Find All Values Of X Such That (6, X, 11) And (5, X, X) Are Orthogonal. (enter Your Answers As A Comma-separated

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Introduction

Understanding the concept of orthogonality in vectors is fundamental in various fields such as mathematics, physics, and engineering. When two vectors are orthogonal, they are perpendicular to each other, which implies their dot product is zero. This property allows us to solve for unknown components in vectors when they are known to be orthogonal.

In this article, we will explore the problem of finding all values of X such that the vectors $\vec{A} = (6, X, 11)$ and $\vec{B} = (5, X, X)$ are orthogonal. Our goal is to derive the values of X that satisfy the orthogonality condition, interpret the results, and understand the implications of these solutions.

Understanding Orthogonality of Vectors

The Dot Product

The key to determining whether two vectors are orthogonal is the dot product. For two vectors $\vec{A} = (a_1, a_2, a_3)$ and $\vec{B} = (b_1, b_2, b_3)$, the dot product is calculated as:

$$
\vec{A} \cdot \vec{B} = a_1b_1 + a_2b_2 + a_3b_3
$$

When vectors are orthogonal, their dot product equals zero:

$$
\vec{A} \cdot \vec{B} = 0
$$

Applying this to our specific vectors, the orthogonality condition becomes:

$$

(6)(5) + (X)(X) + (11)(X) = 0
\]

which simplifies to a quadratic equation in terms of \(X\).

Formulating the Equation for \(X\)

Calculating the Dot Product

Let's explicitly compute the dot product:

\[
6 \times 5 + X \times X + 11 \times X = 0
\]

which simplifies to:

\[
30 + X^2 + 11X = 0
\]

Rearranged into standard quadratic form:

\[
X^2 + 11X + 30 = 0
\]

This quadratic equation forms the basis for finding the values of \(X\).

Solving the Quadratic Equation

To find the roots of the quadratic:

\[
X^2 + 11X + 30 = 0
\]

we can use the quadratic formula:

\[
X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

where \(a = 1\), \(b = 11\), and \(c = 30\).

Calculating the discriminant:

\[

$$
\Delta = b^2 - 4ac = 11^2 - 4 \times 1 \times 30 = 121 - 120 = 1
$$

Since the discriminant is positive, there are two real roots:

$$
X = \frac{-11 \pm \sqrt{1}}{2} = \frac{-11 \pm 1}{2}
$$

Calculating each:

- For the positive root:

$$
X = \frac{-11 + 1}{2} = \frac{-10}{2} = -5
$$

- For the negative root:

$$
X = \frac{-11 - 1}{2} = \frac{-12}{2} = -6
$$

Therefore, the solutions for X are -5 and -6.

Verification of Solutions

Checking $X = -5$

Substitute $X = -5$ into the vectors:

$$
\vec{A} = (6, -5, 11)
$$
$$
\vec{B} = (5, -5, -5)
$$

Compute the dot product:

$$
6 \times 5 + (-5) \times (-5) + 11 \times (-5) = 30 + 25 - 55 = 0
$$

Since the dot product is zero, the vectors are orthogonal for $X = -5$.

Checking $X = -6$

Substitute $X = -6$:

$$
\vec{A} = (6, -6, 11)
$$
$$
\vec{B} = (5, -6, -6)
$$

Compute the dot product:

$$
6 \times 5 + (-6) \times (-6) + 11 \times (-6) = 30 + 36 - 66 = 0
$$

Again, the dot product is zero, confirming the orthogonality for $X = -6$.

Conclusion and Final Answer

In conclusion, the values of X that satisfy the condition for the vectors $(6, X, 11)$ and $(5, X, X)$ to be orthogonal are -5 and -6.

The final answer, entered as a comma-separated list, is:

-5, -6

Additional Insights and Applications

Significance of the Results

Discovering the specific values of X for which vectors are orthogonal has practical applications in computer graphics, physics, and signal processing. For example:

- In physics, orthogonal vectors can represent independent directions or states.
- In computer graphics, they can help in defining coordinate systems and transformations.
- In data analysis, orthogonal vectors can relate to uncorrelated features or principal components.

Extensions and Related Topics

Beyond the scope of this problem, similar techniques are used to:

- Determine angles between vectors using the dot product.
- Find the projection of one vector onto another.
- Solve systems involving multiple vectors and orthogonality constraints.
- Explore vector spaces, basis vectors, and orthogonal complements.

Summary

- Two vectors are orthogonal if their dot product equals zero.
- Setting up the dot product equation yields a quadratic in \(X\).
- Solving the quadratic gives the possible values of \(X\).
- Verification confirms the correctness of the solutions.
- The solutions to the given problem are \(-5\) and \(-6\).

In essence, understanding the properties and calculations related to vector orthogonality enables precise problem-solving and enhances comprehension of multidimensional spaces.

Frequently Asked Questions

How do I determine the value of X such that the vectors (6, X, 11) and (5, X, X) are orthogonal?

To find X, set the dot product of the two vectors to zero: (6)(5) + (X)(X) + (11)(X) = 0, then solve for X.

What is the formula for checking if two vectors are orthogonal?

Two vectors are orthogonal if their dot product equals zero. For vectors (a1, b1, c1) and (a2, b2, c2), the dot product is a1a2 + b1b2 + c1c2.

How do I set up the equation to find X from the vectors (6, X, 11) and (5, X, X)?

Calculate their dot product: 65 + XX + 11X = 0, which simplifies to 30 + X^2 + 11X = 0.

What is the quadratic equation I get when solving for X in this problem?

The quadratic equation is X^2 + 11X + 30 = 0.

How do I solve the quadratic equation $X^2 + 11X + 30 = 0$?

Factor or use the quadratic formula: $X = [-b \pm \sqrt{b^2 - 4ac}] / 2a$. Here, a=1, b=11, c=30.

What are the solutions for X when solving $X^2 + 11X + 30 = 0$?

The solutions are X = -5 and X = -6.

What are the final values of X for which the vectors are orthogonal?

The values are -5 and -6.

Additional Resources

Orthogonality in Vectors: Unlocking the Secrets of Dot Products and Their Applications

When exploring the fascinating world of vectors, one of the fundamental concepts that often come into play is orthogonality. In simple terms, two vectors are orthogonal if they are perpendicular to each other in space, which geometrically means they form a right angle (90 degrees). This property isn't just a mathematical curiosity; it has profound implications in various fields such as physics, engineering, computer graphics, and data science.

Today, we delve into a specific problem involving two vectors: (6, X, 11) and (5, X, X). The question is: Find all values of X such that these two vectors are orthogonal. This problem offers an excellent opportunity to explore the underlying principles of vector dot products, the conditions for orthogonality, and the algebraic techniques used to solve such problems.

Let's embark on this mathematical journey step by step, examining the problem's components, the relevant theories, and the solutions in detail.

Understanding the Core Concepts: Vectors and Orthogonality

What Are Vectors?

A vector is a mathematical object that has both magnitude and direction. In three-dimensional space, vectors are often represented as ordered triples:

$$
\mathbf{A} = (A_x, A_y, A_z)
$$

where A_x, A_y, and A_z are the components along the x, y, and z axes respectively.

In our case, the vectors are:

$$
\mathbf{A} = (6, X, 11)
$$
$$
\mathbf{B} = (5, X, X)
$$

with the variable X being the unknown we need to solve for.

Dot Product and Its Significance

The dot product (also known as the scalar product) between two vectors $\mathbf{A}$ and $\mathbf{B}$ is defined as:

$$
\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z
$$

This operation results in a scalar (a single number) and has the following properties:

- Commutative: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
- Distributive over addition: $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$
- Scalar multiplication: $(k\mathbf{A}) \cdot \mathbf{B} = k(\mathbf{A} \cdot \mathbf{B})$

The significance of the dot product in the context of orthogonality is that two vectors are orthogonal if and only if their dot product is zero:

$$
\mathbf{A} \cdot \mathbf{B} = 0
$$

This criterion allows us to set up an algebraic equation to find the unknowns—in this case, X.

Formulating the Orthogonality Condition

Constructing the Dot Product Equation

Given the vectors:

$$
\mathbf{A} = (6, X, 11)
$$
$$
\mathbf{B} = (5, X, X)
$$

The dot product is:

$$
(6)(5) + (X)(X) + (11)(X) = 0
$$

Expanding and simplifying:

$$
30 + X^2 + 11X = 0
$$

This is a quadratic equation in terms of X:

$$
X^2 + 11X + 30 = 0
$$

Now, our goal is to solve this quadratic for all real values of X that satisfy it.

Solving the Quadratic Equation

Standard Form and Discriminant

The quadratic equation:

$$
X^2 + 11X + 30 = 0
$$

is in standard form $aX^2 + bX + c = 0$, where:

- $a = 1$
- $b = 11$
- $c = 30$

The solutions depend on the discriminant:

$$
D = b^2 - 4ac
$$

Calculating the discriminant:

$$
D = (11)^2 - 4 \times 1 \times 30 = 121 - 120 = 1
$$

Since $D > 0$, there are two distinct real solutions, which can be found using the quadratic formula:

$$
X = \frac{-b \pm \sqrt{D}}{2a} = \frac{-11 \pm \sqrt{1}}{2}
$$

Simplify:

$$
X = \frac{-11 \pm 1}{2}
$$

Thus, the two solutions are:

1. $\displaystyle X = \frac{-11 + 1}{2} = \frac{-10}{2} = -5$

2. $\displaystyle X = \frac{-11 - 1}{2} = \frac{-12}{2} = -6$

Final Answer: All Values of X

The values of \(X\) that make the vectors (6, X, 11) and (5, X, X) orthogonal are:

-6, -5

Expressed as a comma-separated list:

-6, -5

Implications and Practical Applications

Understanding and solving for orthogonality conditions like this one is more than an academic exercise; it has tangible applications across numerous disciplines:

- In Physics: Ensuring vectors representing forces or velocities are perpendicular can be crucial in mechanics and electromagnetism.
- In Computer Graphics: Orthogonal vectors define coordinate systems and projection planes, essential for rendering scenes correctly.
- In Data Science: Orthogonality in feature vectors can reduce multicollinearity, improving model performance.
- In Engineering: Designing components or systems where components need to be perpendicular for safety or functional reasons.

By mastering how to manipulate vector equations and interpret solutions, professionals can optimize designs, improve systems, and deepen their understanding of multidimensional spaces.

Conclusion

The problem of finding all values of \(X\) such that the vectors (6, X, 11) and (5, X, X) are orthogonal elegantly illustrates the power of the dot product in vector algebra. Through straightforward algebraic steps—constructing the dot product equation, solving the quadratic, and interpreting the solutions—we determine that the vector components satisfy the conditions when \(X\) is -6 or -5.

This exercise showcases how foundational mathematical principles underpin complex real-world applications, emphasizing the importance of a solid grasp

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of vector operations. Whether you're an engineer, scientist, or
mathematician, understanding these core techniques enables you to tackle a
broad spectrum of problems with confidence and precision.

Final answer: -6, -5
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