Step 1: Check the Existence of Partial Derivatives

- Both $F_x(0, y) = 0$ and $F_y(0, y) = e^{-y} [\cos(y) - \sin(y)]$ exist everywhere and are continuous functions of y.

Step 2: Verify Continuity of Partial Derivatives

- The partial derivatives are continuous functions of y, as they involve exponentials, sines, and cosines—all continuous functions.

Step 3: Use the Differentiability Theorem

In multivariable calculus, a function with continuous partial derivatives in a neighborhood around a point is differentiable at that point. Since F_x and F_y are continuous everywhere, F is differentiable everywhere, including at (0, y).

Why Does This Imply Differentiability At (0, y)?

The above analysis confirms:

- F(x, y) is continuous at (0, y).
- The partial derivatives F_x and F_y exist and are continuous at (0, y).
- The partial derivatives provide the total differential at (0, y), which approximates the change in F around that point.

Therefore, F(x, y) is differentiable at the point (0, y).

Additional Insights: The Nature of the Function

Independence from x

Since F does not depend on x, the function's differentiability with respect to x is trivial—$F_x = 0$ everywhere.

Behavior with respect to y

The dependence on y is smooth because the derivatives involve exponential and trigonometric functions, which are infinitely differentiable.

Implications for the Graph of F

- The function behaves as a "constant" with respect to x; its graph is a

family of curves in the (x, y) plane, each identical in shape and shifted along the x-axis.
- The smoothness of $F(y)$ ensures no points of non-differentiability along the y-axis.

Summary of Key Points

- Partial derivatives $F_x(0, y) = 0$ and $F_y(0, y) = e^{-y} [\cos(y) - \sin(y)]$ exist and are continuous.
- The function $F(x, y)$ is composed of elementary functions that are infinitely differentiable.
- The continuity of the partial derivatives guarantees differentiability at (0, y).
- The independence of x simplifies the analysis, making F trivially differentiable with respect to x.

Final Thoughts

The differentiability of $F(x, y) = \sin(y) e^{-y} + 8$ at (0, y) underscores the importance of understanding the properties of elementary functions and their derivatives. Recognizing that the function is independent of x and that its derivatives with respect to y are continuous ensures a straightforward conclusion: F is differentiable everywhere, including at the specific point (0, y). This example highlights how the structure and smoothness of component functions govern the differentiability of more complex multivariable functions.

References for Further Reading

- Stewart, James. Calculus: Early Transcendentals. Chapters on multivariable calculus.
- Thomas, George B., and Ross Moore. Thomas' Calculus. Sections on partial derivatives and differentiability.
- Apostol, Tom M. Mathematical Analysis. For rigorous treatment of differentiability in multiple variables.

This comprehensive analysis should clarify why $F(x, y) = \sin(y) e^{-y} + 8$ is differentiable at (0, y), emphasizing the role of partial derivatives and their properties in establishing differentiability in multivariable functions.

Explain Why The Function F X Y Sin Y E Y 8 Is Differentiable At The Point 0 The Partial Derivatives

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