Find The Area Of The Region Bounded By The Graph Of F And The X-axis On The Given Interval. F(x)=x^2-25;[-2,3]

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Understanding how to determine the area of a region bounded by a function and the x-axis is a fundamental concept in calculus. Specifically, when dealing with functions like $F(x) = x^2 - 25$ over a specified interval, it is essential to analyze where the graph lies above or below the x-axis and adjust the calculation accordingly. In this article, we will explore the step-by-step process of finding the area of the region bounded by $F(x) = x^2 - 25$ over the interval $[-2, 3]$, providing clear explanations, methods, and tips for accurate computation.

Understanding the Function and Its Graph

1. The Nature of $F(x) = x^2 - 25$

The given function $F(x) = x^2 - 25$ is a quadratic function, which graphically represents a parabola opening upwards. The key features of this function include:

- Vertex: Since the quadratic is in the form $ax^2 + bx + c$, with $a=1$, $b=0$, and $c=-25$, the vertex occurs at $x=0$, with the value $F(0) = -25$.

- Axis of Symmetry: The parabola is symmetric about the vertical line $x=0$.

- Intercepts:
- x-intercepts: Solve $x^2 - 25 = 0$, yielding $x^2=25$, so $x= \pm 5$. Therefore, the parabola crosses the x-axis at $x= -5$ and $x= 5$.

- Range: Since the parabola opens upward and the vertex is at $(0, -25)$, the minimum point on the graph is at $y= -25$, and it extends upward infinitely.

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2. Graph Behavior on the Interval $[-2, 3]$

Within the interval $[-2, 3]$:

- The function values are:
- $F(-2) = (-2)^2 - 25 = 4 - 25 = -21$
- $F(3) = 9 - 25 = -16$

- The function remains negative throughout this interval because:
- The x-values are within the range where the parabola is below the x-axis, between -5 and 5.

- Since the parabola crosses the x-axis at $x= -5$ and $x= 5$, and our interval $[-2, 3]$ is entirely between these roots, the entire graph of $F(x)$ over $[-2, 3]$ lies below the x-axis.

Calculating the Area of the Bounded Region

1. The Concept of Absolute Area

When calculating the area between a curve and the x-axis, the integral of the function over the interval gives the net area. However, because parts of the graph can lie below the x-axis, the integral might be negative. To find the total area (the absolute measure of the region), we need to consider the absolute value of the function where it is negative.

In this case, since $F(x)$ is negative throughout $[-2, 3]$, the total area is:

$$
\text{Area} = \int_{-2}^{3} |F(x)| dx
$$

But because $F(x) \leq 0$ in this interval, $|F(x)| = -F(x)$. Therefore,

$$
\text{Area} = \int_{-2}^{3} -F(x) dx
$$

---

2. Setting Up the Integral

The integral for the total area is:

\[

\text{Area} = \int_{-2}^{3} - (x^2 - 25) dx

\]

Simplify the integrand:

\[

\text{Area} = \int_{-2}^{3} (-x^2 + 25) dx

\]

Performing the Integration

1. Computing the Integral

The integral becomes:

\[

\int_{-2}^{3} (-x^2 + 25) dx = \int_{-2}^{3} -x^2 dx + \int_{-2}^{3} 25 dx

\]

Calculate each part separately:

- For the first part:

\[

\int -x^2 dx = - \frac{x^3}{3}

\]

- For the second part:

$$
\int 25 dx = 25x
$$

Putting it together, the total is:

$$
\left[- \frac{x^3}{3} + 25x \right]_{-2}^{3}
$$

---

2. Evaluating the Definite Integral

Calculate at the upper limit $x=3$:

$$
- \frac{(3)^3}{3} + 25 \times 3 = - \frac{27}{3} + 75 = -9 + 75 = 66
$$

Calculate at the lower limit $x=-2$:

$$
- \frac{(-2)^3}{3} + 25 \times (-2) = - \frac{-8}{3} - 50 = \frac{8}{3} - 50
$$

Expressed with common denominator:

$$
\frac{8}{3} - \frac{150}{3} = - \frac{142}{3}
$$

Now, subtract the lower limit evaluation from the upper limit:

$$
66 - \left(- \frac{142}{3} \right) = 66 + \frac{142}{3}
$$

Express 66 as a fraction with denominator 3:

$$
66 = \frac{198}{3}
$$

Adding:

$$
\frac{198}{3} + \frac{142}{3} = \frac{340}{3}
$$

Therefore, the total area is:

$$
\boxed{\frac{340}{3}} \text{ square units}
$$

Final Thoughts and Additional Tips

1. Confirming the Sign of the Function

Before integrating, always verify whether the function is positive or negative over the interval. In this case, the entire interval lies below the x-axis, simplifying the absolute value operation.

2. Using Graphs for Visualization

Plotting the function helps visualize the region and confirms the bounds of integration. Graphs can also reveal if the function crosses the x-axis within the interval, which would necessitate splitting the integral.

3. Application of Integral Calculus

The process illustrated demonstrates how integral calculus effectively computes the area of complex regions, especially when the function crosses the x-axis. Remember to account for the absolute value when calculating total shaded areas.

Conclusion

Determining the area of a region bounded by a function and the x-axis involves understanding the behavior of the function, setting up the correct integral, and carefully evaluating it. For the specific case of $F(x) = x^2 - 25$ over $[-2, 3]$, the entire segment lies below the x-axis, and thus, the total bounded area is given by:

$$
\boxed{\frac{340}{3} \text{ square units}}
$$

This method can be extended to other functions and intervals, emphasizing the importance of analyzing the graph, adjusting for negative values, and applying definite integrals accurately. Mastery of these concepts enables precise calculation of areas in various calculus applications, from physics to engineering.

Frequently Asked Questions

What is the first step to find the area of the region bounded by the graph of f(x) = x^2 - 25 and the x-axis on the interval [-2, 3]?

The first step is to identify where the graph intersects the x-axis by solving f(x) = 0, which helps determine the limits of integration and regions where the function is above or below the x-axis.

How do you handle the absolute value when calculating the area bounded by the curve f(x) = x^2 - 25 and the x-axis on [-2, 3]?

Since parts of the graph lie below the x-axis, you take the absolute value of f(x) when integrating to ensure the area is positive, effectively splitting the integral at points where the function crosses the x-axis.

What are the x-values where the graph of f(x) = x^2 - 25 intersects

the x-axis within the interval [-2, 3]?

The function intersects the x-axis at $x = -5$ and $x = 5$, but only $x = -2$ and $x = 3$ are within the interval [-2, 3], so the relevant crossing points inside the interval are at $x = -2$ and $x = 3$.

How do you set up the definite integral to find the area bounded by $f(x) = x^2 - 25$ and the x-axis on [-2, 3]?

You split the integral at the points where the function crosses the x-axis, then take the absolute value of the integrand where the function is negative, resulting in the sum of integrals of $|f(x)|$ over the subintervals within [-2, 3].

What is the final calculated area of the region bounded by $f(x) = x^2 - 25$ and the x-axis on [-2, 3]?

The total area is 25 square units, obtained by integrating the absolute value of $f(x)$ over the interval [-2, 3], considering the regions where the function is below the x-axis.

Additional Resources

Find The Area Of The Region Bounded By The Graph Of F And The X-axis On The Given Interval is a fundamental problem in integral calculus, involving the calculation of the total area enclosed between a curve and the x-axis over a specified interval. In this case, we are given the function $f(x) = x^2 - 25$ over the interval $[-2, 3]$. This task not only requires understanding the geometric interpretation of definite integrals but also involves analyzing the behavior of the function within the interval to accurately determine the bounded area.

This comprehensive review will explore the problem step-by-step, covering the key concepts of the function's behavior, the process of finding the x-intercepts, setting up the integral, and computing the total enclosed area. Additionally, the article will discuss common pitfalls, techniques for dealing with functions crossing the x-axis, and the significance of understanding such problems in broader mathematical contexts.

Understanding the Function $f(x) = x^2 - 25$

Before calculating the area, it's crucial to analyze the properties of the given function. The quadratic nature of $f(x) = x^2 - 25$ plays a significant role in understanding where the function is positive or negative within the interval $[-2, 3]$.

Graphical Behavior of $f(x)$

- Shape and Symmetry: Since $f(x) = x^2 - 25$ is a parabola opening upwards with its vertex at the origin, shifted downward by 25 units, the parabola's lowest point (vertex) is at $(0, -25)$.
- Intercepts:
- Y-intercept: When $x=0$, $f(0) = -25$.
- X-intercepts: Found by solving $x^2 - 25 = 0$:

$$
x^2 = 25 \Rightarrow x = \pm 5
$$

These are the points where the graph crosses the x-axis, at $(-5, 0)$ and $(5, 0)$.

Behavior Within $[-2, 3]$

- Since the interval $[-2, 3]$ is within the range of x-values between -5 and 5, the graph will:
- Be below the x-axis where $f(x) < 0$.
- Be above the x-axis where $f(x) > 0$.

Evaluating $f(x)$ at key points:

- $f(-2) = (-2)^2 - 25 = 4 - 25 = -21$ (below x-axis)
- $f(3) = 9 - 25 = -16$ (below x-axis)
- $f(0) = -25$ (below x-axis)

Thus, on the entire interval $[-2, 3]$, the function remains below the x-axis. This simplifies the problem to calculating the area between the curve $f(x)$ and the x-axis, which, since $f(x)$ is negative throughout, will be the integral of $-f(x)$ over $[-2, 3]$.

Identifying the Regions and x-Intercepts

Even though the interval $[-2, 3]$ lies entirely below the x-axis, understanding the points where the function crosses the x-axis is essential for accurate area calculation, especially in more complex scenarios.

Finding the X-Intercepts

- As previously determined, the roots of $f(x) = x^2 - 25$ are at:
$$
x = \pm 5
$$
- Since $-2 > -5$, the entire interval $[-2, 3]$ is to the right of the left root at -5 and to the left of the right root at 5.
- Therefore, within $[-2, 3]$, the function is negative (below the x-axis), and the area calculation involves integrating the negative of the function to obtain the positive area measure.

Setting Up the Integral for the Area Calculation

The goal is to compute the total area bounded between $f(x)$ and the x-axis over $[-2, 3]$. Since $f(x)$ remains negative within this interval, the area A is:

$$
A = \int_{-2}^{3} |f(x)| \, dx = \int_{-2}^{3} -f(x) \, dx
$$

which simplifies to:

$$
A = \int_{-2}^{3} -(x^2 - 25) \, dx = \int_{-2}^{3} (25 - x^2) \, dx
$$

This integral captures the total area because $25 - x^2$ is positive over $[-2, 3]$, given the function's values.

Calculating the Integral

Step-by-Step Integration

- Write the integral:

$$
A = \int_{-2}^{3} (25 - x^2) \, dx
$$

- Split the integral:

$$
A = \int_{-2}^{3} 25 \, dx - \int_{-2}^{3} x^2 \, dx
$$

- Calculate each integral separately:

Integral of 25:

$$
\int 25 \, dx = 25x
$$

Evaluated from -2 to 3:

$$
25(3) - 25(-2) = 75 + 50 = 125
$$

Integral of x^2:

$$
\int x^2 \, dx = \frac{x^3}{3}
$$

Evaluated from -2 to 3:

$$
\frac{3^3}{3} - \frac{(-2)^3}{3} = \frac{27}{3} - \frac{-8}{3} = 9 + \frac{8}{3} = \frac{27}{3} + \frac{8}{3} = \frac{35}{3}
$$

Putting it all together:

$$
A = 125 - \frac{35}{3} = \frac{375}{3} - \frac{35}{3} = \frac{340}{3}
$$

Final Area:

$$

$$
A = \frac{340}{3} \text{ square units}
$$

Interpretation of the Result

The computed area, $\frac{340}{3}$, is approximately 113.33 square units. This represents the total region enclosed between the graph of $f(x) = x^2 - 25$ and the x-axis over the interval $[-2, 3]$. Since the function remains negative throughout, the area calculation is straightforward—simply integrating the negative of the function over the interval.

Key Takeaways and Features

- Understanding the Graph: Recognizing the parabola's shape and intercepts is crucial for accurate area calculations.
- Sign of the Function: Determining whether the function is above or below the x-axis helps decide whether to integrate $f(x)$ or $-f(x)$.
- Integral Setup: Properly setting up the definite integral ensures an accurate measurement of the bounded area.
- No Subintervals Needed: Since the function does not cross the x-axis within $[-2, 3]$, the calculation is straightforward. If it had, we'd split the integral at the crossing points.

Pros and Cons of the Approach

Pros:

- Simplicity: The integral setup is straightforward because the function does not change sign within the interval.
- Clarity: Recognizing the behavior of $f(x)$ simplifies the process, avoiding unnecessary complications.
- Educational Value: Demonstrates how to handle functions entirely below or above the x-axis in area calculations.

Cons:

- Limited to Sign Behavior: If the function crosses the x-axis within the interval, the method would require splitting the integral, increasing complexity.
- Potential for Sign Errors: Failing to consider the sign of $f(x)$ could lead to incorrect area measurements, especially if the function crosses the axis.

Conclusion

Calculating the area bounded by $f(x) = x^2 - 25$ and the x-axis over $[-2, 3]$ exemplifies the importance of understanding the function's behavior. Since the entire interval lies below the x-axis, the task simplifies to integrating $-f(x)$ over the interval, leading to an area of $\frac{340}{3}$. This exercise highlights the significance of analyzing the graph beforehand, recognizing where the function crosses the x-axis, and carefully setting up integrals accordingly. Such problems are foundational in calculus, illustrating the connection between geometric intuition and algebraic computation, and reinforcing the importance of sign analysis in integral calculus.

In summary:

- The total bounded area is \(\boxed{\frac{340}{3}}\) square units.

- The process involved analyzing the function

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